

1 Flanders Mathematics Olympiad 1999-2000: First Round.

The first round consists of 30 multiple choice problems which are selected by the FMO jury. Scores are computed as follows: a correct answer yields 5 points, a blank answer 1 point and no points are given for a wrong answer. Participants may spend 3 hours solving the problems.

1.1 The problems

1. If $\frac{1}{x} - \frac{1}{b} = \frac{1}{a}$, then x equals

(A) $a + b$	(B) $\frac{b-a}{ab}$	(C) $\frac{1}{a} + \frac{1}{b}$	(D) $\frac{ab}{a+b}$	(E) $a - b$
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2. A regular n -gon has an equal number of sides and diagonals. In this case n equals

(A) 4	(B) 5	(C) 6
(D) 8	(E) n can have different values	

3. There are 10 red, 10 blue and 10 green cards, each one numbered from 1 to 10, in a box. One removes every red card that is a multiple of 3, every even blue card and every green card that is a prime. What is the percentage of the cards remaining in the box?

(A) 0,6%	(B) 6%	(C) 18%	(D) 30%	(E) 60%
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4. Filip is rather clumsy in using his calculator. Instead of multiplying a positive number by 3, he divides it by 3 and instead of taking the square root of this result, he squares it. His (incorrect) result is 16. What is the right answer?

(A) 6	(B) 12	(C) 16	(D) 18	(E) 36
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5. For $x \in \mathbb{R}$, the smallest value of $\left|x + \frac{1}{x}\right|$ equals

(A) $\frac{1}{\sqrt{2}}$	(B) $\sqrt{2}$	(C) 2	(D) $\frac{3}{2}$	(E) $2\sqrt{2}$
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6. A magic square of order 3 with sum 0 is a 3×3 matrix consisting of real numbers so that the sum of each row, of each column and of the two diagonals is 0.

Given:

$$\begin{bmatrix} 1 & -\frac{1}{2} & \cdot \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \end{bmatrix}$$

In how many ways can this matrix be completed in order to become a magic square?

(A) 0	(B) 1	(C) 2
(D) 3	(E) infinitely many	

7. How many strictly positive integers cannot be written as a sum with only terms 5, 7 or 11? (Note that sums consisting of one term are allowed.)

(A) 6	(B) 8	(C) 10	(D) 12	(E) at least 14
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8. The distance between two villages is 20 cm on a topographic map with scale 1 : 10.000. What is the distance between those villages on a map with scale 1 : 25.000?

(A) 2 cm	(B) 4 cm	(C) 8 cm	(D) 10 cm	(E) 50 cm
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9. A student is either healthy or ill. Assume that if a student is healthy today, the probability of him (her) being healthy tomorrow is 95 % and, if a student is ill today, the probability of him (her) still being ill tomorrow is 55 %. If 20 % of the students are ill today, then what is the percentage of ill students we can expect tomorrow?

(A) 11%	(B) 15%	(C) 50%	(D) 55%	(E) 60%
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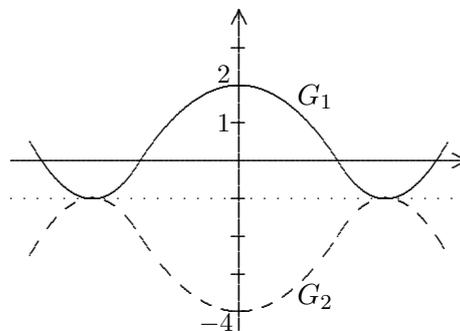
10. The number of solutions in \mathbb{R} of the equation

$$|1 - x^2| = 1 - x$$

equals

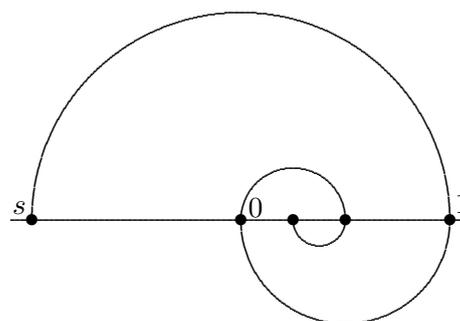
(A) 0	(B) 1	(C) 2	(D) 3	(E) 4
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11. If G_1 is the graph of $y = f(x)$, then G_2 is the graph of



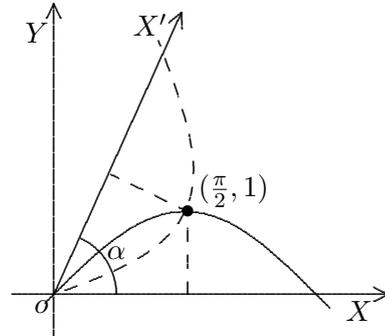
(A) $y = -f(x)$	(B) $y = f(-x)$	(C) $y = f(x) - 6$
(D) $y = -f(x) - 1$	(E) $y = -f(x) - 2$	

12. Start with the point s and construct a halfcircle of radius 1. Continue this curve with a halfcircle of radius $\frac{1}{2}$ as shown in the figure. Keep on doing this, making sure that the radius of each halfcircle is half the radius of the previous halfcircle. Determine the distance between s and the “endpoint”.



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|-------------------|-------------------|---------------------|----------------|---------------------|
| (A) $\frac{4}{3}$ | (B) $\frac{3}{2}$ | (C) $\frac{\pi}{3}$ | (D) $\sqrt{2}$ | (E) $\frac{\pi}{2}$ |
|-------------------|-------------------|---------------------|----------------|---------------------|

13. Reflect the X -axis of an orthonormal coordinate system and the graph of $y = \sin x$ with respect to the line through the origin and the point $(\frac{\pi}{2}, 1)$. What can be said about the angle α between the reflected X -axis and the X -axis itself?



- | | | |
|---|--|--|
| (A) $\sin \alpha = \frac{2}{\pi}$ | (B) $\sin \alpha = \frac{2}{\sqrt{4 + \pi^2}}$ | (C) $\sin \alpha = \frac{4\pi}{4 + \pi^2}$ |
| (D) $\sin \alpha = \frac{4\sqrt{\pi^2 - 4}}{\pi^2}$ | (E) $\sin \alpha = 1$ | |

14. A teacher wants to compute the mean score m of three tests for each student. First he computes the mean score of the two lowest results and then the mean of this result and the best score. For each student, he finds a number that is

- | | |
|------------------------------|---------------------------------|
| (A) strictly less than m . | (B) strictly greater than m . |
| (C) equal to m . | (D) not less than m . |
| (E) not greater than m . | |

15. $P = (1 + \frac{1}{2})(1 + \frac{1}{3}) \dots (1 + \frac{1}{1998})(1 + \frac{1}{1999})(1 + \frac{1}{2000})$. We have that

- | | | |
|----------------|----------------|-----------------------|
| (A) $P < 1000$ | (B) $P = 1000$ | (C) $1000 < P < 2000$ |
| (D) $P = 2000$ | (E) $P > 2000$ | |

16. $\sqrt{2000^{2000}}$ is a number that ends with a great amount of zeros. Starting from the right, the first nonzero digit is

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|-------|-------|-------|-------|---------|
| (A) 2 | (B) 4 | (C) 6 | (D) 8 | (E) odd |
|-------|-------|-------|-------|---------|

17. How many of the following statements are correct in \mathbb{Z} ?

- $\forall y, \exists x : x^2 = y$
- $\exists y, \forall x : x^2 = y$
- $\forall x, \exists y : x^2 = y$
- $\exists x, \forall y : x^2 = y$

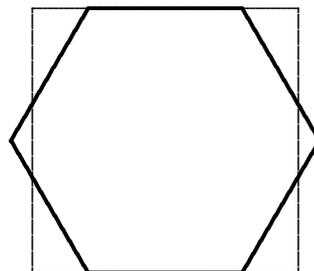
$$\exists x, \exists y : x^2 = y$$

- | | | | | |
|-------|-------|-------|-------|-------|
| (A) 0 | (B) 1 | (C) 2 | (D) 3 | (E) 4 |
|-------|-------|-------|-------|-------|

18. One of the angles of a right triangle measures 25° . The angle between the median and the altitude line, drawn from the right angle equals

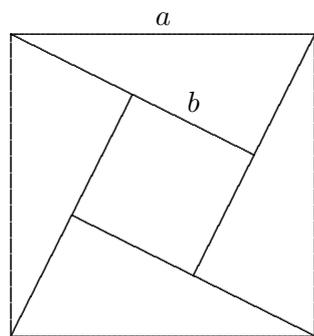
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|----------------|----------------|----------------|----------------|----------------|
| (A) 25° | (B) 30° | (C) 35° | (D) 40° | (E) 45° |
|----------------|----------------|----------------|----------------|----------------|

19. Starting with a square of side 1, a regular hexagon is constructed, concentric with the square (see figure). Determine the area of the intersection of both figures.



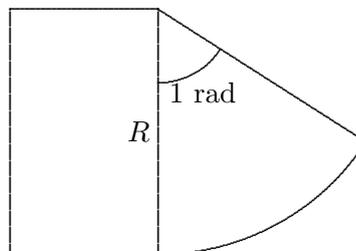
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|------------------------------|-------------------------------|------------------------------|--------------------------|--------------------------|
| (A) $\frac{2}{\sqrt{3}} - 1$ | (B) $1 - \frac{1}{2\sqrt{3}}$ | (C) $2 - \frac{2}{\sqrt{3}}$ | (D) $\frac{\sqrt{3}}{2}$ | (E) $\frac{2}{\sqrt{3}}$ |
|------------------------------|-------------------------------|------------------------------|--------------------------|--------------------------|

20. In the figure, a big square of side a is divided into a small square and four rectangular triangles. If all triangles have the same area as the small square, the side b of the small square equals



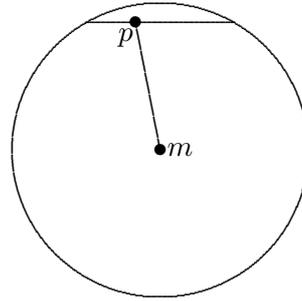
- | | | | | |
|--------------------------|-------------------|--------------------|-----------------------------|------------------------------------|
| (A) $\frac{a}{\sqrt{5}}$ | (B) $\frac{a}{2}$ | (C) $\frac{2a}{5}$ | (D) $a\frac{\sqrt{5}-1}{2}$ | (E) $a\frac{\sqrt{5}-1}{\sqrt{5}}$ |
|--------------------------|-------------------|--------------------|-----------------------------|------------------------------------|

21. A sector of a circle of radius R and angle 1 (in radians) has the same area as a rectangle of length R . What is the width of the rectangle?



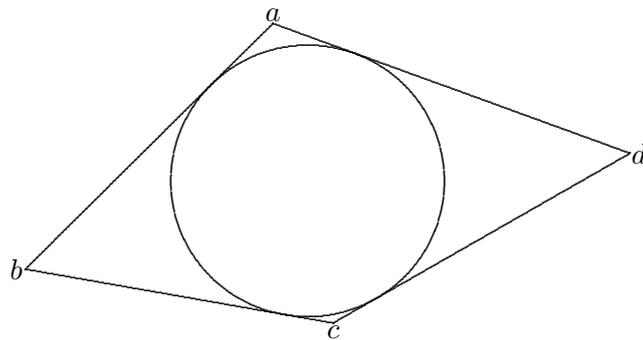
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|-----------------------|-------------------|-------------------|----------------------|---------|
| (A) $\frac{\pi R}{4}$ | (B) $\frac{R}{2}$ | (C) $\frac{R}{3}$ | (D) $\frac{2R}{\pi}$ | (E) R |
|-----------------------|-------------------|-------------------|----------------------|---------|

22. The diameter of a circle with center m is 110 cm. A point p lying on a chord divides the chord in pieces of 30 and 60 cm. Then $|pm|$ (in cm) equals



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|--------|--------|--------|--------|--------|
| (A) 28 | (B) 30 | (C) 32 | (D) 33 | (E) 35 |
|--------|--------|--------|--------|--------|

23. In the figure, the quadrangle $abcd$ is circumscribed to a circle. If $|ab| = 4$, $|bc| = 5$ and $|cd| = 3$, then $|ad|$ equals



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|-------|-------|---------|-------|----------|
| (A) 1 | (B) 2 | (C) 2,4 | (D) 3 | (E) 3,75 |
|-------|-------|---------|-------|----------|

24. 9 white and 18 black cubes with edge 1 are used to form one big cube with edge 3. The partial area of the new cube that is white, is at most

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|-------------------|---------------------|---------------------|-------------------|-------------------|
| (A) $\frac{1}{2}$ | (B) $\frac{13}{27}$ | (C) $\frac{25}{54}$ | (D) $\frac{4}{9}$ | (E) $\frac{1}{3}$ |
|-------------------|---------------------|---------------------|-------------------|-------------------|

25. p , q , r and s are points on a line. They are situated at a distance of 1 m from each other, as shown in the picture below.



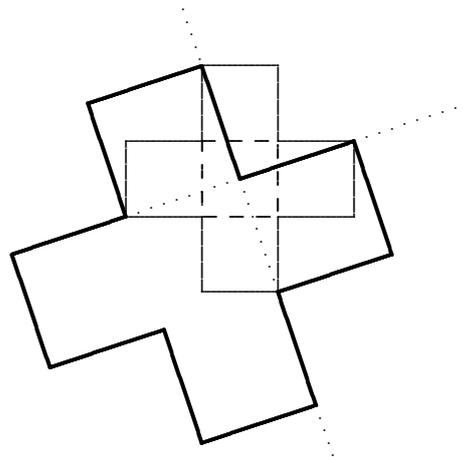
Determine the length (in meter) of the shortest way from p to s so that there's at least 1 meter between yourself and q and r at any time.

- | | | | | |
|---------------------|-------|---------------|----------------------|-------|
| (A) $1 + 2\sqrt{2}$ | (B) 4 | (C) $1 + \pi$ | (D) $\frac{4}{3}\pi$ | (E) 5 |
|---------------------|-------|---------------|----------------------|-------|

26. I buy 6 pencils, 5 coloured pencils, 8 notebooks and 12 pieces of coloured paper. A pencil costs 14 BEF, a coloured pencil 20 BEF. I forgot the other prices, but they are whole numbers. Which of the following amounts is the possible total cost?

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|-------------|-------------|-------------|-------------|-------------|
| (A) 150 BEF | (B) 200 BEF | (C) 250 BEF | (D) 300 BEF | (E) 350 BEF |
|-------------|-------------|-------------|-------------|-------------|

27. Starting with a small Greek cross, made of 5 congruent squares, a bigger one is constructed. Some of the sides lie on the orthogonal diagonals (and their extensions) of the small Greek cross (see figure). Determine the ratio of the area of the big cross versus the area of the small cross.



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|-------|---------------------------|-------------------|-------------------|-------|
| (A) 2 | (B) $\frac{\sqrt{10}}{2}$ | (C) $\frac{5}{2}$ | (D) $\frac{8}{3}$ | (E) 3 |
|-------|---------------------------|-------------------|-------------------|-------|

28. One night the king couldn't sleep and went to the royal kitchen to find a box of biscuits. He ate $\frac{1}{8}$ of the biscuits. A little later, the queen felt hungry and ate $\frac{1}{6}$ of the remaining biscuits. Later still, the princess went down and took $\frac{1}{7}$ of what was left in the box. Then came the prince who ate $\frac{1}{5}$ of the remainders. Finally his dog stole $\frac{1}{4}$ of the leftovers. Who ate most of the biscuits?

- | | | |
|----------------|---------------|------------------|
| (A) the king | (B) the queen | (C) the princess |
| (D) the prince | (E) the dog | |

29. A jogger is running laps of 400 m in a parc. He runs the first lap with a constant speed v in exactly 2 minutes. Every following lap, he increases his speed with 5% of v , without exceeding his personal limit of 1 minute and 20 seconds for one lap. How many laps, at most, will he run?

- | | | | | |
|-------|-------|-------|--------|--------|
| (A) 7 | (B) 8 | (C) 9 | (D) 10 | (E) 11 |
|-------|-------|-------|--------|--------|

30. Fill in the blanks with a digit from 1 to 9 so that

$$\square\square\% \text{ of } \square\square\square\square$$

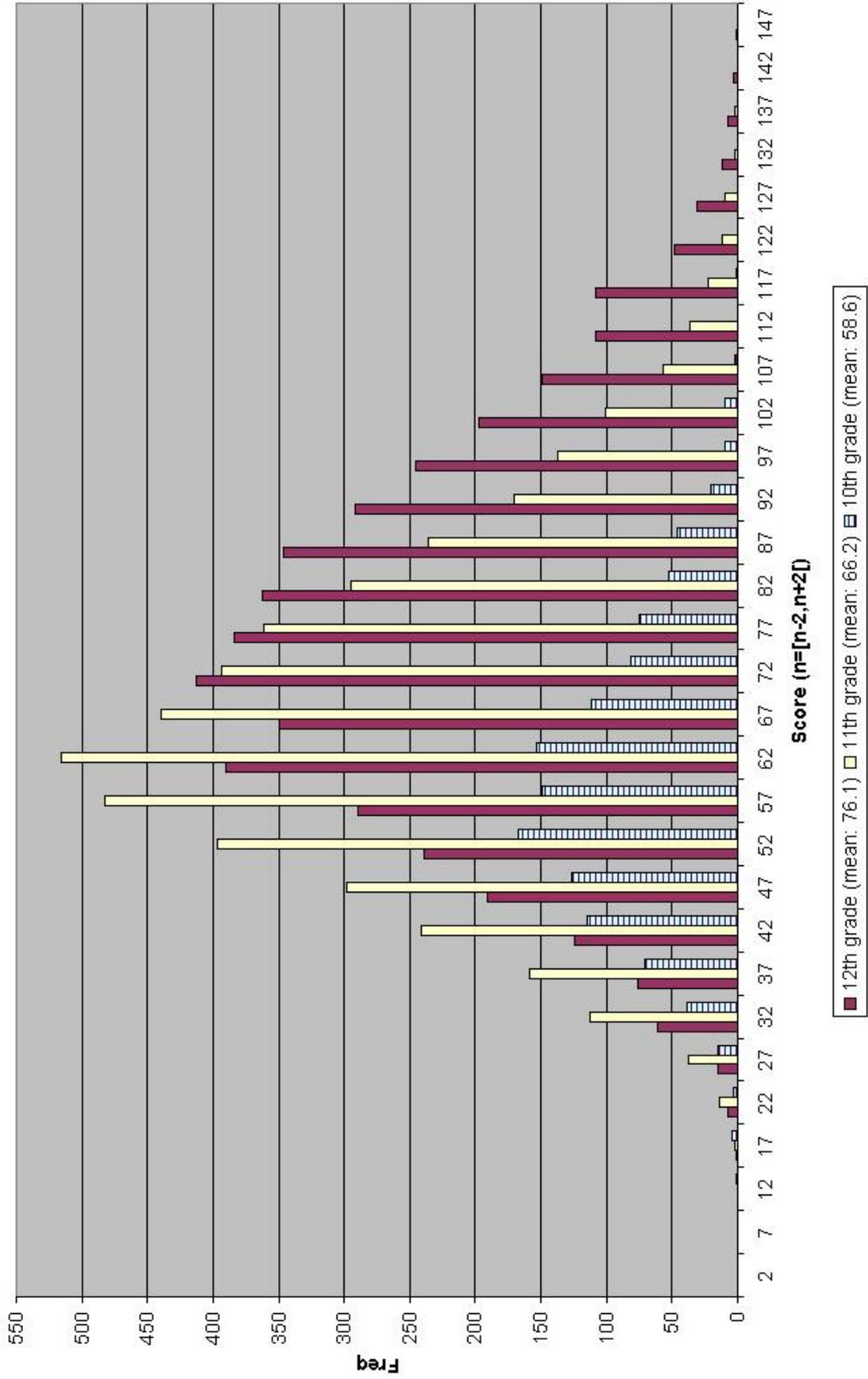
equals 2000. Then the box on the left contains

- | | | |
|-------|---------------------------|-------|
| (A) 2 | (B) 4 | (C) 6 |
| (D) 8 | (E) un undefinable digit. | |

1.2 Answer Patterns of the First Round.

Flanders Mathematics Olympiad 2000 – First Round Answer Patterns (all participants)									
Problem	Correct	A	B	C	D	E	Correct	Wrong	Blank
1	D	16.05	1.00	4.21	75.01	0.73	75.01	22.00	2.99
2	B	2.06	73.41	2.81	2.65	11.87	73.41	19.40	7.19
3	E	0.57	1.02	1.03	2.30	90.84	90.84	5.00	4.16
4	A	84.76	10.92	0.66	0.54	1.81	84.76	13.95	1.28
5	C	6.82	3.97	43.02	13.05	3.38	43.02	27.24	29.74
6	B	25.54	21.74	3.39	2.01	9.80	21.74	40.74	37.53
7	B	3.82	27.12	10.09	5.07	23.82	27.12	42.82	30.05
8	C	0.27	3.29	87.77	0.32	6.69	87.77	10.57	1.66
9	B	19.75	67.99	0.86	1.73	1.51	67.99	23.88	8.13
10	D	3.12	15.18	40.84	26.49	5.78	26.49	64.94	8.57
11	E	10.16	5.32	3.33	8.15	56.40	56.40	26.99	16.61
12	A	25.25	4.62	8.32	19.42	6.52	25.25	38.87	35.88
13	C	3.34	9.87	12.31	3.40	2.69	12.31	19.29	68.39
14	D	5.41	32.70	4.10	42.42	7.03	42.42	49.26	8.33
15	C	19.36	2.65	27.88	2.29	17.89	27.88	42.19	29.93
16	C	23.85	17.25	25.23	4.24	4.57	25.23	49.93	24.84
17	C	2.85	11.96	32.13	26.80	2.82	32.13	44.46	23.41
18	D	18.68	4.58	6.66	40.92	7.50	40.92	37.46	21.62
19	C	2.20	10.98	4.65	4.84	1.84	4.65	19.88	75.47
20	A	45.44	10.79	5.04	3.13	1.70	45.44	20.66	33.90
21	B	9.78	30.28	1.68	6.98	6.01	30.28	24.44	45.28
22	E	3.14	2.96	3.29	4.82	27.93	27.93	14.24	57.84
23	B	1.05	7.50	9.00	2.99	9.82	7.50	22.87	69.63
24	B	5.76	38.94	7.49	5.98	22.54	38.94	41.79	19.27
25	C	13.61	3.42	36.71	12.89	8.64	36.71	38.57	24.72
26	D	0.18	1.74	5.63	63.17	15.41	63.17	23.01	13.82
27	C	3.68	17.11	31.89	5.14	2.56	31.89	28.51	39.60
28	B	5.34	62.96	2.02	4.40	14.10	62.96	25.89	11.15
29	E	17.04	12.94	13.55	8.52	11.33	11.33	52.07	36.59
30	C	6.73	6.23	4.03	2.76	35.07	4.03	50.80	45.17

Flanders Mathematics Olympiad - First round 2000

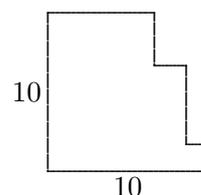


2 Flanders Mathematics Olympiad 1999-2000: Second Round.

Scores are computed in the same way as for the first round. Participants may spend 2 hours solving the problems.

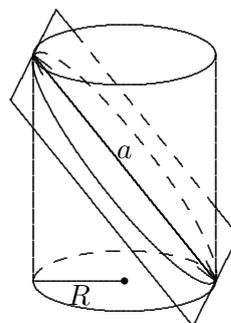
2.1 The problems

1. The perimeter of the given figure is



- | | | |
|------------------|-----------------|-----------------------|
| (A) less than 40 | (B) 40 | (C) between 40 and 80 |
| (D) 85 | (E) undefinable | |

2. A cylinder of radius R is divided by a plane, as shown in the figure. The plane is tangent to both circles at the base and at the top of the cylinder. The distance between the tangent points is a . Determine the volume of the part of the cylinder underneath the plane.



- | | |
|---|--------------------------------|
| (A) $\frac{1}{2}\pi R^2\sqrt{a^2 - 4R^2}$ | (B) $\pi R^2\sqrt{a^2 - 4R^2}$ |
| (C) $\pi R^2\sqrt{a^2 - R^2}$ | (D) $\frac{1}{2}\pi R^2 a$ |
| (E) $\frac{1}{2}\pi R(a^2 - 4R^2)$ | |

3. Which of one the following sets is finite?

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|---|
| (A) $\left\{\frac{x}{y} \mid x, y \in \mathbb{N}_0, \frac{x}{y} \geq 2000, x \geq 2000\right\}$ |
| (B) $\left\{\frac{x}{y} \mid x, y \in \mathbb{N}_0, \frac{x}{y} \leq 2000, x \leq 2000\right\}$ |
| (C) $\left\{\frac{x}{y} \mid x, y \in \mathbb{N}_0, \frac{x}{y} \leq 2000, y \leq 2000\right\}$ |
| (D) $\left\{\frac{x}{y} \mid x, y \in \mathbb{N}_0, \frac{x}{y} \leq 2000, x \geq 2000\right\}$ |
| (E) $\left\{\frac{x}{y} \mid x, y \in \mathbb{N}_0, \frac{x}{y} \leq 2000, y \geq 2000\right\}$ |

4. $\sqrt{2 + \sqrt{3}} + \sqrt{2 - \sqrt{3}}$ equals

- | | | | | |
|-------|-----------------|----------------|-----------------|----------------|
| (A) 2 | (B) $2\sqrt{2}$ | (C) $\sqrt{5}$ | (D) $2\sqrt{5}$ | (E) $\sqrt{6}$ |
|-------|-----------------|----------------|-----------------|----------------|

5. If $f : \mathbb{R}_0^+ \rightarrow \mathbb{R}$ satisfies the property $\forall x, y \in \mathbb{R}_0^+ : f(xy) = f(x) + f(y)$, then $f(0,5) + f(1) + f(2)$ equals

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|-------------------|--|-------|
| (A) 0 | (B) 1 | (C) 2 |
| (D) $\frac{7}{2}$ | (E) undefinable by lack of information | |

6. For a subset A of a set S one defines the characteristic function k_A as follows:

$$\begin{aligned} k_A(x) &= 1 & \text{if } x \in A \\ k_A(x) &= 0 & \text{if } x \in S \setminus A \end{aligned}$$

Consider the following expressions on arbitrary subsets A and B of S :

- I. $k_A \cdot k_B$ is the characteristic function of $A \cap B$.
- II. $k_A + k_B$ is the characteristic function of $A \cup B$.
- III. $k_A - k_{A \cap B}$ is the characteristic function of $A \setminus B$.

The correct expressions are

- | | | | | |
|-------|--------------|---------------|----------------|-------------------|
| (A) I | (B) I and II | (C) I and III | (D) II and III | (E) I, II and III |
|-------|--------------|---------------|----------------|-------------------|

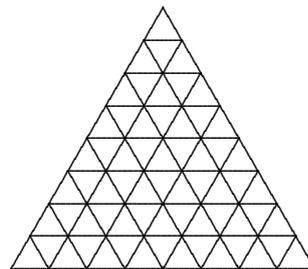
7. One chooses a fixed point a and a variable point b on a circle. The probability that the length of the chord $[ab]$ is less than the radius equals

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|-------------------|-------------------|---------------------|-------------------|-------------------|
| (A) $\frac{1}{2}$ | (B) $\frac{1}{3}$ | (C) $\frac{1}{\pi}$ | (D) $\frac{1}{4}$ | (E) $\frac{1}{6}$ |
|-------------------|-------------------|---------------------|-------------------|-------------------|

8. Which one of the following statements is false?

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|---|
| (A) All rectangles are similar. |
| (B) All circles are similar. |
| (C) All squares are similar. |
| (D) All regular pentagons are similar. |
| (E) All isosceles triangles with a top angle of 50° are similar. |

9. Divide every side of a given triangle into 2000 equal parts and join the points on all sides, as shown in the figure (with 8 instead of 2000 equal parts). How many triangles of the smallest area are there?



- (A) 1.999.000 (B) 2.000.000 (C) 2.001.000 (D) 3.998.001 (E) 4.000.000

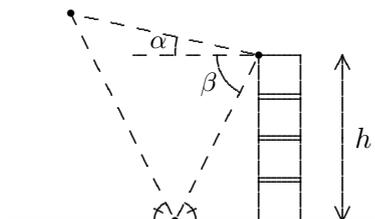
10. If $x \in \mathbb{R}$, then the smallest value of $x^2 + 3x$ is

- (A) $-\frac{9}{4}$ (B) $-\frac{3}{2}$ (C) 0 (D) $\frac{3}{2}$ (E) $\frac{9}{4}$

11. The number of real roots of the equation $x^4 - 2x + 3 = 0$ is

- (A) 0 (B) 1 (C) 2 (D) 3 (E) 4

12. An ornithologist is located at a height h above the water level of a lake. Above her, she sees a bird at an angle α and its reflection in the water at an angle β . At what height (with respect to the lake) is the bird flying?



- (A) $2h \sin \beta$ (B) $2h \cos \beta$ (C) $h \sin(\alpha + \beta)$
 (D) $\frac{h}{\cos(\beta - \alpha)}$ (E) $h \frac{\sin(\alpha + \beta)}{\sin(\beta - \alpha)}$

13. How many of the following expressions are true for $x = 2000^\circ$?

- $\sin x - \cos x < 0$
 $\sin x - \tan x < 0$
 $\cos x - \tan x < 0$
 $\cotan x - \tan x < 0$

- (A) 0 (B) 1 (C) 2 (D) 3 (E) 4

14. The smallest natural number with exactly 15 natural divisors belongs to the interval

- (A) $]0, 50]$ (B) $]50, 100]$ (C) $]100, 150]$ (D) $]150, 200]$ (E) $]200, 250]$

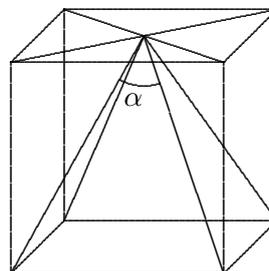
15. We define a “prime triplet” as a triplet of primes (a, b, c) where $c - b = b - a = 2$ ($a, b, c \in \mathbb{N}$). How many “prime triplets” can be found?

(A) 0	(B) 1
(C) 2	(D) a finite number greater than 2
(E) infinitely many	

16. $0,5^{0,5} =$

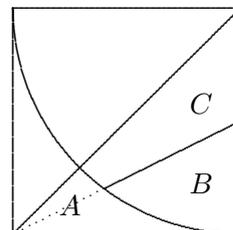
(A) 1	(B) 0,25	(C) $0,1^{0,1}$	(D) 5^5	(E) $0,25^{0,25}$
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17. In a cube one joins the centre of the top face with the four vertices of the base. Thus, a regular pyramid is constructed. The cosine of the top angle α of one of the lateral faces equals



(A) $\frac{\sqrt{3}}{3}$	(B) $\frac{2}{3}$	(C) $\frac{\sqrt{2}}{2}$	(D) $\frac{7}{9}$	(E) $\frac{\sqrt{3}}{2}$
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18. In the figure we see a square, a quarter of a circle, a diagonal of the square and a line segment that joins a vertex to the midpoint of a side. What can be said about the areas A , B and C of the indicated parts?



(A) $A < B < C$	(B) $A < C < B$	(C) $B < A < C$
(D) $B < C < A$	(E) $C < A < B$	

19. If you join all vertices of a regular 11-gon two at a time, how many line segments of different length do you get?

(A) 4	(B) 5	(C) 10	(D) 12	(E) 55
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20. Consider a box with dimensions $30 \times 40 \times 50$ made of 60 cubes with dimensions $10 \times 10 \times 10$. How many of these cubes are intersected by an inner diagonal of the box?

(A) 8	(B) 9	(C) 10	(D) 11	(E) 12
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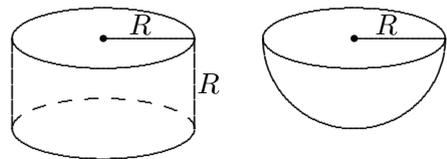
21. Assume you've already answered 15 of the past 20 problems correctly. How many problems of this round do you still have to solve correctly in order to have 80% correct answers?

(A) 6 (B) 7 (C) 8 (D) 9 (E) alle 10

22. Take two fixed real numbers a and b ($a \neq b$). The equation $|x - a| + |x - b| = k$ in the variable x ($\in \mathbb{R}$) where $k > 0$ has

(A) exactly one solution for exact one value of k .
 (B) exactly one solution for infinitely many values of k .
 (C) infinitely many solutions for exactly one value of k .
 (D) infinitely many solutions for some values of k .
 (E) two solutions for every value of k .

23. Two pots, one in the shape of a cylinder of height R and radius of the base R , one in the shape of a half sphere of radius R , both without lid, are evenly painted (in- and outside). If the cylinder needs k times more paint than the half sphere, then k equals

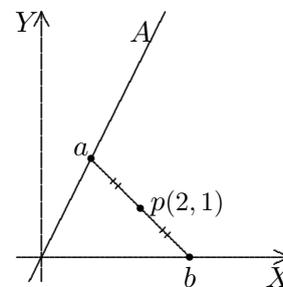


(A) 1 (B) $\sqrt{2}$ (C) 1,5 (D) $\frac{\pi}{2}$ (E) 2

24. Five customers each have to pay a different amount to a company. A non attentive accountant writes down the names on the invoices in an arbitrary way. A sleepy secretary puts the invoices in five adressed envelopes in an arbitrary way. A lazy courier arbitrarily drops the five envelopes in the five letterboxes. What is the probability for each customer to find the right amount on his invoice in his letterbox?

(A) $\frac{1}{5}$ (B) $\frac{1}{5^3}$ (C) $\frac{1}{5^{15}}$ (D) $\frac{1}{5!}$ (E) $(\frac{1}{5!})^3$

25. In an orthonormal coordinate system, we consider the line A with equation $y = 2x$ and a fixed point $p(2, 1)$. We draw a line through p that intersects A in a and the X -axis in b such that p is the midpoint of $[ab]$. The length of the line segment $[ab]$ equals



(A) $2\sqrt{2}$ (B) 3 (C) $3\sqrt{2}$ (D) 4 (E) $2\sqrt{5}$

26. Consider the numbers consisting of two digits, that, after multiplication by the sum of their digits, yield a product which equals the sum of the third powers of the digits. How many suchlike numbers can be found?

(A) 0	(B) 1	(C) 2	(D) 3	(E) 4
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27. How many of the following equations have real solutions?

$$\begin{aligned} \sin^2 x + \tan^2 x &= 1 \\ \cos^2 x + \tan^2 x &= 1 \\ \sin^2 x + \cotan^2 x &= 1 \\ \cos^2 x + \cotan^2 x &= 1 \\ \tan^2 x + \cotan^2 x &= 1 \end{aligned}$$

(A) 1	(B) 2	(C) 3	(D) 4	(E) 5
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28. The greatest integer less than or equal to x is represented by $\lfloor x \rfloor$. Examples are: $\lfloor \pi \rfloor = 3$, $\lfloor -\pi \rfloor = -4$ and $\lfloor 2000 \rfloor = 2000$.

Let $m, n \in \mathbb{Z}$, $m \leq n$. How many even numbers i where $m \leq i \leq n$ can be found ?

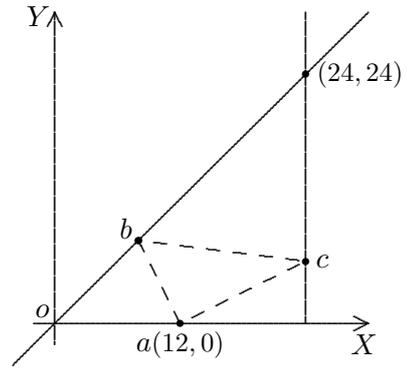
(A) $\lfloor \frac{n-m}{2} \rfloor$	(B) $\lfloor \frac{n-m+1}{2} \rfloor$	(C) $\lfloor \frac{n}{2} \rfloor - \lfloor \frac{m}{2} \rfloor$
(D) $\lfloor \frac{n}{2} \rfloor - \lfloor \frac{m}{2} \rfloor + 1$	(E) $\lfloor \frac{n}{2} \rfloor - \lfloor \frac{m-1}{2} \rfloor$	

29. The altitude line drawn from the right angle of a rectangular triangle abc in which the radian measure of the angle in a is α ($0 < \alpha < \frac{\pi}{2}$), divides the rectangular triangle into two parts with areas A and B . Let A be the area of the part in which a is a vertex.

The quotient $\frac{A}{B}$ equals

(A) $\cotan \alpha$	(B) $\tan \alpha$	(C) $\sin \alpha$	(D) $\tan^2 \alpha$	(E) $\cotan^2 \alpha$
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30. In an orthonormal coordinatesystem one considers the point $a(12, 0)$, a variable point b on the line $y = x$ and a variable point c on the line $x = 24$. If triangle abc has a minimal perimeter, then the coordinates of the point b are

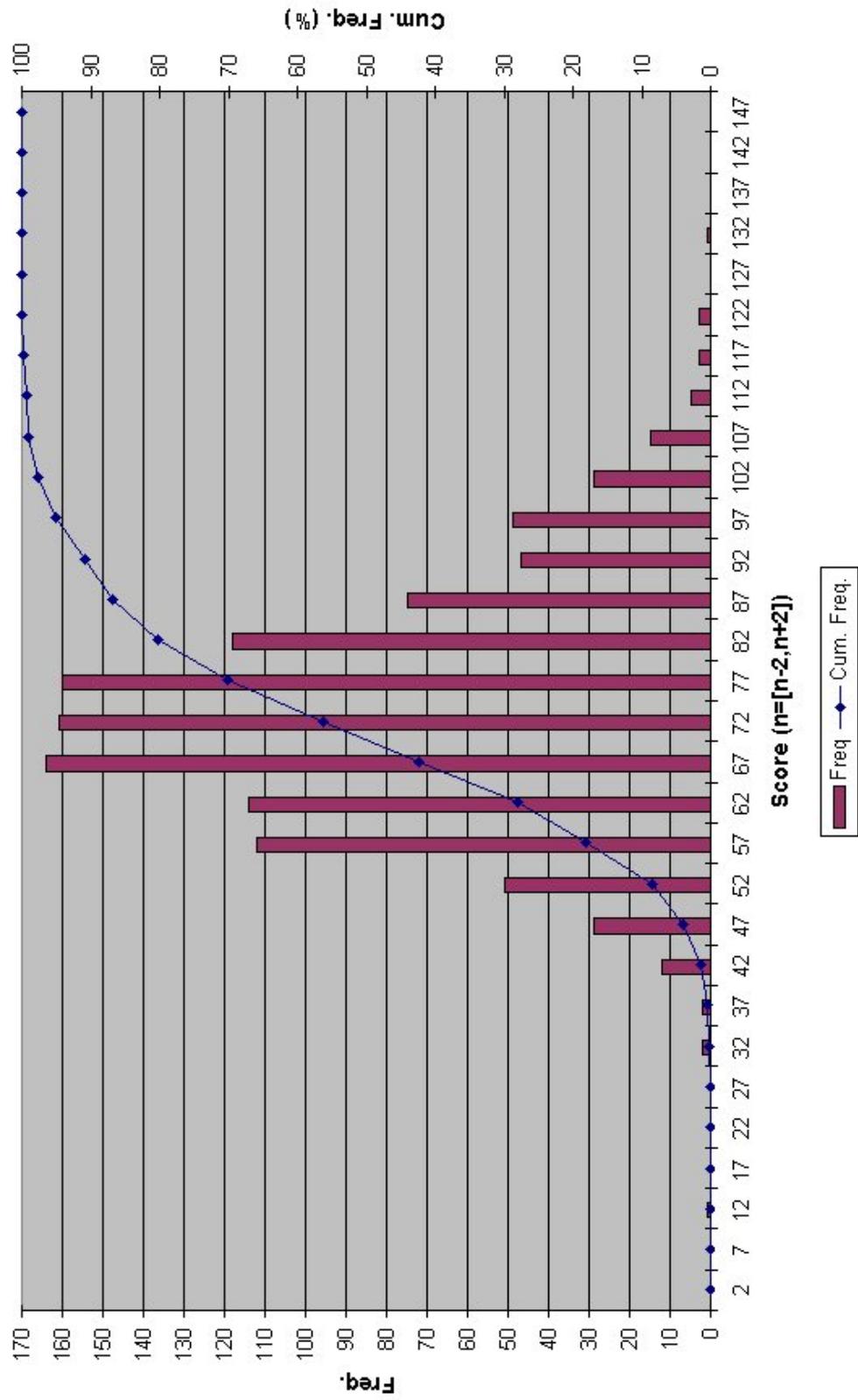


- | | |
|---|--------------|
| (A) (6, 6) | (B) (8, 8) |
| (C) (9, 9) | (D) (12, 12) |
| (E) There is no point b for which the perimeter is minimal. | |

2.2 Answer Patterns of the Second Round.

Flanders Mathematics Olympiad 2000 – Second Round Answer Patterns (all participants)									
Problem	Correct	A	B	C	D	E	Correct	Wrong	Blank
1	B	2.95	81.61	7.03	1.21	5.20	81.61	16.39	1.99
2	A	76.32	11.19	2.25	0.61	1.21	76.32	15.26	8.41
3	C	1.65	10.23	57.68	5.46	3.21	57.68	20.56	21.77
4	E	9.45	11.36	7.81	2.08	45.45	45.45	30.79	23.76
5	A	13.70	19.34	1.39	2.08	40.16	13.70	63.14	23.16
6	C	5.12	4.42	18.82	9.11	4.94	18.82	23.59	57.59
7	B	3.21	58.46	16.22	3.38	8.93	58.46	31.74	9.80
8	A	86.56	4.51	0.35	2.43	2.78	86.56	10.15	3.30
9	E	0.26	3.82	3.73	3.90	83.26	83.26	11.71	5.03
10	A	66.09	24.11	5.38	0.61	0.69	66.09	30.79	3.12
11	A	50.48	4.25	4.77	1.56	4.34	50.48	15.00	34.52
12	E	1.73	2.43	4.68	5.03	11.01	11.01	13.88	75.11
13	C	2.08	9.45	66.52	13.70	1.91	66.52	27.15	6.33
14	C	2.60	2.60	30.79	9.45	8.59	30.79	23.24	45.97
15	B	4.16	44.75	24.28	5.20	9.37	44.75	43.10	12.14
16	E	0.69	5.29	3.99	0.78	80.75	80.75	10.75	8.50
17	B	3.30	34.61	8.41	3.99	10.49	34.61	26.19	39.20
18	A	32.35	5.81	9.19	7.03	2.52	32.35	24.54	43.10
19	B	23.50	54.90	5.12	1.56	1.30	54.90	31.48	13.62
20	C	6.76	5.46	14.31	7.81	12.40	14.31	32.52	53.17
21	D	0.35	0.78	1.30	95.58	0.69	95.58	3.12	1.30
22	C	9.45	5.98	5.98	13.01	31.83	5.98	60.28	33.74
23	C	5.20	4.60	28.88	7.98	7.11	28.88	24.89	46.23
24	D	13.79	37.29	4.51	11.88	21.16	11.88	77.02	11.10
25	A	55.59	4.16	2.34	2.17	4.42	55.59	13.10	31.31
26	C	18.73	7.46	7.81	4.68	1.65	7.81	32.52	59.67
27	D	2.86	13.36	7.46	13.36	5.20	13.36	28.88	57.76
28	E	7.37	10.84	6.07	9.45	20.99	20.99	33.82	45.19
29	E	7.03	7.03	2.95	7.72	26.02	26.02	24.72	49.26
30	C	7.03	4.77	5.38	11.88	8.50	5.38	32.18	62.45

Flanders Mathematics Olympiad - Second Round 2000 - All scores
 (mean: 72.87)



3 The 2000 Final Round.

1. A number n consists of 7 different digits and is divisible by each of those digits. n cannot contain the digits ...

2. Consider 2 triangles. The edges of the second triangle have the same length as the medians of the first triangle. Determine the ratio of the areas of both triangles.

3. We denote p_n for the n -th prime number ($p_1 = 2$). The row (f_j) is determined as follows:

- $f_1 = 1, f_2 = 2$;
- $\forall j \geq 2$: if $f_j = kp_n$ with $k < p_n, (k \in \mathbb{N}_0)$ then $f_{j+1} = (k+1)p_n$;
- $\forall j \geq 2$: if $f_j = p_n^2$ then $f_{j+1} = p_{n+1}$.

- (a) Prove that all elements of the row (f_j) are different.
- (b) At what position in the row (f_j) does the last element with less than three digits appear?
- (c) Describe the natural numbers which do not appear in the row (f_j) .
- (d) How many numbers that contain less than three digits appear in the row (f_j) .

4. Solve:

$$\sin x < \cos x < \tan x < \cotan x$$

with $0 \leq x < 2\pi$.
