1 Flanders Mathematics Olympiad 1996–1997: First Round

The first round consists of 30 multiple choice problems which are selected by the FMO jury. Scores were computed as follows: a correct answer yields 5 points, a blanc answer 1 point and no points are given for a wrong answer. Participants can spend 3 hours to solve the problems.

1.1 The problems

1. Which one of the following numbers is not rational?

(A) -1997	(B) $8^{2/3}$	(C) $\sqrt{0{,}49}$	(D) $100^{0{,}5}$	(E) $1000^{0{,}1}$

2. The sum of two numbers is the same as the sum of the roots of $x^2 + 6x + 1 = 0$, while their product equals the product of the roots of $x^2 + 8x + 7 = 0$. The largest of the two numbers equals

(A) $-3+\sqrt{2}$	(B) -1	(C) $-4+\sqrt{15}$	(D) $3+\sqrt{2}$	(E) 7

3. Let $f(x) = 3x - 2$. The graph of the inverse function f^{-1} is a line that intercepts the y-axis at

(A) $-\dfrac{2}{3}$	(B) $-\dfrac{1}{2}$	(C) -2	(D) $\dfrac{1}{2}$	(E) $\dfrac{2}{3}$

4. A cage contains rabbits and pheasants. Together they have 35 heads and and 94 legs. The difference between the number of pheasants and the number of rabbits amounts to

(A) 7	(B) 9	(C) 11	(D) 13	(E) 15

5. By extending all faces of a cube, the space is divided into a number of regions. How many regions?

(A) 9	(B) 16	(C) 24	(D) 27	(E) 32

6. Which of the following functions of x has a graph that is not symmetrical with respect to the y-axis.

(A) $1+\sin^2 x$	(B) $\sin(1+x^2)$	(C) $\sin(x+1)^2$
(D) $1-\sin\lvert x\rvert$	(E) $\sin(1+x)+\sin(1-x)$	

7. How many of the following four statements about natural numbers are true?

(1) Out of three successive odd numbers, exactly two are prime.
(2) Out of three successive odd numbers, at least two are prime.
(3) Out of three successive odd numbers, at least one is prime.
(4) Out of three successive odd numbers, at least one is not prime.

(A) 0	(B) 1	(C) 2	(D) 3	(E) 4

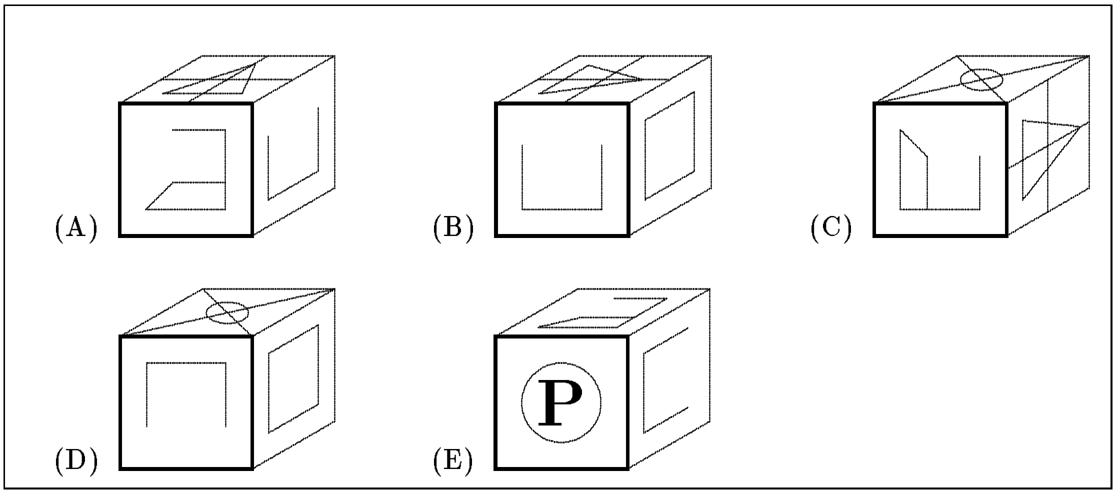
8. P and Q are real polynomials in x, respectively of degree m and n with $0 < n < m$. The degree of $(P-Q)(P+Q)$ equals

(A) $2m$	(B) m^2	(C) n^2	(D) mn	(E) $m^2 - n^2$

9. Let A be the set of all teachers, B the set of all people having a luxurious car and C the set of all people that have a rich partner. Which of the following statements is equivalent to the statement that amongst the teacher only those with a rich partner can afford a luxurious car?

(A) $B \subset A \cup C$	(B) $A \cap C \subset B$	(C) $A \cap B \subset C$
(D) $B \subset A \cap C$	(E) $C \subset A \cap B$	

10. Here are 4 pictures of the same die. One picture is from another die. Which one?



11. The sequence $0, -1, 0, 1, -2, -1, 0, 1, 2, -3, -2, -1, 0, 1, 2, 3, \dots$ is formed by reading the piramid below from top to bottom, and each line from left to right.

$$\begin{array}{ccccccccccc} & & & & & 0 & & & & & \\ & & & & -1 & 0 & 1 & & & & \\ & & & -2 & -1 & 0 & 1 & 2 & & & \\ & & -3 & -2 & -1 & 0 & 1 & 2 & 3 & & \\ & -4 & -3 & -2 & -1 & 0 & 1 & 2 & 3 & 4 & \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \end{array}$$

The 1997-th term in this sequence equals

(A) 14	(B) 15	(C) 16	(D) 17	(E) 18

12. Let $a, b \in \mathbb{N}_0$ with $a \neq b$ and let V be the smallest subset of $\mathbb{N}$ that has the properties:

$$(1)\ a, b \in V \quad \text{and} \quad (2)\ \forall x, y : x, y \in V \Rightarrow x + y \in V.$$

The set V equals

(A) $\{a, b, a+b\}$
(B) $\{na + mb \mid n, m \in \mathbb{N}_0\}$
(C) $\mathbb{N}_0$
(D) $\{na + mb \mid n, m \in \mathbb{N}\}$
(E) $\{na + mb \mid n, m \in \mathbb{N}, (m, n) \neq (0, 0)\}$

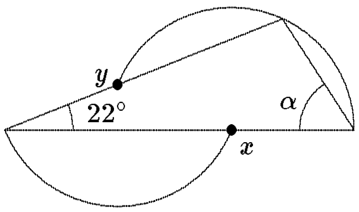
13. Consider the points $p(-2, -1)$ and $q(2, 2)$ in the plane. If $r(k, 1)$ is a point for which $|pr| + |rq|$ is minimal, then k equals

(A) $\frac{1}{3}$	(B) $\frac{3}{4}$	(C) $\frac{2}{3}$	(D) 1	(E) $\frac{4}{3}$

14. For each $n \in \mathbb{N}$, $n > 1$, the number $(2n)^{2n} - 1$ is divisible by

(A) 2	(B) 3	(C) n	(D) $n - 1$	(E) $2n + 1$

15. The figure shows arcs with center points x and y. The angle α measures



(A) 44°	(B) 46°	(C) 57°	(D) 60°	(E) 68°

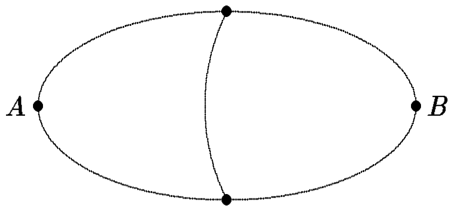
16. Let a and b be two different numbers. The line through the points (a, a^3) and (b, b^3) intersects the curve $y = x^3$ at a third point. The y-coordinate of this point equals

(A) $a+b$	(B) $a-b$	(C) $-a-b$	(D) $-(a+b)^3$	(E) $(a-b)^3$

17. The number of solutions to the equation $|x-2| + |x-3|^{-1} = 4$ equals

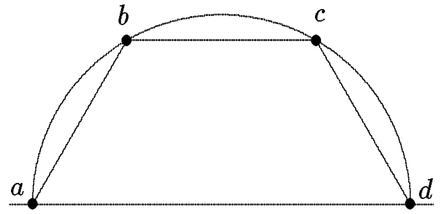
(A) 1	(B) 2	(C) 3	(D) 4	(E) 6

18. A park has two entries A and B and a network of paths as in the picture. A jogger wishes to run a circuit, starting at one of the entries, in which he covers each path exactly once in each direction, without ever making a U-turn. In how many ways is this possible?



(A) 0	(B) 2	(C) 4	(D) 6	(E) 8

19. One half of a circle is cut into three equal parts, which creates a quadrangle $abcd$ (see figure). If the radius of the circle measures 2, the area of the quadrangle measures



(A) 4	(B) $3\sqrt{2}$	(C) $3\sqrt{3}$	(D) $6\sqrt{3}$	(E) $\frac{3}{2}\sqrt{3}$

20. An office employs 18 people: one of them speaks Dutch, French and English; 3 speak French and English; 13 speak Dutch, of which 5 also speak English; 9 speak French; no one speaks only English. How many people speak only French?

(A) 2	(B) 3	(C) 4	(D) 5	(E) 6

21. Two regular octagons with sides measuring z and Z have the same centre and their sides are parallel. The area of the biggest octagon (with side Z) is twice the area of the smallest one. The smallest distance between their sides equals

(A) $\frac{z}{3}$	(B) $\frac{z}{2}$	(C) $\frac{Z}{3}$	(D) $\frac{Z}{2}$	(E) z

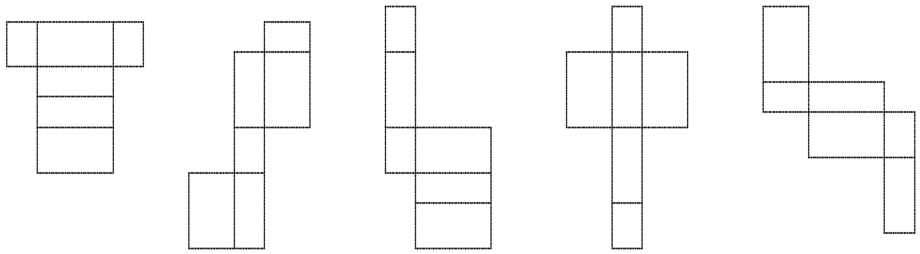
22. Consider the following equations:

$$\begin{aligned}
\cos^2 a + \cos^2 b + \cos^2 c &= 1\\
\sin^2 a + \sin^2 b + \sin^2 c &= 2\\
\cos^2 a + \cos^2 b + \cos^2 c + \sin^2 a + \sin^2 b + \sin^2 c &= 3
\end{aligned}$$

If m is the number of these equations which hold in each rectangular triangle and n the number of these equations which hold in each equilateral triangle, then $m + n$ equals

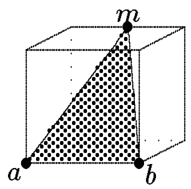
(A) 1	(B) 2	(C) 3	(D) 4	(E) 5

23. How many of the following developments can be formed into a closed box, while folding on the edges?



(A) 1	(B) 2	(C) 3	(D) 4	(E) 5

24. One connects the middle m of an edge of a cube with the two furthermost corners a and b. The cosine of $\widehat{amb}$ equals



(A) $\frac{1}{2}$	(B) $\frac{\sqrt{3}}{3}$	(C) $\frac{\sqrt{2}}{2}$	(D) $\frac{7}{9}$	(E) $\frac{\sqrt{3}}{2}$

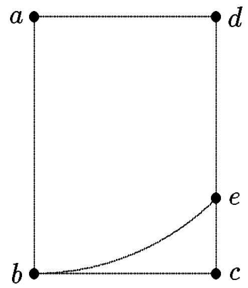
25. The area of the union of all squares with side 1 whose midpoints lie in a fixed square with side 1, equals

(A) $3 + \frac{\pi}{4}$	(B) $\frac{\pi}{2} + 2\sqrt{2} + 1$	(C) $\frac{7}{2}$
(D) 4	(E) $2 + 2\sqrt{2}$	

26. A number with eight digits is a multiple of 73 and also a multiple of 137. Moreover, the second digit from the left equals 7. The sixth digit from the left then equals

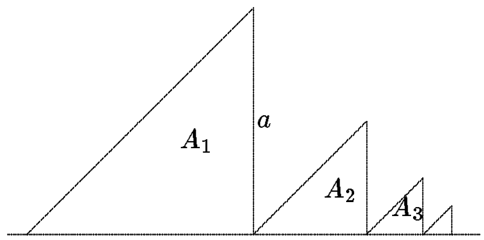
(A) 1	(B) 3	(C) 5	(D) 7	(E) 9

27. A rectangular piece of paper $abcd$ has the A4-format, i.e. $\frac{|ab|}{|ad|} = \sqrt{2}$ (see figure). An arc is drawn with a as centre and $|ab|$ as radius; it intersects cd at a point e. If $|ab| = \lambda$ and $|ad| = \mu$, then $|de|$ equals



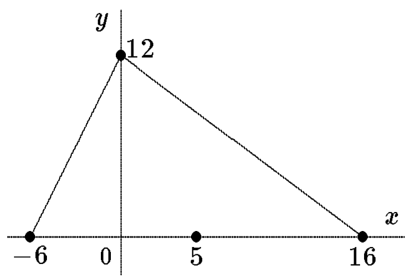
(A) λ	(B) μ	(C) $\lambda - \mu$	(D) $\frac{\lambda^2}{\mu}$	(E) $\frac{\mu^2}{\lambda}$

28. An isosceles rectangular triangle A_1 has rectangular side a. We prolong one rectangular side, on which we place a second, congruent triangle A_2 with half the dimensions of A_1 (see figure). In the same way we construct A_3 from A_2, and so on, ad infinitum. The resulting figure resembles a saw-blade. The total area of this figure equals



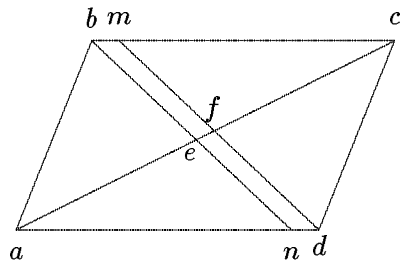
(A) a^2	(B) $\frac{3a^2}{4}$	(C) $\frac{2a^2}{3}$	(D) $\frac{\sqrt{3}a^2}{2}$	(E) $\frac{\sqrt{2}a^2}{2}$

29. Consider the triangle with vertices $(-6, 0)$, $(0, 12)$ and $(16, 0)$. How many points with integer coordinates lie on the sides of this triangle and form an obtuse triangle with the points $(0, 0)$ and $(5, 0)$?



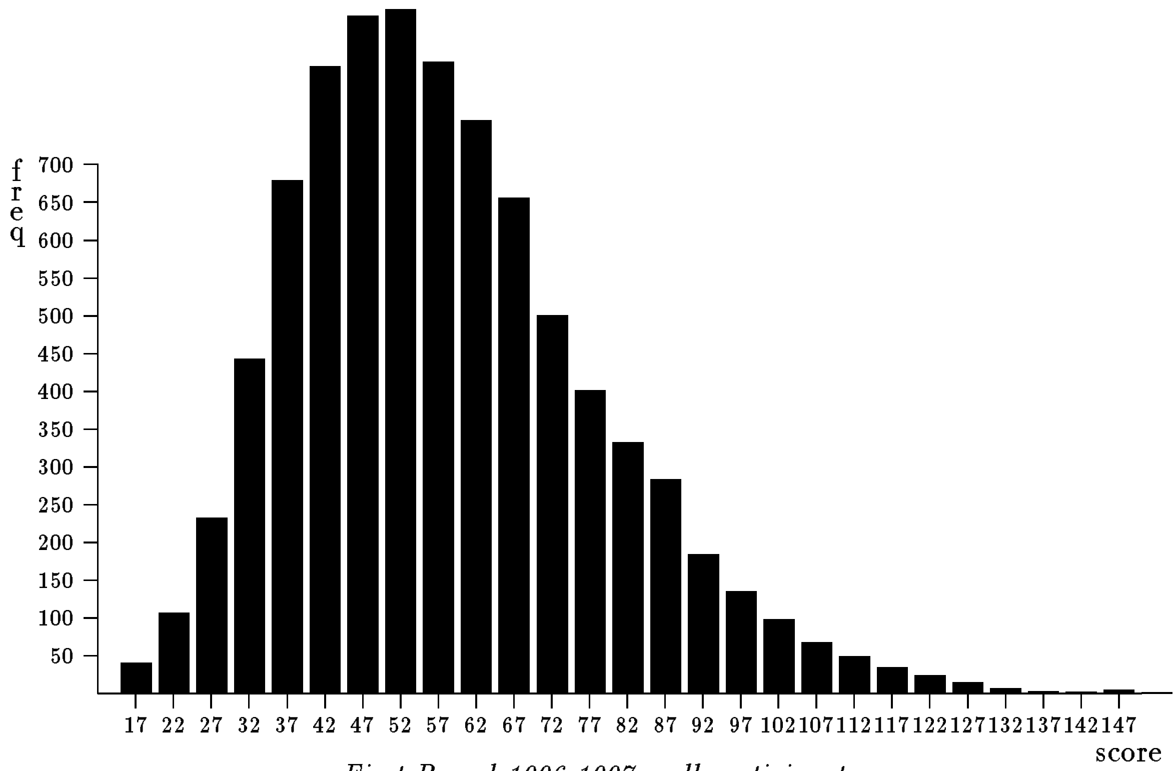
(A) 2	(B) 5	(C) 7	(D) 9	(E) 15

30. We choose a point m on the side $[bc]$ of a parallellogram $abcd$ such that $\overrightarrow{bm} = 0{,}1\overrightarrow{mc}$. We choose n on the side $[ad]$ such that $\overrightarrow{an} = 10\overrightarrow{nd}$. The lines bn and md intersect ac at the points e and f. The vector $\overrightarrow{ef}$ equals



(A) $\frac{1}{10}\overrightarrow{ac}$	(B) $\frac{1}{11}\overrightarrow{ac}$	(C) $\frac{1}{19}\overrightarrow{ac}$	(D) $\frac{1}{20}\overrightarrow{ac}$	(E) $\frac{1}{21}\overrightarrow{ac}$

1.2 Histogram of the first round.



First Round 1996-1997 : all participants.

1.3 Answer patterns of the First Round

Flanders Mathematics Olympiad 1997 – First Round Answer Patterns (all participants)							
Problem	Correct	A	B	C	D	E	Blank
1	E	9.05	19.19	17.83	2.05	44.73	7.04
2	A	34.73	6.28	2.51	8.69	13.73	33.97
3	E	6.02	12.77	13.43	2.04	55.01	10.51
4	C	1.73	3.43	83.93	3.23	0.68	6.76
5	D	11.82	10.68	14.91	32.62	4.30	25.61
6	C	4.85	2.99	27.96	6.47	5.62	52.05
7	A	12.96	45.12	29.12	6.09	2.01	4.53
8	A	38.59	22.11	0.77	2.44	23.51	12.50
9	C	3.56	45.42	19.77	17.31	4.44	9.30
10	D	0.16	0.65	2.09	93.33	1.75	1.84
11	C	2.84	4.34	16.16	9.08	5.55	62.01
12	E	22.45	8.31	11.48	6.41	12.47	38.76
13	C	9.30	8.86	35.63	6.93	7.09	32.15
14	E	2.94	13.77	4.60	18.43	40.65	19.50
15	C	2.64	3.56	30.74	8.61	15.17	39.19
16	D	4.22	3.52	4.49	17.81	8.96	60.96
17	D	12.40	22.45	3.79	15.84	4.70	40.46
18	E	17.18	9.46	27.86	3.10	32.65	9.48
19	C	2.77	4.32	50.95	8.68	6.06	26.18
20	B	17.66	25.29	4.98	27.77	10.30	13.85
21	B	3.39	6.76	6.94	9.21	7.19	66.49
22	D	4.87	8.15	10.81	21.56	1.53	53.04
23	C	2.79	10.87	74.77	7.91	1.44	2.04
24	D	8.50	3.66	9.78	13.60	11.92	52.39
25	B	2.94	8.55	1.20	16.26	2.16	68.80
26	D	3.75	5.35	2.83	20.31	2.75	64.95
27	B	1.71	44.64	8.75	6.48	7.30	30.19
28	C	14.39	7.93	14.04	6.79	8.91	47.89
29	C	4.72	9.06	23.61	7.47	16.65	38.45
30	E	5.31	6.76	12.86	12.92	10.81	51.18

2 Flanders Mathematics Olympiad 1996-1997: Second Round.

Scores are computed in the same way as in the first round but participants may only spend 2 hours to solve the problems.

2.1 The problems

1. The inverse of $3+\sqrt{10}$ equals

(A) $3-\sqrt{10}$	(B) $-3-\sqrt{10}$	(C) $\sqrt{10}+3$	(D) $\sqrt{10}-3$	(E) $\frac{1}{3}+\frac{1}{\sqrt{10}}$

2. The number $\sqrt{12^{12}}$ equals

(A) 6^6	(B) $(2\sqrt{3})^{12}$	(C) 6^{12}
(D) $12^{2\sqrt{3}}$	(E) none of the previous	

3. The space-diagonal of a cube measures 3. The volume of that cube then equals

(A) 1	(B) $\sqrt{3}$	(C) 3
(D) $3\sqrt{3}$	(E) none of the previous	

4. In which regular polygon does the shortest diagonal have same length as the radius of the circumscribed circle?

(A) 6-gon	(B) 8-gon	(C) 10-gon
(D) 12-gon	(E) 15-gon	

5. The triangle abc has $|bc| = 6$, $|ac| = 9$ and $\hat{a} = 30°$. Which of the following statements is correct?

(A) The angle $\hat{b}$ is acute.	(B) The angle $\hat{b}$ is obtuse.
(C) The angle $\hat{b}$ is rectangular.	(D) The angle $\hat{b}$ can be acute or obtuse.
(E) Such a triangle can not be constructed.	

6. Person A has 16 sapphires (of same value), person B has 10 rubies (of same value) and person C has 8 diamonds (also of same value). Each of them gives the other persons two of his original stones. Now they each have a collection of stones worth 270 000 belgian francs. What is the value of 1 diamond?

(A) 20 000 BEF	(B) 40 000 BEF	(C) 50 000 BEF
(D) 100 000 BEF	(E) 120 000 BEF	

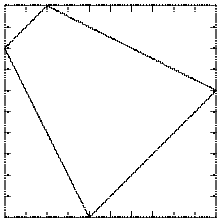
7. Compute $\left(\frac{1999}{6}\right)^2 - \left(\frac{1997}{6}\right)^2$.

(A) $\frac{1}{9}$	(B) $\frac{2}{3}$	(C) 111	(D) 222	(E) 1332

8. How many prime numbers can be written in two different ways as the sum of two prime numbers? The ordering of terms plays no role.

(A) 0	(B) 1	(C) 2	(D) 3	(E) more than 3

9. An isosceles trapezium is inscribed in a square with side 10 (see figure). The area of the trapezium equals



(A) 46	(B) 48	(C) 50	(D) 52	(E) 54

10. Suppose the equation $x^2 + ax + b = 0$ has two integer solutions. Consider the following statements.

I. both solutions are odd $\Longleftrightarrow$ a and b are odd
II. both solutions are odd $\Longleftrightarrow$ a is even and b is odd
III. both solutions are odd $\Longleftrightarrow$ a is odd and b is even
IV. one solution is even, the other odd $\Longleftrightarrow$ a is odd and b is even
V. one solution is even, the other odd $\Longleftrightarrow$ a is even and b is odd
VI. one solution is even, the other odd $\Longleftrightarrow$ a and b are odd

Which statements are correct?

(A) I and IV	(B) II and IV	(C) I and V
(D) III and V	(E) II and VI	

11. The set of solutions in $\mathbb{R}$ to the inequality $\sqrt{\dfrac{x^3+8}{x}} > x - 2$ equals

(A) $\mathbb{R}$	(B) $\mathbb{R}_0^+$	(C) $\mathbb{R}^+$
(D) $]-\infty,-2] \cup]0,+\infty[$	(E) $]-\infty,-2] \cup [2,+\infty[$	

12. The graph of a function $y = f(x)$, using orthonormal axes, is shifted parallelly with the first bissectrice over a distance of $\sqrt{2}$, such that x- and y-coordinates increase in each point. The new function is given by

(A) $y = f(x+1)+1$	(B) $y = f(x-1)+1$
(C) $y = f(x)+\sqrt{2}$	(D) $y = f(x+\sqrt{2})+\sqrt{2}$
(E) $y = f(x-\sqrt{2})+\sqrt{2}$	

13. How many real numbers differ exactly 1 with their inverse?

(A) 0	(B) 1	(C) 2	(D) 4	(E) infinitely many

14. For $x \in [0, \frac{\pi}{2}[$, the function $f(x) = \dfrac{\sin^2 x}{\cos x}$ takes the following number of integer values.

(A) 0	(B) 1
(C) 2	(D) finitely many, but more than 2
(E) infinitely many	

15. Let $a, b, c \in \mathbb{N}_0$. Division of a by b yields quotient q and remainder r, and division of q by c yields quotient q' and remainder r'. The division of a by bc then has remainder

(A) r	(B) r'	(C) rr'
(D) $br' + r$	(E) not defined from these data	

16. In how many ways does one can write $\frac{1}{14}$ as $\frac{a}{7} + \frac{b}{2}$, with a and b integers?

(A) 0	(B) 1	(C) 2	(D) 3	(E) more than 3

17. Which of the following inequalities is correct? (Observe that $a^{b^c} = a^{(b^c)}$.)

(A) $2^{3^2} < 2^{3^3} < 3^{2^2} < 3^{2^3} < 3^{3^2}$	(B) $3^{2^2} < 3^{2^3} < 2^{3^2} < 2^{3^3} < 3^{3^2}$
(C) $3^{2^2} < 3^{2^3} < 2^{3^2} < 3^{3^2} < 2^{3^3}$	(D) $3^{2^2} < 2^{3^2} < 3^{2^3} < 2^{3^3} < 3^{3^2}$
(E) $3^{2^2} < 2^{3^2} < 3^{2^3} < 3^{3^2} < 2^{3^3}$	

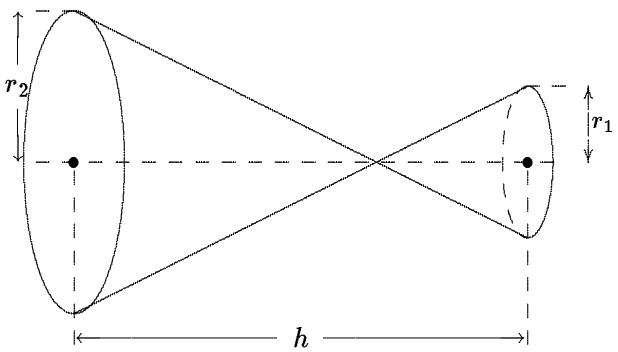
18. Let $A = \{a, b, c\}$. The number of mappings $f : A \to A$ that satisfy $f \circ f = f$ equals

(A) 1	(B) 4	(C) 7	(D) 10	(E) 13

19. You divide a piece of paper into five equal parts. You keep two parts, throw away one part, and repeat the same procedure with the two parts left. While infinitely repeating this procedure, the fraction of paper you keep will tend to

(A) $\frac{7}{15}$	(B) $\frac{2}{3}$	(C) $\frac{1}{2}$	(D) $\frac{3}{5}$	(E) $\frac{4}{5}$

20. The figure shows a rotational double cone. It's volume measures



(A) $\frac{h}{3}\pi(r_1^2 + r_2^2)$	(B) $\frac{h}{3}\pi(r_2^2 + r_2 r_1 + r_1^2)$
(C) $\frac{h}{3}\pi(r_2^2 - r_1 r_2 + r_1^2)$	(D) $\frac{h}{3}\pi(r_2 + r_1)^2$
(E) $\frac{h}{3}\pi(r_2 - r_1)^2$	

21. An, Bert and Chris each wear jeans of a different color (grey, blue or black). Moreover, one knows that the following statements are true

- If An wears the blue one, then Bert wears grey.
- If An wears the black one, then Chris wears grey.
- If Bert doesn't wear black, then Chris wears a blue one.

This implies

(A) Chris wears a blue jeans.
(B) Chris wears a grey jeans.
(C) Chris wears a black jeans.
(D) The statements contradict.
(E) The statements do not determine the color of Chris jeans, there is more than one possibility.

22. How many non-congruent triangles do exist such that each of its sides has integer length less or equal than 5?

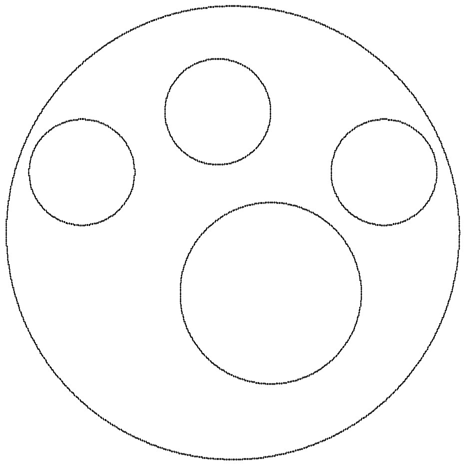
(A) 10	(B) 21	(C) 22	(D) 25	(E) 35

23. The unknowns a, b and c are natural numbers. How many solutions does the following system of equations have?

$$\begin{cases} a^3 - b^3 - c^3 &= 3abc \\ a^2 &= 2(b+c) \end{cases} ?$$

(A) 1	(B) 2	(C) 3	(D) 4	(E) infinitely

24. The 'rugo' is the traditional housing in Burundi. It consists of a circular hedge which encloses a domain with four huts: one for the parents (the family hut) and three smaller ones: the boy's hut, the girl's hut and the washing hut. All huts are circular and the diameter of the smaller ones equals half the diameter of the family hut. As Burundese people spend most of their time outside, they want to maximalize the open area inside the hedge. Let D be the diameter of the domain. What is the diameter of the family hut, if $\frac{2}{3}$ of the domain is to be kept free?



(A) $\frac{\sqrt{3}}{\sqrt{14}}D$	(B) $\frac{\sqrt{2}}{\sqrt{21}}D$	(C) $\frac{2}{\sqrt{21}}D$	(D) $\frac{2\sqrt{2}}{\sqrt{21}}D$	(E) $\frac{2}{\sqrt{15}}D$

25. Let a, b and c be the lengths of the medians of a rectangular triangle, such that $a \geq b \geq c$. Now $\frac{a^2+b^2}{c^2}$ equals

(A) 2	(B) $\frac{5}{2}$	(C) 3	(D) 4	(E) 5

26. René gets the following question in his math test:

> *Fill out on the dots, $||$ or $\perp$, such that the following expressions hold for all lines A, B and C in the plane.*
>
> *(a)* $A \ldots A$
>
> *(b)* $A \,||\, B$ *and* $B \ldots C \Longrightarrow A \ldots C$

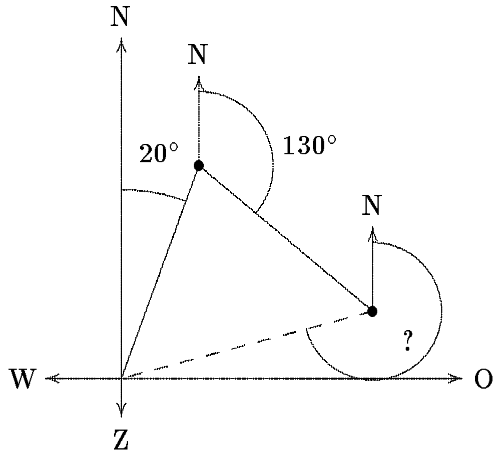
René didn't prepare his test and takes a random guess. What is the probability of René having a correct test?

(A) 0	(B) $\frac{1}{8}$	(C) $\frac{1}{6}$	(D) $\frac{1}{4}$	(E) $\frac{1}{2}$

27. If $\tan x = \sqrt[3]{\dfrac{b}{a}}$, with $a, b \in \mathbb{R}_0^+$ and $x \in]0, \dfrac{\pi}{2}[$, then $\dfrac{a}{\cos x} + \dfrac{b}{\sin x}$ equals

(A) $(a^3 + b^3)^{1/3}$	(B) $(a^{3/2} + b^{3/2})^{2/3}$	(C) $a + b$
(D) $(a^{2/3} + b^{2/3})^{3/2}$	(E) $(a^{1/3} + b^{1/3})^3$	

28. An expedition leaves it's base and walks 10 km in 20° NE direction. After this it walks 10 km again, but in 130° SE direction. In which direction (expressed in degrees) does the expedition have to go in order to return to the base in a straight line?

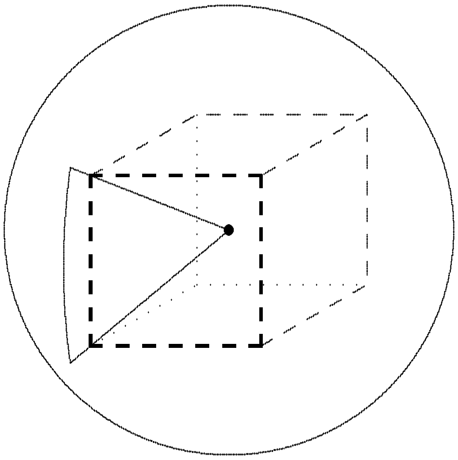


(A) 235°	(B) 240°	(C) 250°	(D) 255°	(E) 260°

29. Consider all integers consisting of n digits, each chosen from the set $\{1, 2, 3, 4\}$, such that no digit 3 appears to the right of a digit 4. E.g., when n is 6, the integers 123314 and 113424 satisfy, while 114234 doesn't. Let a_n be the number of such n-digit integers. Which expression holds for all $n \in \mathbb{N}_0$?

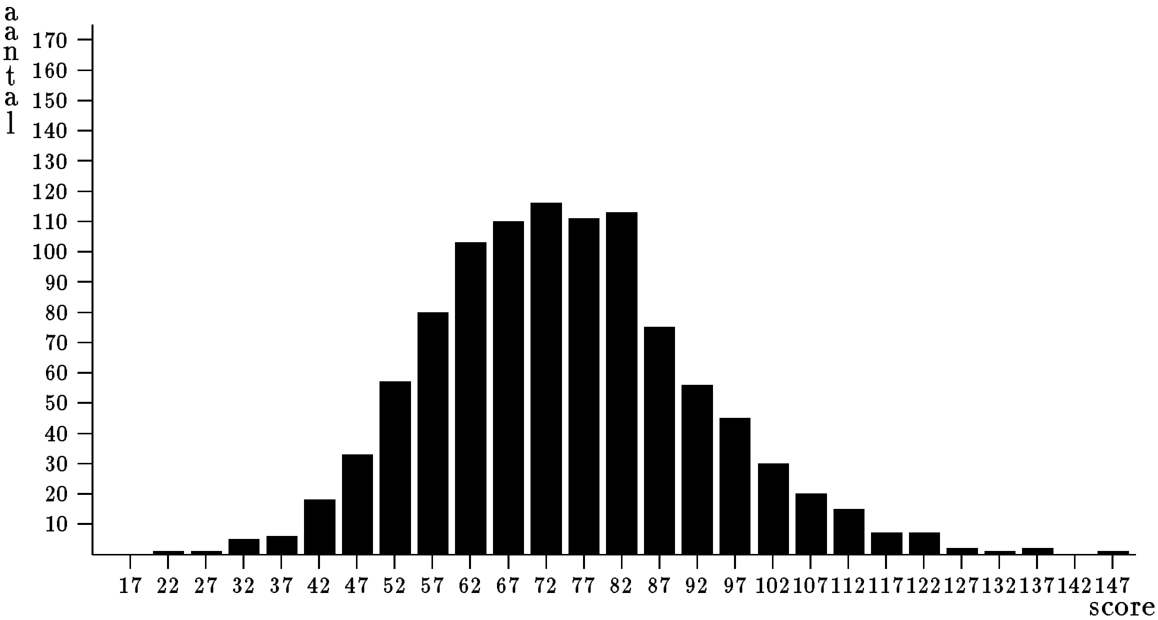
(A) $a_{n+1} = 4a_n - 1$	(B) $a_{n+1} = 4a_n - 6^{n-1}$
(C) $a_{n+2} = 2a_{n+1} + 6a_n$	(D) $a_{n+2} = 4a_{n+1} - \dfrac{3}{2}a_n$
(E) $a_{n+1} = 3a_n + 3^n$	

30. Consider a sphere of radius R and a cube with the same centre. The twelve edges of the cube are projected from the centre onto the surface of the sphere. This yields twelve arcs that divide the surface of the sphere into six congruent parts. Let α and β be the acute angles that satisfy $\sin\alpha = \frac{\sqrt{3}}{3} = \cos\beta$. The perimeter of each part equals



(A) $4R\alpha$	(B) $4R\beta$	(C) $4R(\alpha+\beta)$
(D) $8R\alpha$	(E) $8R\beta$	

2.2 Histogram of the second round.



Second Round 1996-1997 : all participants

2.3 Answer patterns of the Second Round

Flanders Mathematics Olympiad 1997 – Second Round Answer patterns (all participants)							
Problem	Correct	A	B	C	D	E	Blank
1	D	2.07	5.52	0.89	70.34	15.86	5.32
2	B	0.10	75.86	0.59	1.28	21.28	0.89
3	D	10.64	7.88	2.56	58.62	8.77	11.53
4	D	10.34	2.27	2.86	52.02	2.36	30.05
5	D	18.82	13.50	1.28	54.78	2.36	9.26
6	C	1.28	4.53	72.32	3.65	0.89	17.34
7	D	0.59	0.20	4.24	83.05	5.91	6.01
8	A	55.67	2.66	1.58	2.17	11.63	26.21
9	B	3.05	86.70	1.18	2.07	1.38	5.62
10	B	3.74	43.25	1.67	3.45	4.14	43.74
11	D	3.35	10.15	1.87	60.10	9.16	15.37
12	B	18.13	41.58	3.74	2.36	5.52	28.67
13	D	20.00	4.63	28.37	32.41	1.97	12.61
14	E	4.33	33.10	11.63	4.04	16.75	30.15
15	D	2.66	2.36	2.96	43.15	14.88	33.99
16	E	5.22	2.17	3.94	0.30	78.82	9.56
17	E	2.07	0.89	2.46	5.62	68.87	20.10
18	D	11.72	7.29	4.24	2.76	1.18	72.81
19	B	5.12	45.32	8.08	13.89	4.33	23.25
20	C	7.68	3.74	19.70	3.05	3.55	62.27
21	A	17.54	1.28	3.35	53.40	15.17	9.26
22	C	7.78	7.49	9.26	5.81	8.37	61.18
23	D	3.55	3.15	3.35	2.17	14.58	73.10
24	C	1.08	4.53	49.95	9.06	3.55	31.82
25	E	2.76	3.94	3.55	3.65	32.12	53.99
26	D	0.20	38.13	4.04	43.94	2.36	11.23
27	D	1.28	2.96	1.48	7.00	0.99	86.31
28	D	3.25	2.76	4.93	56.06	4.53	28.47
29	E	1.58	2.76	2.17	2.66	5.81	85.02
30	D	2.07	1.87	3.25	2.36	1.18	89.26

3 Flanders Mathematics Olympiad 1996-1997: the Final Round

1. Write 1997 as a sum of natural numbers in such a way that the product of these numbers is as big as possible. Prove that your solution is the best one.

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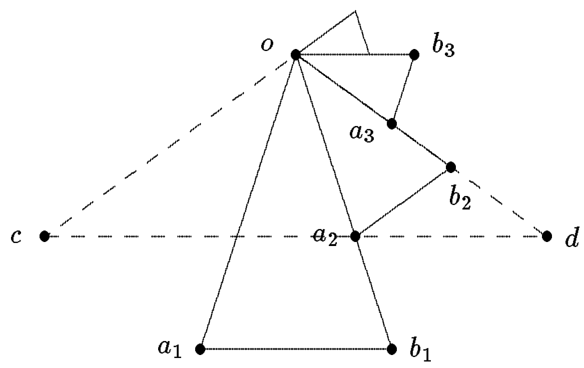
2. With respect to an orthonormal basis we consider the curves determined by the equations
$$x^2 + y^2 = r^2 \text{ (with } r \in \mathbb{R}_0^+\text{) and } (xy)^2 = 1.$$
Let us write F_r for the convex polygon having the intersection points of these curves (assuming that there are intersection points) as vertices.
   (a) Find an expression $f(r)$ which describes the area of the polygon F_r.
   (b) For which values of r is F_r a regular polygon?

---

3. Assume that ℓ is a given length. Consider the isosceles triangle $\Delta_1 = oa_1b_1$, having top-angle $\widehat{a_1ob_1} = 36°$ and two sides of length ℓ.
On the line ob_1 we take a point a_2 such that $|oa_2| = |a_1b_1|$ and we construct - on the outer side of triangle Δ_1 - a second isosceles triangle $\Delta_2 = oa_2b_2$ having top-angle $\widehat{a_2ob_2} = 36°$ (see figure).
We continue working in this direction and constructing a third isosceles triangle $\Delta_3 = oa_3b_3$ with top-angle $\widehat{a_3ob_3} = 36°$ and $|oa_3| = |a_2b_2|$.
Keep repeating this procedure.
Prove that the area of the union of all the triangles, constructed in this way, is smaller than or equal to the area of the isosceles triangle $\Delta = ocd$ with top-angle $\widehat{cod} = 108°$ and two sides $|oc| = |od| = \ell$.



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4. Consider thirteen birds arriving (and sitting) on a flat surface in such a way that from each 5-tuple of these birds it is known that 4 birds are sitting on a circle.
   (a) Prove that there exists a circle such that at least 6 birds are sitting on it.
   (b) Can this result be sharpened?
