1 Flanders Mathematics Olympiad 1994–1995 : First Round.

1.1 The problems.†

1. Which of the following polynomials does **NOT** divide $(x-1)^2(x^3+x)$?

(A) x^3-x^2+x-1	(B) x^2-2x+1	(C) x^2-x
(D) x^3-x	(E) $x^4-2x^2(x-1)-2x+1$	

2. If $z \in \mathbb{Z}_0$, then z^2-z is

(A) sometimes even, sometimes odd	(B) always strictly positive
(C) never a prime number	(D) always even
(E) always odd	

3. Which of the following statements is true?

(A) 8^8 is the second power of 4^4	(B) 8^8 is the third power of 4^4
(C) 8^8 is the fourth power of 4^4	(D) 8^8 is the eighth power of 4^4
(E) 8^8 is the sixteenth power of 4^4	

4. How many of the following six numbers consisting of 10 digits can be divided by 6?

1515151515 1994001994 2222222224 2333333334 3888888888 9999999999

(A) 1	(B) 2	(C) 3	(D) 4	(E) 5

5. The centre of a face of a cube with edges of length 2 has the same distance to each of the four vertices of the opposite face of the cube. This distance equals

(A) $\sqrt{5}$	(B) $\sqrt{6}$	(C) $\sqrt{7}$	(D) $2\sqrt{2}$	(E) 3

6. A test containing 3 questions has a maximal quotation of 10 points. For each question one can get at least 2 points. In how many different ways can the 10 points be divided over the three questions, when each question gets an integral amount of points?

(A) 4	(B) 13	(C) 15	(D) 20	(E) 30

7. In a acute-angled triangle abc an

(A) altitude line	(B) inner bisector	(C) median
(D) perpendicular bisector	(E) parallel bisector	

divides the triangle into two parts of equal area.

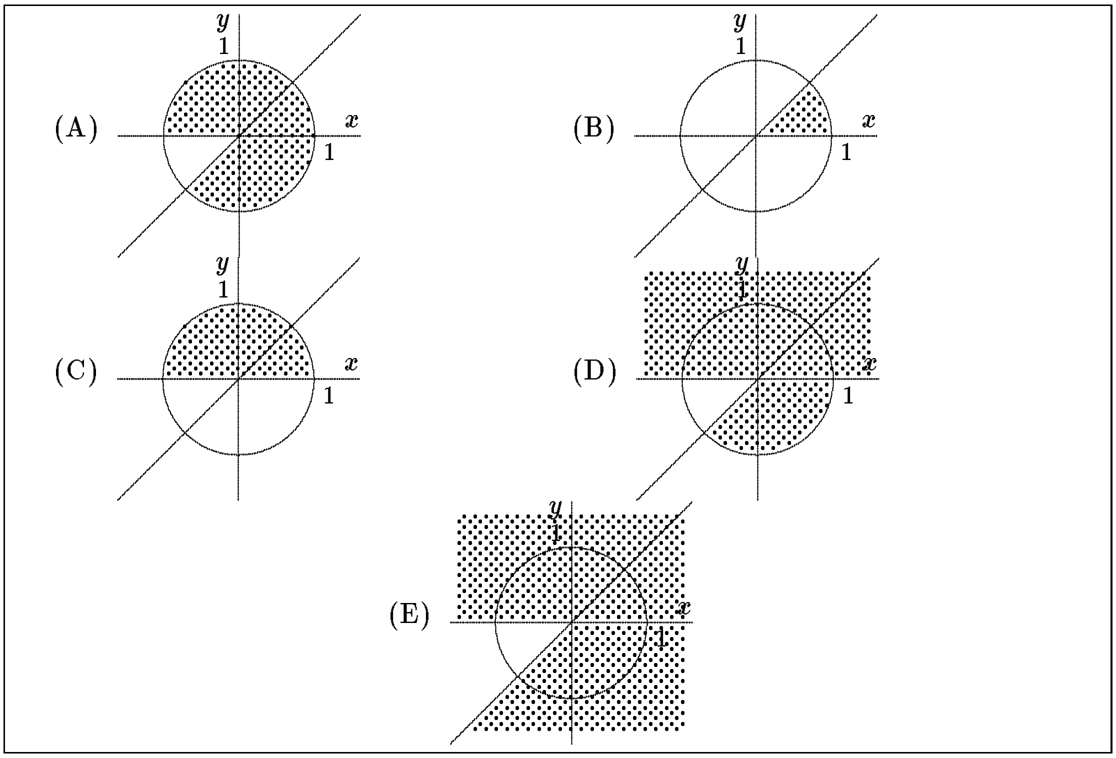
8. Within the set $\mathbb{R}_0$, the equation $\dfrac{a}{\frac{b}{c}} = \dfrac{\frac{a}{b}}{c}$ holds if and only if

(A) $a = b = c$	(B) $a = b = c = 1$	(C) $c = 1$
(D) $c = 1$ or $c = -1$	(E) The equation holds for all $a, b, c \in \mathbb{R}_0$	

9. A computer is sending a message consisting of three digits (each 0 or 1) to an other computer. How many of the following statements are true?
   a. If the first digit is 0, the sum of the digits is less than or equal to 3.
   b. If the sum of the digits is odd, the first or the third digit equals 1.
   c. The sum of the digits is odd if and only if at least one digit equals 1.
   d. If the sum of the digits is odd, then at least one digit equals 1.

(A) 0	(B) 1	(C) 2	(D) 3	(E) 4

10. Within which of the following pictures does the shaded part (with edges included) represent the set $\{(x,y) \in \mathbb{R}^2 \parallel (x^2+y^2 \leq 1 \text{ and } x \geq y) \text{ or } (y \geq 0)\}$



11. Let $a < b$ and $c < d$ be elements of $\mathbb{R}_0$. How many of the following 4 statements are true?

$a - c < b - d$; $\qquad a - d < b - c$; $\qquad ac < bd$; $\qquad \dfrac{1}{b} < \dfrac{1}{a}$.

(A) 0	(B) 1	(C) 2	(D) 3	(E) 4

12. The smallest value the function $x^2+7x+12$ attains on the set $\{x \in \mathbb{R} \parallel x^2-7x+12 \leq 0\}$ equals

(A) -4	(B) $-\dfrac{1}{4}$	(C) 3	(D) 42	(E) $\dfrac{195}{4}$

13. Which of the following 5 statements does not hold in each triangle abc?

(A) $\sin a = \sin(b + c)$	(B) $\cos 2a = \cos 2(b + c)$
(C) $\tan \frac{a}{2} = \tan \frac{b+c}{2}$	(D) $\cos(a - b + c) = \cos(b - a - c)$
(E) $\cos \frac{a}{2} = \sin \frac{b+c}{2}$	

14. How many possible values can one get by multiplying two different numbers out of the set $\{4, 8, 9, 16, 27, 32, 64, 81, 243\}$?

(A) 72	(B) 36	(C) 32	(D) 20	(E) 12

15. A cone has altitude 4 and a base with radius 3. The angle of the development of this cone equals

(A) $\frac{3\pi}{2}$	(B) $\frac{4\pi}{3}$	(C) $\frac{4\pi}{5}$	(D) $\frac{5\pi}{6}$	(E) $\frac{6\pi}{5}$

16. The equation $|4 - 5x| = -x$ has (within $\mathbb{R}$)

(A) two solutions, one positive, one negative
(B) one solution, which is negative
(C) two solutions, which are negative
(D) two solutions, which are positive
(E) no solutions

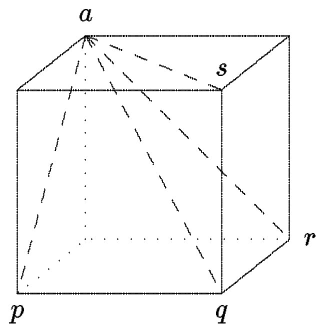
17. Write a number consisting of six digits chosen in $\{0, 1\}$. For instance, 010111, 000000, 110010, ... The chance that a sequence of at least three zeros appears equals

(A) $\frac{1}{2}$	(B) $\frac{5}{16}$	(C) $\frac{1}{8}$	(D) $\frac{9}{32}$	(E) $\frac{1}{3}$

18. The number of solutions (x, y) $(x,\ y \in \mathbb{Z})$, from the equation $x^2 + y^2 + x + y = 3$ is

(A) 0	(B) 1	(C) 2	(D) 3	(E) > 3

19. Four diagonals are drawn through the vertex a of a cube, as in the figure. How many different values does one get when computing the angles $\widehat{paq}$, $\widehat{par}$, $\widehat{pas}$, $\widehat{qar}$, $\widehat{qas}$, $\widehat{ras}$?



(A) 1	(B) 2	(C) 3	(D) 4	(E) 5

20. Let $f : \mathbb{R} \longrightarrow \mathbb{R}$ be a periodic function with period 3, such that $f(x) = x^2$ for each $-1 < x \leq 2$. Then $f(-4)$ equals

(A) -1	(B) 0	(C) 1	(D) 4	(E) 16

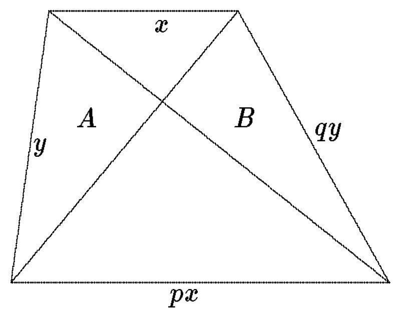
21. In an orthonormal coordinate system we only allow movements that are translations over the vectors $(0,1)$ and $(1,0)$. In how many different ways can one get from $(0,0)$ to $(3,3)$, while avoiding the points $(2,0)$, $(3,0)$ and $(3,1)$?

(A) 10	(B) 11	(C) 12	(D) 13	(E) 14

22. The negation of " $\forall x$ even : $x^2 + x$ is even " is

(A) $\forall x$ even: $x^2 + x$ is odd
(B) $\forall x$ odd: $x^2 + x$ is even
(C) $\forall x$ odd: $x^2 + x$ is odd
(D) $\exists x$ even: $x^2 + x$ is odd
(E) $\exists x$ odd: $x^2 + x$ is even

23. The ratio of the length of the large base to the length of the small base of a trapezoid equals p. The ratio of the greater to the smaller of the two remaining sides equals q. By drawing the two diagonals, the trapezoid is split into four triangles. The ratio A/B of the areas of the two triangles that do not contain the bases (see figure), equals

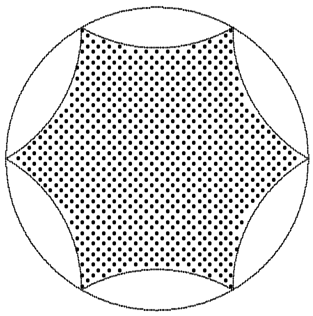


(A) $\frac{p}{q}$	(B) $\frac{p^2}{q^2}$	(C) 1	(D) $\frac{q}{p}$	(E) $\frac{q^2}{p^2}$

24. How many triangles can be constructed such that the lengths of its sides are three consecutive odd integers, and its perimeter is strictly smaller than 1000?

(A) 165	(B) 166	(C) 167	(D) 331	(E) 332

25. Consider a circle with radius 1, and six equal arcs, also with radius 1, which intersect on the circle (see figure). The area of the shaded region on the figure equals



(A) $2\pi - 3\sqrt{3}$	(B) $\frac{3\sqrt{3}}{2}$	(C) $3\sqrt{3} - \pi$
(D) $\frac{\pi\sqrt{3}}{2}$	(E) none of the previous	

26. Assuming the real numbers x and y satisfy $\frac{x}{y} = \frac{2y}{x+y}$, how many values can $\frac{y}{x}$ possibly attain?

(A) 1	(B) 2	(C) 3	(D) 4	(E)infinitely many

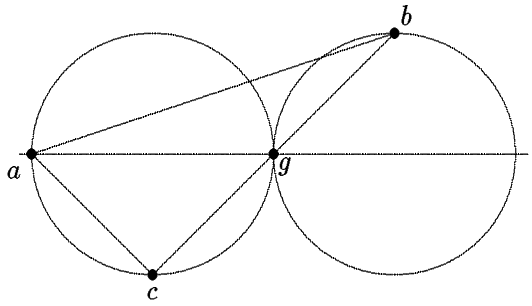
27. Let $\lfloor x \rfloor$ be the greatest integer less than or equal to x, e.g. $\lfloor \pi \rfloor = 3$, $\lfloor -\pi \rfloor = -4$. Consider the following 4 equations:

$\lfloor 7x \rfloor = 7$; $\quad \lfloor 7x \rfloor = 7\lfloor x \rfloor$; $\quad \lfloor x+7 \rfloor = x+7$; $\quad \lfloor x+7 \rfloor = \lfloor x \rfloor + 7$.

How many of those equations are satisfied by all $x \in]1, \frac{11}{10}[$?

(A) 0	(B) 1	(C) 2	(D) 3	(E) 4

28. Two circles with radius 1 touch each other in the point g. Their common diameter intersects one circle also in a. The points b and c lie each on a different circle such that bc contains g en meets the common diameter at an angle of 45°. The area of the triangle abc equals

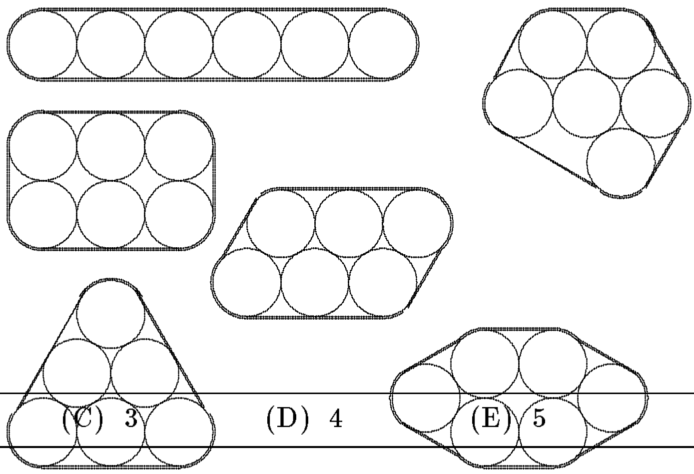


(A) 1.5	(B) $\frac{\pi}{2}$	(C) 2	(D) $\sqrt{5}$	(E) 2.5

29. A box contains 900 cards, each having a different integer between 100 and 999 written on it. Mary draws randomly a number of cards from the box and computes for each card the sum of the digits of the number on it. How many cards does Mary have to draw to be sure that at least three of those cards give the same sum of digits?

(A) 53	(B) 54	(C) 55	(D) 56	(E) 57

30. The figure gives a view from above on six ways to put six soft drink cans together. The cans are tightened by a non-elastic ribbon. In some cases the ribbon has the same length. In how many of those cases does one get the smallest length possible?



(A) 1	(B) 2	(C) 3	(D) 4	(E) 5

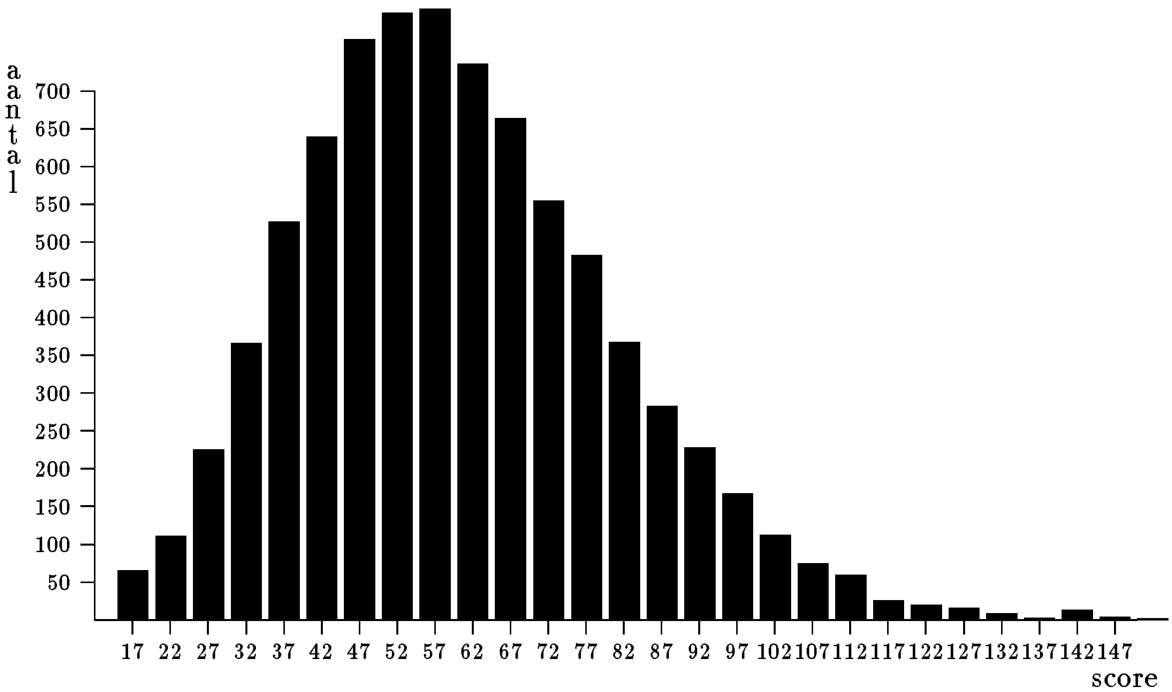
2 Flanders Mathematics Olympiad 1994-1995: some statistics.

2.1 Answer-patterns of the first round.

Flanders Mathematics Olympiad 1995
First Round – *:correct answer

Problem	Correct	A	B	C	D	E	Blank
1	D	7.87	2.27	4.67	43.84*	10.39	30.91
2	D	7.11	15.11	5.66	67.94*	0.66	3.38
3	B	3.50	72.41*	12.32	2.63	2.68	6.45
4	B	17.93	55.55*	14.10	4.42	0.47	7.53
5	B	10.78	51.85*	1.70	13.63	2.57	19.46
6	C	11.14	9.80	62.63*	3.72	5.26	7.43
7	C	3.40	11.85	48.85*	6.12	7.38	22.39
8	D	1.93	11.12	9.51	63.74*	8.03	5.61
9	C	3.71	26.04	46.27*	17.22	0.88	5.85
10	D	4.67	21.04	8.73	19.59*	2.05	43.89
11	B	8.07	26.24*	27.97	31.45	2.80	3.46
12	D	17.91	9.77	19.85	19.67*	3.41	29.37
13	C	4.08	5.74	32.15*	6.78	6.47	44.75
14	C	10.12	42.61	27.99*	3.03	0.87	15.34
15	E	5.93	5.37	6.87	3.37	8.55*	69.92
16	E	6.31	13.63	1.33	14.56	49.92*	14.24
17	B	10.06	15.35*	11.34	8.40	7.33	47.51
18	A	35.38*	3.19	8.11	1.30	7.94	44.07
19	B	2.57	27.66*	19.15	23.38	5.93	21.31
20	D	2.03	5.16	12.96	22.05*	11.15	46.64
21	E	6.41	5.29	9.27	8.98	27.64*	42.38
22	D	14.04	19.79	6.61	27.67*	7.36	24.51
23	C	5.85	3.38	13.87*	4.58	2.19	70.13
24	A	11.20*	24.43	6.76	6.57	6.10	44.91
25	C	6.33	2.32	8.29*	3.01	9.46	70.58
26	B	16.88	9.65*	1.33	1.45	20.22	50.46
27	D	2.08	5.31	9.81	36.02*	3.01	43.78
28	C	3.01	2.86	44.72*	5.09	4.11	40.18
29	A	5.87*	7.15	7.06	3.89	2.63	73.39
30	A	18.90*	25.28	10.90	5.36	4.02	35.54

2.2 Histogram of the first round.



3 Flanders Mathematics Olympiad 1994-1995: Final Round.

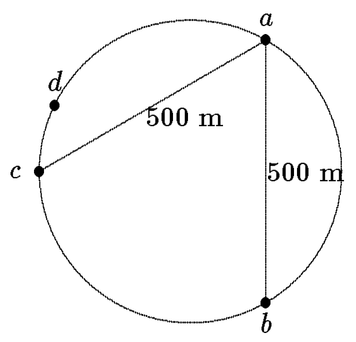
Question 1. Four married couples come together for a game of chess. They draw lots to form four groups of two players. Knowing that:

Beatrice plays against Eddy;
Alice plays against Clara's husband;
Freddy plays against Guy's wife;
Doris plays against Alice's husband;
Guy plays against Eddy's wife,

can one conclude with whom Hubert is married? Give a proof of your answer.

Question 2.
How many real numbers x belonging to $[1,3]$ have the property that x^2 has the same decimal part as x? Give a proof of your answer.

Question 3.
The points a, b, c and d lie on the edge of a circular lake with radius R, as indicated on the figure. The distances between a and c and between a and b both equal 500 meter. The time a swimmer needs to get from d to c, from d to a and from a to b with the same constant velocity are in proportion to 1, 5 and 7. Determine R.



Question 4.

1. Prove that for all $n \in I\!N$ and for all $\alpha \in]0, \pi[$:

$$\sum_{k=0}^{n} \sin k\alpha = \frac{\sin \dfrac{(n+1)\alpha}{2} \sin \dfrac{n\alpha}{2}}{\sin \dfrac{\alpha}{2}}.$$

2. Let $G(n)$ be the average length of the diagonals of a regular n-gon ($n \geq 4$), inscribed in a circle of radius 1. Compute:

$$\lim_{n \to +\infty} G(n).$$