1 Flanders Mathematics Olympiad 1992-1993.

1.1 The problems of the first round.

1. $\sqrt[6]{a}\,\sqrt[3]{a}$, where $a > 0$, equals

(A) $\sqrt{a}$	(B) $\sqrt[9]{a}$	(C) $\sqrt[12]{a}$	(D) $\sqrt[9]{a^2}$	(E) $\sqrt[18]{a^2}$

2. How many of the following four assertions are true?

$$\begin{aligned} 2^{10} + 2^{10} &= 2^{11} \\ 2^{10} - 2^{10} &= 0^{10} \\ 2^{10} \cdot 2^{10} &= 2^{20} \\ 2^{10} : 2^{10} &= 10^{0} \end{aligned}$$

(A) 0	(B) 1	(C) 2	(D) 3	(E) 4

3. $1 - \dfrac{1}{\dfrac{1}{x-1} + 1}$ equals

(A) 2	(B) $\dfrac{1}{x}$	(C) $-\dfrac{1}{x}$	(D) $x - 1$	(E) $\dfrac{x-1}{x}$

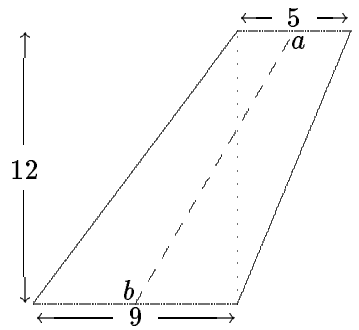
4. For how many integers n is $\sqrt{1 - (n+2)^2}$ a real number?

(A) 0	(B) 1	(C) 2	(D) 3	(E) infinitely many

5. Which of the following numbers is the smallest?

(A) $0{,}4^2$	(B) $0{,}5^2$	(C) $0{,}5^{-1}$	(D) 5^{-1}	(E) $\sqrt{0{,}25}$

6. The large base of an obtuse trapezium (see figure) measures 9, while the small base measures 5. The height equals 12. Moreover, the perpendicular through one vertex of the small base also contains one vertex of the large base. When connecting the midpoints of the small and the large base, we construct a segment with length x which satisfies



(A) $x \leq 13{,}5$	(B) $13{,}5 < x < 14$	(C) $x = 14$
(D) $14 < x < 14{,}5$	(E) $14{,}5 \leq x$	

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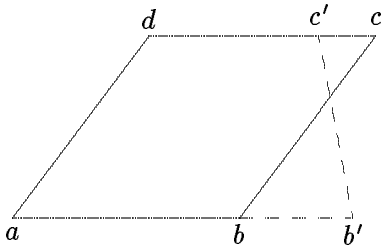
7. How many positive integers n (with $1 < n < 1000$) are a power with integer exponent m ($m > 1$) of an odd non-prime number.

(A) 2	(B) 3	(C) 4	(D) 5	(E) 6

8. Consider a parallellogram $abcd$ (see figure). The side $[dc]$ is shortened by 25%, the side $[ab]$ lengthened by 50%. This yields a trapezium $ab'c'd$. By what percentage the area of this trapezium is larger than the area of the parallellogram $abcd$?



(A) 0%	(B) 12,5%	(C) 20%	(D) 25%	(E) 40%

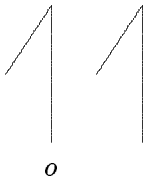
9. What is the right most digit of $2^{1993} - 1992$?

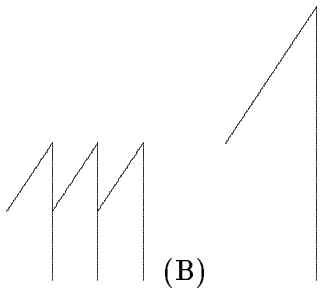
(A) 0	(B) 2	(C) 4	(D) 6	(E) 8

10. The product $(1 + x + x^2 + \cdots + x^{100})(1 + x + x^2 + \cdots + x^{25})$ is a polynomial in the variable x. The coefficient of x^{50} is

(A) 1	(B) 25	(C) 26	(D) 50	(E) 51

11. What does the figure which is composed of the endpoints of all vectors $o\vec{v} + o\vec{w}$ look like, if v runs through the left "1" and w runs through the right "1" on the next figure?



(A)  (B) (C) (D) (E)

12. On its way from Bombay to New York, a plane makes an intermediate stop at London. During this stop the following tasks have to be fulfilled:

(1) unloading the luggage (time needed: 20 minutes)
(2) unboarding the passengers (time needed: 10 minutes)
(3) loading the luggage (time needed: 20 minutes, carried out after (1))
(4) cleaning the plane (time needed: 15 minutes, carried out after (2))
(5) refueling (time needed: 20 minutes)
(6) technical check up (time needed: 30 minutes)
(7) bringing the meals on board (time needed: 10 minutes, carried out after (4))
(8) boarding the passengers (time needed: 25 minutes, carried out after (4) and (5))

Determine the minimal time the plane should stay on the ground for this stop.

(A) 30 min	(B) 50 min	(C) 1 u	(D) 1 u 20 min	(E) 2 u 30 min

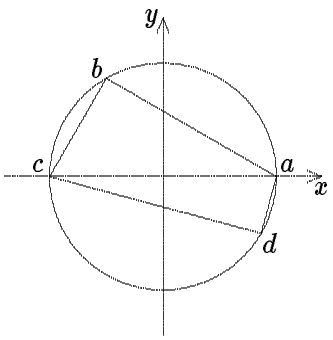
13. The paper-format A4 is defined in such a way that the ratio v of the long side with respect to the short side is unaltered when the paper is cut in two along a line that connects the midpoints of the long sides. Which of the following equations does v satisfy?

(A) $v = 4$	(B) $v^2 = 4$	(C) $v^3 = 4$	(D) $v^4 = 4$	(E) $v^5 = 4$

14. On a goniometric circle the following points are given:

a, the image of $0°$
b, the image of $120°$
c, the image of $180°$
d, the image of $330°$

The area of the quadrangle $abcd$ equals



(A) $\sqrt{3}+1$	(B) $\sqrt{3}-1$	(C) $\frac{\sqrt{3}+1}{2}$	(D) $\frac{\sqrt{3}-1}{2}$	(E) $\frac{2\sqrt{3}+3}{6}$

15. During the summer holidays five pupils (An, Bart, Carla, Doris and Eric) went swimming regularly. By accident, each time they went, exactly one of them was missing. An went least often (5 times), Eric most often (8 times). Wat can one tell about the number of times Bart, Carla and Doris went swimming?

(A) each went six times	(B) each went seven times
(C) two went six times, the other seven	(D) one went six times, the others seven
(E) there is not enough information to solve	

16. The line having equation $y = ax$ and the line with equation $y = -x + b$ intersect in a point with strictly negative coordinates. It follows that

(A) $a > 0$ and $b > 0$	(B) $a > 0$ and $b < 0$	(C) $a < 0$ and $b > 0$
(D) $a < 0$ and $b < 0$	(E) $a \cdot b = 0$	

17. Let a, b, $c \in \mathbb{N}_0$. Which of the following propositions is false?

(A) a divides $b \Longrightarrow a$ div. bc	(B) a divides 0
(C) a divides b and $c \Longrightarrow a$ div. $2b + 5c$	(D) a divides b and $c \Longrightarrow a$ div. bc
(E) a divides $bc \Longrightarrow a$ div. b or a div. c	

18. Two congruent cones with height equal to the double of the radius R of their base, share a common axis and are placed in such a way that each base contains the top of the other cone. The volume of the part of the space that is taken up by the union of those cones amounts to

(A) $\frac{7}{12}\pi R^3$	(B) $\frac{2}{3}\pi R^3$	(C) $\frac{7}{6}\pi R^3$	(D) $\frac{5}{4}\pi R^3$	(E) $\frac{4}{3}\pi R^3$

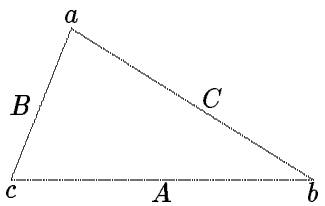
19. We have a red, a blue and a green pencil at our disposal and we want to colour the sides of an equilateral triangle. In how many ways can this be done? Two colourings of the triangle are considered as equal if one coincides with the other after a displacement **within** the plane.

(A) 6	(B) 10	(C) 11	(D) 12	(E) 27

20. Consider the expression $(a^b)^c$. If we subsitute a, b and c with three different values from the set $\{0, 1, 2, 3\}$, how many different values can you possibly get?

(A) 2	(B) 3	(C) 4	(D) 5	(E) 6

21. In the triangle abc, the medians through b and c are perpendicular. It follows that $B^2 + C^2$ equals

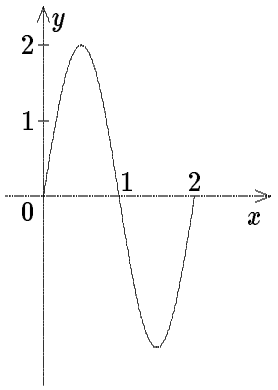


(A) A^2	(B) $2A^2$	(C) $3A^2$	(D) $4A^2$	(E) $5A^2$

22. A palindrome is a non-negative integer that stays unaltered when its digits are read from the right to the left or from the left to the right. E.g. 121, 0, 2002 and 4 are palindromes. The number of palindromes that are smaller than 1 000 000 is

(A) 900	(B) 1991	(C) 1993	(D) 1999	(E) 2220

23. In this figure the graph shows a full period of a sine-function. This function is given by

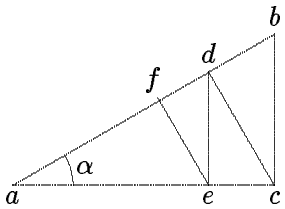


(A) $y = 2\sin\frac{x}{\pi}$	(B) $y = 2\sin\frac{\pi}{2}x$	
(C) $y = \sin 2x$	(D) $y = \sin \pi x$	
	(E) $y = 2\sin \pi x$	

24. The domain D of the function $f : x \mapsto \dfrac{x^2}{2x-1}$ is $\mathbb{R} \setminus \{\frac{1}{2}\}$. The image $f(D)$ is

(A) $\mathbb{R}\setminus]-1, 0[$	(B) $\mathbb{R}\setminus]0, 1[$	(C) $\mathbb{R}\setminus]-2, 0[$
(D) $\mathbb{R}\setminus]0, 2[$	(E) $\mathbb{R}$	

25. In a right triangle abc, $|ab| = 4$ and $\widehat{a} = \alpha$. Moreover, $cd \perp ab$, $de \perp ac$ and $ef \perp ab$. The length of $[ef]$ equals



(A) $4\sin^4\alpha$	(B) $4\sin^3\alpha \cdot \cos\alpha$	(C) $4\sin^2\alpha \cdot \cos^2\alpha$
(D) $4\sin\alpha \cdot \cos^3\alpha$	(E) $4\cos^4\alpha$	

26. If the angles α, β and γ of a triangle form the successive terms of an arithmetic sequence, then

$$\frac{\sin\alpha + \sin\beta + \sin\gamma}{\cos\alpha + \cos\beta + \cos\gamma} =$$

(A) $3\sqrt{3}$	(B) $\sqrt{3}$	(C) $\frac{\sqrt{3}}{3}$	(D) 1	(E) $\frac{1}{3\sqrt{3}}$

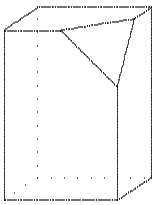
27. A given genealogical family consists of a pair of ancestors with three children from which two marry and one does not. For each of the successive married couples the same situation returns (they have three children from which two marry). Determine the maximal number of persons occurring in the genealogical, up to and including the tenth generation after the ancestors (all partners are included in the counting)?

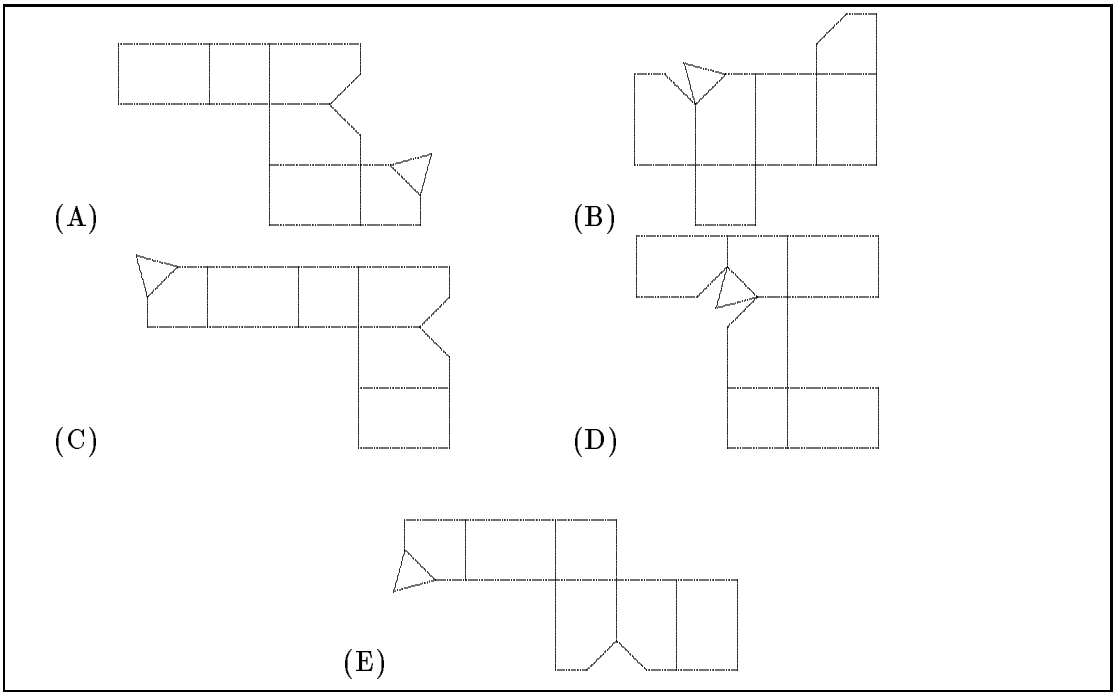
(A) 4095	(B) 4102	(C) 4104	(D) 5115	(E) 5117

28. Let $\lfloor x \rfloor$ represent the greatest integer smaller than or equal to x. Let $x, y \in \mathbb{R}$. Which one of the following propositions is true?

(A) $\lfloor x+y \rfloor = \lfloor x \rfloor + \lfloor y \rfloor$	(B) $\lfloor x+y \rfloor = \lfloor x \rfloor + \lfloor y \rfloor + 1$
(C) $\lfloor x+y \rfloor > \lfloor x \rfloor + \lfloor y \rfloor$	(D) $\lfloor x+y \rfloor \leq \lfloor x \rfloor + \lfloor y \rfloor$
(E) $\lfloor x+y \rfloor \leq \lfloor x \rfloor + \lfloor y \rfloor + 1$	

29. The next figure represents a box from which a piramid is left out in one of the corners. Which one of the following developments (drawn to scale) is the one of this body?



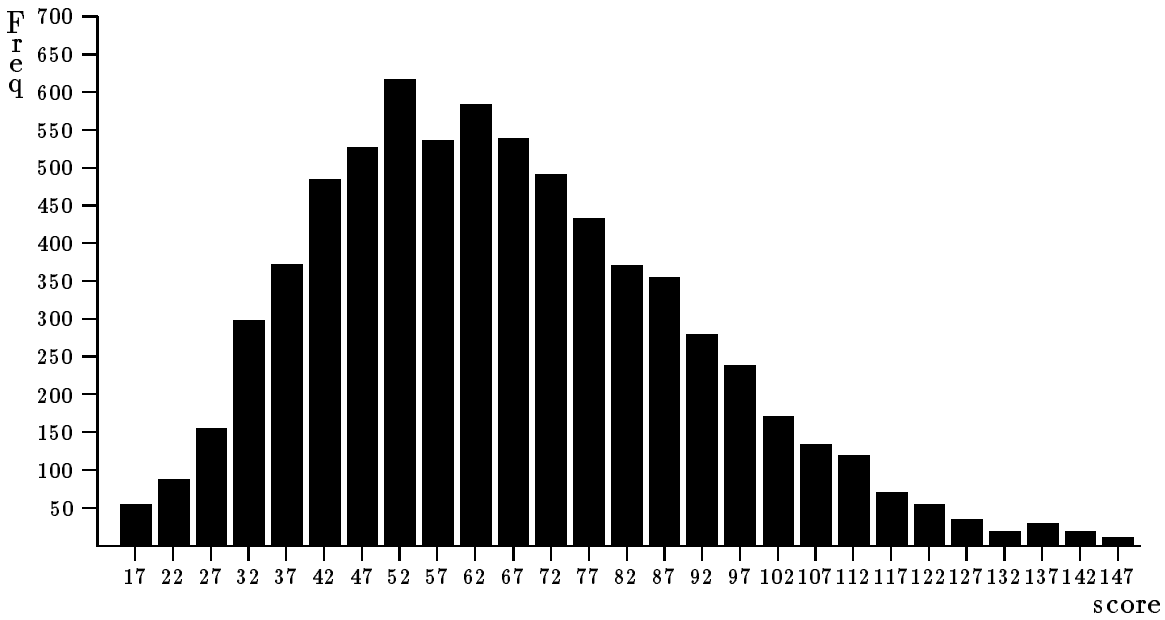


30. For how many integers n (with $100 \leq n \leq 200$) the fraction $\dfrac{n^2-3}{n^2-1}$ is reducible?

(A) 0	(B) 25	(C) 50	(D) 76	(E) 101

2 VWO. 1992-1993: some statistics.

2.1 Histogram of the first round.



2.2 Answer-patterns of the first round.

Flanders Mathematics Olympiad 1992-1993 First Round - (*: correct answer)						
Problem	A	B	C	D	E	Blank
1	69.24*	14.30	0.62	7.04	5.00	3.80
2	0.41	3.60	13.57	29.71	51.76*	0.92
3	0.95	64.41*	23.40	4.22	3.59	3.43
4	12.66	7.45	11.35	47.64	13.33*	7.57
5	78.94*	1.71	4.74	4.12	9.08	1.40
6	5.56	31.24*	17.94	3.42	4.11	37.73
7	3.06	3.04	5.01	16.46*	19.92	52.51
8	1.45	55.00*	4.28	9.88	1.79	27.60
9	39.19*	5.76	5.62	14.26	2.84	32.33
10	7.81	6.75	40.20*	3.19	6.54	35.50
11	1.47	7.78	10.38	3.12	19.21*	58.05
12	0.97	48.66*	13.38	16.16	10.15	10.66
13	2.67	15.01	6.37	27.84*	2.10	46.02
14	3.44	1.69	27.39*	6.93	2.12	58.43
15	1.64	3.11	35.02*	5.14	32.65	22.44
16	3.30	47.59*	4.32	8.72	1.93	34.13
17	3.46	19.97	11.80	2.41	39.77*	22.54
18	3.25	6.59	12.24*	2.94	10.74	64.23
19	18.18	16.80	17.15*	6.97	7.92	32.96
20	1.89	5.97	61.82*	7.67	11.56	11.10
21	16.30	6.04	2.80	2.78	8.60*	63.48
22	5.62	4.49	2.71	22.97*	6.93	57.27
23	6.41	7.67	6.18	3.11	37.80*	38.82
24	1.40	18.11*	1.34	1.38	26.78	50.99
25	1.65	2.05	3.39	14.37*	1.72	76.82
26	1.78	14.03*	3.15	11.50	1.12	68.42
27	3.66	2.85	3.19	12.73	33.92*	43.65
28	5.82	3.06	5.75	11.58	34.61*	39.16
29	5.27	14.99	2.53	1.64	57.74*	17.84
30	13.02	2.50	20.41*	3.78	3.13	57.16

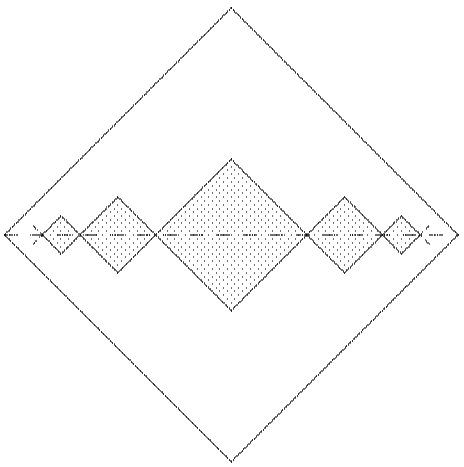
3 The final round.

Vraag 1 *In December, each of the 20 students of the same class sends 10 greeting cards to 10 different fellow class members (different from himself).*

a) Show that there are at least 2 students who send each other a greeting card.

b) Suppose a class consisting of n students, each of them sending m greeting cards to m different fellow class members (different from himself). How do n and m have to be in order to be sure that there will be at least 2 students sending each other a greeting card?

Vraag 2 *A jeweller covers the diagonal of a silver square (of side 1) by putting an "infinite number" of little golden squares on it in the following way:*



- *the sides of the golden squares are always parallel to those of the given (silver) square;*
- *the sides of the golden squares have a length which is exactly half or twice the length of the sides of the neighbouring golden square; moreover, the middle golden square is the largest;*
- *each of the midpoints of the golden squares lies on one diagonal of the silver square and having a distance to one of the vertices (of the silver square) equal to $1/2$, $1/4$, $1/8$, $1/16, \ldots$ of the length of that diagonal;*
- *neighbouring golden squares have one common vertex;*

a) Determine the side-length of the middle golden square.

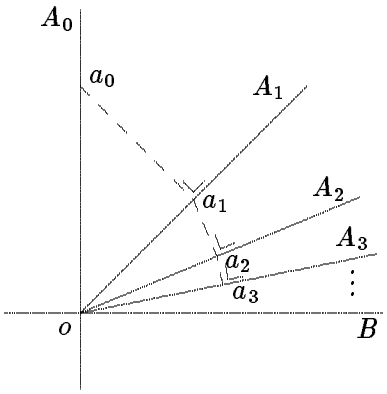
b) Determine the total area covered with gold.

Vraag 3 *Prove the following inequality:*

$$-1 < \left(\frac{a-b}{a+b}\right)^{1993} + \left(\frac{b-c}{b+c}\right)^{1993} + \left(\frac{c-a}{c+a}\right)^{1993} < 1$$

(a, b and c are strictly positive numbers)

Vraag 4 *Consider the following picture:*



A_0 and B are two lines perpendicular to each other and intersecting in o. A_n is the line from o such that the angle between A_{n+1} and B is half of the angle between A_n and B.
a_0 lies on A_0 and satisfies $|oa_0| = 1$ while a_{n+1} is the orthogonal projection of a_n on A_{n+1} $(n \in \mathbb{N})$.

Calculate

$$\lim_{n \to +\infty} |oa_n|.$$