# 8 Flanders Mathematics Olympiad 1997-1998: First Round.

The first round consists of 30 multiple choice problems which are selected by the FMO jury. Scores are computed as follows: a correct answer yields 5 points, a blanc answer 1 point and no points are given for a wrong answer. Participants may spend 3 hours solving the problems.

## 8.1 The problems

The following approximate values may be useful.

$\sqrt{2} \approx 1,4142$ ; $\sqrt{3} \approx 1,7321$ ; $\sqrt{5} \approx 2,2361$ ; $\sqrt{7} \approx 2,6458$ ; $\pi \approx 3,1416$

1. Consider the following table. The sum of all numbers of line 1 up to and including line 1998 equals

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| Line 1 | | | | | 1 | | | | |
| Line 2 | | | | 1 | | -1 | | | |
| Line 3 | | | 1 | | -1 | | 1 | | |
| Line 4 | | 1 | | -1 | | 1 | | -1 | |
| Line 5 | 1 | | -1 | | 1 | | -1 | | 1 |
| ... | | | | | ... | | | | |

(A) 0 (B) 1 (C) 999 (D) 1998 (E) 1999

2. At a university there are $S$ students and $P$ professors. Which of the following relations expresses that there are 6 times more students than professors?

(A) $S + 6 = P$ (B) $S - 6 = P$ (C) $6S = P$
(D) $6S \geq P$ (E) none of the preceding

3. $\sqrt{25^{4a^2}} =$

(A) $25^{2a}$ (B) $25^{2|a|}$ (C) $25^{2a^2}$ (D) $5^{2|a|}$ (E) $5^{2a^2}$

4. Which of the following amounts is different from the other amounts?

(A) 10% of 5000 BEF (B) 20% of 2500 BEF (C) 25% of 2000 BEF
(D) 33% of 1500 BEF (E) 40% of 1250 BEF

5. Diophantus was a happy child during 1/6 of his life. After another 1/12 he began to grow a beard. Then it lasted only 1/7 before he got married. In the fifth year after his marriage he got a son, who died 4 years before his father did and who became but half as old as his father. Which age did Diophantus reach?

(A) 44 years (B) 54 years (C) 64 years (D) 74 years (E) 84 years

6. The following grid is filled up to a Latin square (i.e. every number 1, 2, 3 and 4 occurs exactly once in every line and in every column). Which number occurs in the square with the question mark?

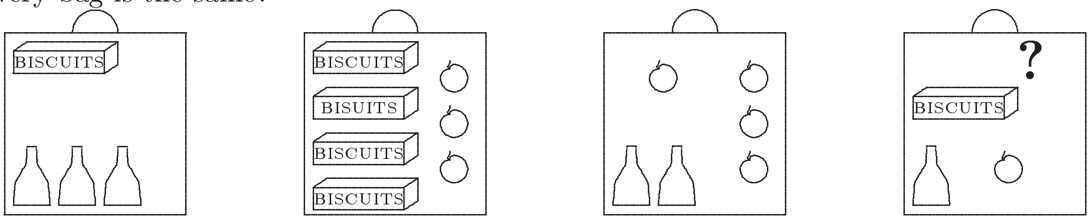
| | | | |
|---|---|---|---|
| 1 | 2 | | |
| | | | |
| ? | 3 | | |
| | | 4 | |

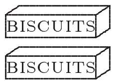
| | | |
|---|---|---|
| (A) 1 | (B) 2 | (C) 3 |
| (D) 4 | (E) several answers are possible | |

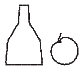
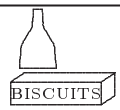
7. The number of solutions in $\mathbb{R}$ of the equation $x^2 + \dfrac{1}{x^2} = \dfrac{1}{1998^2} + 1998^2$ equals

| | | | | |
|---|---|---|---|---|
| (A) 0 | (B) 1 | (C) 2 | (D) 4 | (E) 8 |

8. What do we have to put in the fourth shopping bag so that the price of the contents of every bag is the same?




(A)  (B)  (C) 

(D)  (E) 


9. The number of intersections of the graph of $f : [0, 1000\pi] \to \mathbb{R} : x \mapsto \cos x$ and the graph of $g : [0, 1000\pi] \to \mathbb{R} : x \mapsto \sin x$ equals

| | | |
|---|---|---|
| (A) 500 | (B) 1000 | (C) 1500 |
| (D) 2000 | (E) infinitely many | |

10. How many natural numbers consisting of two digits equal the sum of their digits added up with the product of their digits?

| | | | | |
|---|---|---|---|---|
| (A) 5 | (B) 6 | (C) 7 | (D) 8 | (E) 9 |

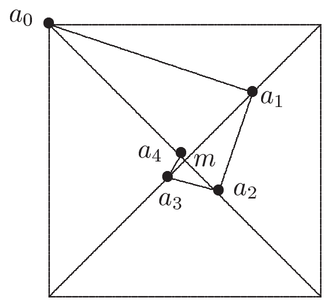
11. In an acute-angled triangle the smallest angle measures 1/5 of the largest one. Each angle measures an integer number of degrees. The sum of the two largest angles equals

| | | |
|---|---|---|
| (A) 157° | (B) 160° | (C) 163° |
| (D) 166° | (E) several solutions are possible | |

12. Paul has got a collection of coins. If he places the coins in piles of 6, then 3 coins are remaining. When the coins are placed in piles of 8, 7 coins are remaining and in piles of 5, 4 coins are remaining. If the number of coins $n$ is less than 100, then:

| | | |
|---|---|---|
| (A) $n \leq 20$ | (B) $20 < n \leq 40$ | (C) $40 < n \leq 60$ |
| (D) $60 < n \leq 80$ | (E) $80 < n$ | |

13. Consider a square with centre $m$ and with a diagonal of length 2. We construct a "spiral" $a_0a_1a_2a_3\ldots$ as shown in the picture. The distance from $a_{n+1}$ to $m$ is half of the distance from $a_n$ to $m$. The total length of the spiral equals




| | | | | |
|---|---|---|---|---|
| (A) 2 | (B) $\sqrt{5}$ | (C) 4 | (D) 8 | (E) infinity |

14. Consider a bar code (known as EAN-13). The right most digit, $c$, is a (so-called) check digit. One computes $c$ by subtracting the last digit of

$$(a_1 + a_3 + \ldots + a_{11}) + 3(a_2 + a_4 + \ldots + a_{12})$$

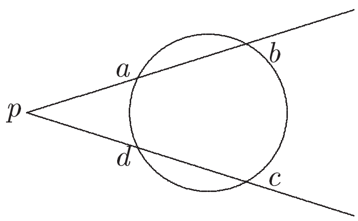
from 10. The left most digit of the bar code shown here is unreadable due to water damage. That digit equals

? 4 1 0 0 5 6 0 1 1 5 7 5

$a_1$ $a_2$ $a_3$ $a_4$ $a_5$ $a_6$ $a_7$ $a_8$ $a_9$ $a_{10}$ $a_{11}$ $a_{12}$ $c$

| | | | | |
|---|---|---|---|---|
| (A) 1 | (B) 2 | (C) 3 | (D) 4 | (E) 5 |

15. A circle is divided into four arcs. Each of the three adjoining arcs $\overset{\frown}{ab}$, $\overset{\frown}{bc}$ and $\overset{\frown}{cd}$ measures 100° (see figure). The angle $\hat{p}$ formed by the straight lines $ab$ and $dc$ measures




| | | | | |
|---|---|---|---|---|
| (A) 15° | (B) 20° | (C) 25° | (D) 30° | (E) 40° |

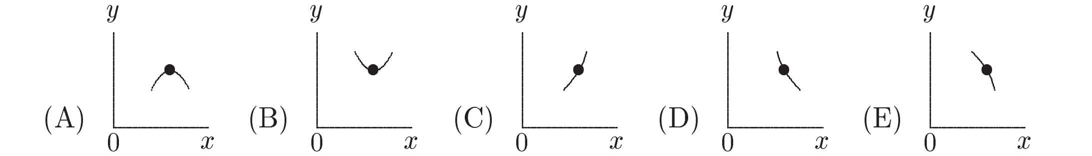
16. Consider the real function $f(x) = |x+1| + |x-3|$ and the following four statements:

- $f$ is strictly increasing over $\mathbb{R}^+$,
- $f$ is strictly decreasing over $\mathbb{R}^-$,
- $f$ is constant over an interval,
- $f$ is strictly positive over its domain.

How many of these statements are correct?

| (A) 0 | (B) 1 | (C) 2 | (D) 3 | (E) 4 |
|---|---|---|---|---|

17. Which picture shows part of the graph of $y = (10 - x)^2 + 10$? (each time • is the point with coordinates $(10, 10)$)




18. How many of the following six polynomials are a divisor of the polynomial $x^7 - x$?

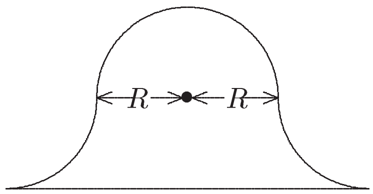
$$x^2 + x + 1, \quad x^3 - 1, \quad x^2 - 1, \quad x^4 + x^2 + 1, \quad x^4 + x, \quad x^2 - x$$

| (A) 2 | (B) 3 | (C) 4 | (D) 5 | (E) 6 |
|---|---|---|---|---|

19. The page numbers in a book counting 320 pages are cut into single digits. These digits are put into a bag. Determine the probability that, upon picking one digit out of the bag, the digit will be 1.

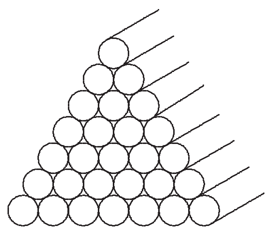
| (A) $\frac{1}{10}$ | (B) $\frac{11}{100}$ | (C) $\frac{43}{213}$ | (D) $\frac{43}{240}$ | (E) $\frac{40}{213}$ |
|---|---|---|---|---|

20. The picture shows a cross section of a tunnel, formed by four identical quarter-circles and a horizontal bottom. Determine the area of this cross-section.




| (A) $2R^2$ | (B) $\pi R^2$ | (C) $4R^2$ | (D) $6R^2$ | (E) $2\pi R^2$ |
|---|---|---|---|---|

21. 28 tubes are piled as in the picture. The diameter of each tube equals 10 cm. The best approximation of the height of the pile is



| (A) 51 cm | (B) 52 cm | (C) 61 cm | (D) 62 cm | (E) 70 cm |
|---|---|---|---|---|

22. How many integers ($100 < n < 1000$), can not be divided by 2 nor by 5?

| | | | | |
|---|---|---|---|---|
| (A) 270 | (B) 360 | (C) 540 | (D) 630 | (E) 810 |

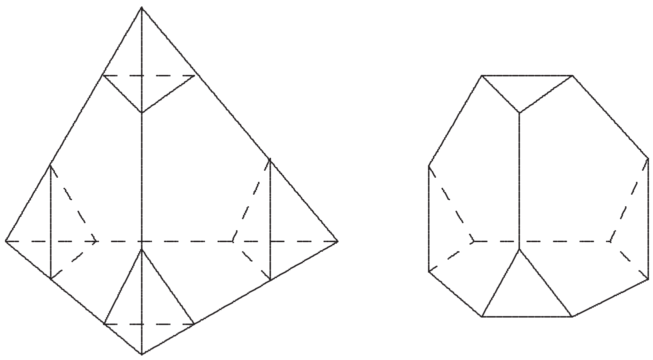
23. Consider the following three statements about the real function $f(x) = \sqrt{\dfrac{1}{x} - 1}$.

**I.** $f$ is defined for all $x$ greater than 0.
**II.** $f$ is defined for some negative values of $x$.
**III.** $f$ takes all positive real numbers as value.

The correct statements are:

| | | | | |
|---|---|---|---|---|
| (A) I | (B) II | (C) III | (D) I and III | (E) II and III |

24. The four vertices of a regular tetrahedron $V$ are cut off, such that the remaining body $L$ has eight faces: four regular triangles and four regular hexagons (see figure). The ratio of the area of $L$ to the area of $V$ equals

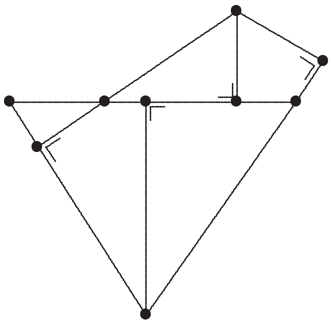


| | | | | |
|---|---|---|---|---|
| (A) $\frac{1}{3}$ | (B) $\frac{1}{2}$ | (C) $\frac{2}{3}$ | (D) $\frac{7}{9}$ | (E) 1 |

25. The altitudes of an acute-angled triangle $abc$ intersect the circumscribed circle in the points $a'$, $b'$ and $c'$. These altitudes correspond with the following special lines of the triangle $a'b'c'$.

| | |
|---|---|
| (A) the altitudes | (B) the medians |
| (C) the bisectors | (D) the perpendicular bisectors |
| (E) none of the preceding | |

26. The number of circles going through at least four of the points denoted with • in the picture, equals



| | | | | |
|---|---|---|---|---|
| (A) 0 | (B) 1 | (C) 2 | (D) 3 | (E) 4 or more |

27. Four cards are lying on a table. Each card carries a character on one side and a digit on the other side. The following is visible

| | | | |
|---|---|---|---|
| $A$ | $K$ | 4 | 7 |

We want to verify the following statement:

> "If there is a vowel on one side of a card, then there is an even number on the other side."

Which cards do we certainly have to turn to examine the statement?

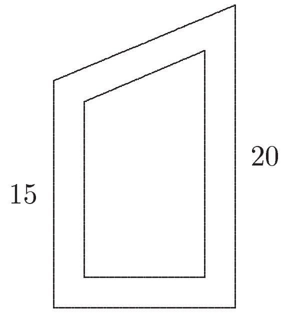
| (A) A | (B) A and 4 | (C) A and 7 | (D) A, 4 and 7 | (E) all cards |
|---|---|---|---|---|

28. The number of the following five equations having at least one solution in $\mathbb{N}_0$ equals

$$
\begin{aligned}
\sqrt{x} &= \sqrt{x} \\
\sqrt{x}.\sqrt{x} &= \sqrt{x}+\sqrt{x} \\
\sqrt{x}.\sqrt{x}.\sqrt{x} &= \sqrt{x}+\sqrt{x}+\sqrt{x} \\
\sqrt{x}.\sqrt{x}.\sqrt{x}.\sqrt{x} &= \sqrt{x}+\sqrt{x}+\sqrt{x}+\sqrt{x} \\
\sqrt{x}.\sqrt{x}.\sqrt{x}.\sqrt{x}.\sqrt{x} &= \sqrt{x}+\sqrt{x}+\sqrt{x}+\sqrt{x}+\sqrt{x}
\end{aligned}
$$

| (A) 1 | (B) 2 | (C) 3 | (D) 4 | (E) 5 |
|---|---|---|---|---|

29. A piece of land having the shape of a right-angled trapezium, has an area of 210 $m^2$ and has bases of 15 $m$ and 20 $m$. Building on the piece of land is restricted to the trapezium with sides parallel to, and on a distance of 2 $m$ from the sides of the given trapezium (see picture). The area of this inner trapezium (measured in $m^2$) equals




| (A) $\frac{160}{3}$ | (B) $\frac{320}{3}$ | (C) $\frac{324}{3}$ | (D) $\frac{372}{3}$ | (E) $\frac{400}{3}$ |
|---|---|---|---|---|

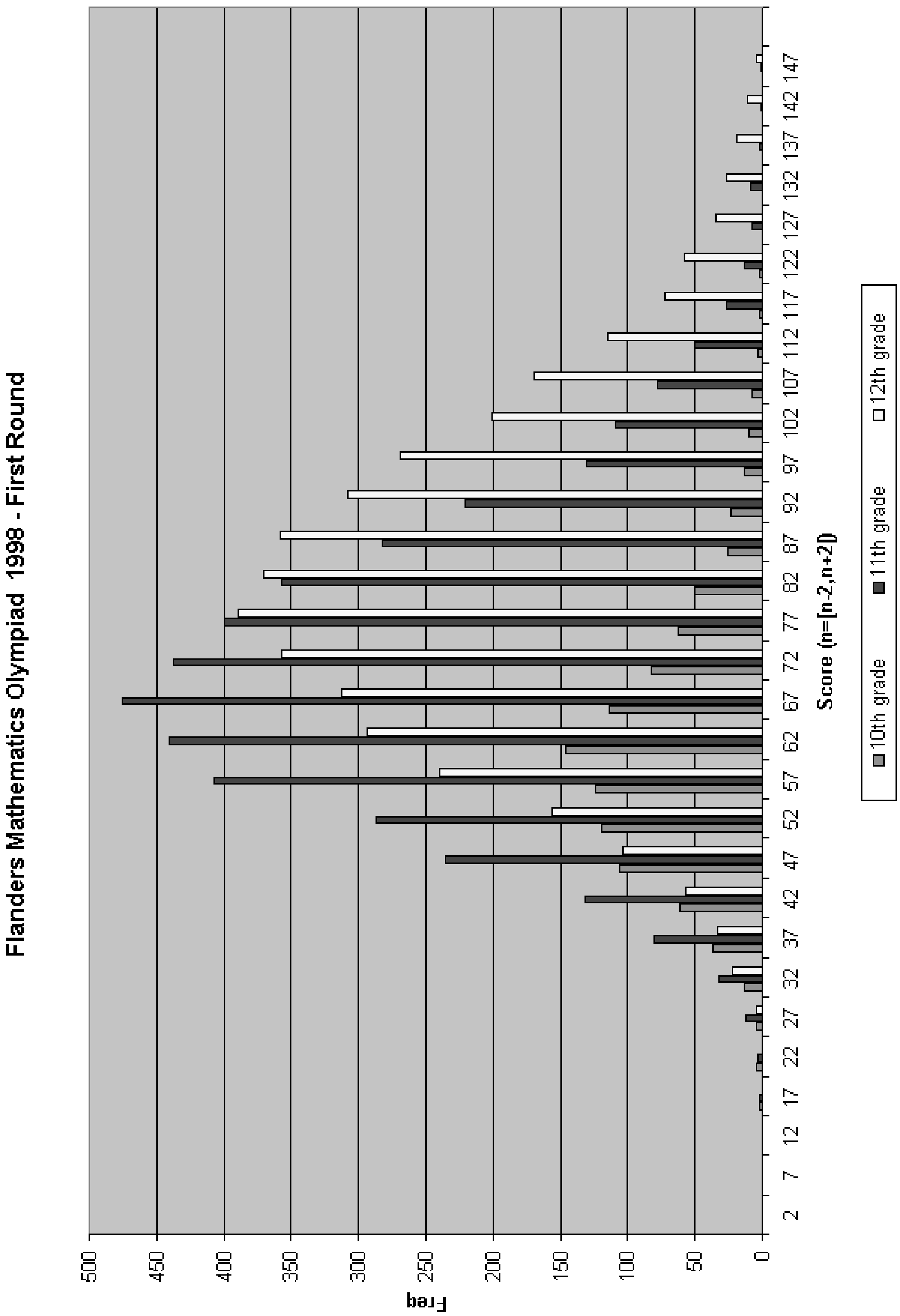
30. I am leaving for a meeting 15 minutes too late according to my watch, which runs ahead 5 minutes. The meeting starts 10 minutes too late. I arrive ...

(A) just in time
(B) 10 minutes early
(C) 10 minutes late
(D) 20 minutes late
(E) 30 minutes late

## 8.2 Answer patterns of the First Round.

| Flanders Mathematics Olympiad 1998 – First Round<br>Answer Patterns (all participants, i.e. 9298) | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| Problem | Correct | A | B | C | D | E | Correct | Wrong | Blank |
| 1 | C | 5.99 | 1.97 | 86.18* | 1.60 | 0.70 | 86.18 | 10.84 | 2.98 |
| 2 | E | 0.12 | 0.20 | 27.33 | 3.14 | 67.28* | 67.28 | 31.71 | 1.01 |
| 3 | C | 15.05 | 14.58 | 50.84* | 11.77 | 4.08 | 50.84 | 45.56 | 3.60 |
| 4 | D | 0.44 | 1.60 | 0.69 | 91.02* | 4.83 | 91.02 | 7.89 | 1.09 |
| 5 | E | 2.04 | 4.40 | 6.54 | 4.68 | 47.46* | 47.46 | 17.72 | 34.81 |
| 6 | D | 0.49 | 2.19 | 0.54 | 87.49* | 7.25 | 87.49 | 10.62 | 1.89 |
| 7 | D | 5.18 | 12.33 | 32.03 | 26.81* | 0.74 | 26.81 | 50.40 | 22.79 |
| 8 | E | 2.80 | 3.80 | 3.70 | 5.64 | 46.82* | 46.82 | 16.02 | 37.16 |
| 9 | B | 6.18 | 40.90* | 2.42 | 12.22 | 7.98 | 40.90 | 28.86 | 30.24 |
| 10 | E | 2.47 | 1.43 | 1.28 | 3.31 | 64.70* | 64.70 | 8.53 | 26.77 |
| 11 | C | 2.46 | 8.03 | 18.11* | 1.48 | 39.84 | 18.11 | 51.88 | 30.01 |
| 12 | B | 3.52 | 48.95* | 2.38 | 3.11 | 5.78 | 48.95 | 14.82 | 36.23 |
| 13 | B | 12.49 | 25.36* | 7.74 | 1.48 | 18.33 | 25.36 | 40.11 | 34.53 |
| 14 | A | 61.48* | 7.69 | 3.68 | 2.39 | 4.81 | 61.48 | 18.66 | 19.86 |
| 15 | B | 1.65 | 24.26* | 3.65 | 12.55 | 6.98 | 24.26 | 24.87 | 50.87 |
| 16 | C | 3.04 | 16.49 | 40.08* | 10.35 | 1.63 | 40.08 | 31.56 | 28.36 |
| 17 | B | 3.91 | 73.64* | 4.51 | 5.00 | 1.85 | 73.64 | 15.42 | 10.94 |
| 18 | E | 12.07 | 9.10 | 9.34 | 8.92 | 18.90* | 18.90 | 39.45 | 41.65 |
| 19 | C | 3.38 | 4.24 | 33.04* | 7.08 | 5.23 | 33.04 | 20.22 | 46.74 |
| 20 | C | 4.32 | 7.77 | 44.47* | 2.37 | 10.32 | 44.47 | 24.90 | 30.63 |
| 21 | D | 1.89 | 6.67 | 33.08 | 25.65* | 15.62 | 25.65 | 57.38 | 16.97 |
| 22 | B | 8.91 | 57.54* | 8.40 | 3.48 | 6.76 | 57.54 | 27.67 | 14.79 |
| 23 | C | 10.08 | 4.90 | 51.02* | 12.05 | 3.12 | 51.02 | 30.24 | 18.74 |
| 24 | D | 5.23 | 2.51 | 14.97 | 25.88* | 1.33 | 25.88 | 24.08 | 50.04 |
| 25 | C | 10.68 | 12.52 | 33.62* | 4.68 | 8.44 | 33.62 | 36.38 | 30.00 |
| 26 | E | 24.65 | 9.02 | 6.42 | 4.48 | 4.59* | 4.59 | 44.64 | 50.76 |
| 27 | C | 19.34 | 22.55 | 17.80* | 10.20 | 20.20 | 17.80 | 72.43 | 9.77 |
| 28 | C | 12.99 | 31.62 | 19.72* | 4.71 | 6.97 | 19.72 | 56.36 | 23.92 |
| 29 | B | 1.76 | 9.24* | 27.60 | 7.08 | 3.57 | 9.24 | 40.04 | 50.72 |
| 30 | A | 86.26* | 0.71 | 8.66 | 1.22 | 0.33 | 86.26 | 10.95 | 2.80 |

### 8.3 Histogram of the first round.



## 9 Flanders Mathematics Olympiad 1997-1998: Second Round.

Scores are computed in the same way as for the first round. Participants may spend 2 hours solving the problems.

### 9.1 The problems

1. A mother says: “All of my children have at least one brother and at least one sister ”. How many children does she have at least?

| | | | | |
|---|---|---|---|---|
| (A) 2 | (B) 3 | (C) 4 | (D) 5 | (E) 6 |

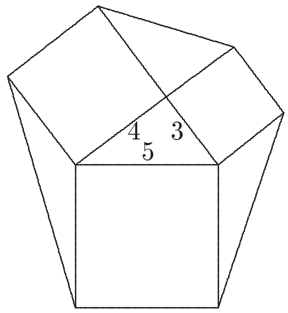
2. The number of different integer values $3 - 3\,|\cos x|$ can take equals

| | | | | |
|---|---|---|---|---|
| (A) 2 | (B) 3 | (C) 4 | (D) 6 | (E) 7 |

3. $\lfloor a \rfloor$ stands for the greatest integer not exceeding $a$. For example $\lfloor 1998 \rfloor = 1998$, $\lfloor \pi \rfloor = 3$ and $\lfloor -\pi \rfloor = -4$. The set of solutions in $\mathbb{R}$ of the equation $\lfloor 2x \rfloor = 3$ is

| | | | | |
|---|---|---|---|---|
| (A) $[1, \frac{3}{2}[$ | (B) $[\frac{3}{2}, 2[$ | (C) $[2, 3[$ | (D) $[3, 4[$ | (E) $[6, 8[$ |

4. A hexagon is divided into four triangles and three squares (see picture). The sides of the inside triangle have a length of 3, 4 and 5. How many of these seven figures have an area of 6?




| | | | | |
|---|---|---|---|---|
| (A) 1 | (B) 2 | (C) 3 | (D) 4 | (E) none |

5. In how many ways can the area of a rectangle be bisected by a straight line?

| | | | | |
|---|---|---|---|---|
| (A) 2 | (B) 4 | (C) 6 | (D) 8 | (E) more than 8 |

6. Twelve positive numbers are numbered. The fourth number is 4, the twelfth number is 12. The sum of three successive numbers equals 333. The seventh number equals

| | | |
|---|---|---|
| (A) 4 | (B) 7 | (C) 12 |
| (D) 317 | (E) not defined from these data | |

7. Which of the following five numbers is the smallest one?

(A) $\sqrt[30]{30}$ (B) $\sqrt[6]{2}$ (C) $\sqrt[10]{3}$ (D) $\sqrt[12]{4}$ (E) $\sqrt[15]{5}$

8. Which of the following sign tables is *not* the sign table of a polynomial function of the third degree?

(A)
$$\begin{array}{c|ccccccc} x & & x_1 & & x_2 & & x_3 & \\ \hline f(x) & - & 0 & + & 0 & - & 0 & + \end{array}$$

(B)
$$\begin{array}{c|ccccc} x & & x_1 & & x_2 & \\ \hline f(x) & + & 0 & + & 0 & - \end{array}$$

(C)
$$\begin{array}{c|ccccc} x & & x_1 & & x_2 & \\ \hline f(x) & - & 0 & - & 0 & + \end{array}$$

(D)
$$\begin{array}{c|ccc} x & & x_1 & \\ \hline f(x) & - & 0 & + \end{array}$$

(E)
$$\begin{array}{c|ccc} x & & x_1 & \\ \hline f(x) & - & 0 & - \end{array}$$

9. Consider the following statement about the natural numbers:

$$\forall\, x \text{ even: } x^2 + x \text{ is even.}$$

The negation of this statement is:

(A) $\forall\, x$ even: $x^2 + x$ is odd .
(B) $\forall\, x$ odd : $x^2 + x$ is even.
(C) $\forall\, x$ odd : $x^2 + x$ is odd .
(D) $\exists\, x$ even: $x^2 + x$ is odd .
(E) $\exists\, x$ odd : $x^2 + x$ is odd .

10. $\dfrac{\cos^3 15^\circ + \sin^3 15^\circ}{\cos 15^\circ + \sin 15^\circ}$ equals

(A) $0$ (B) $\frac{1}{4}$ (C) $\frac{1}{2}$ (D) $\frac{3}{4}$ (E) $1$

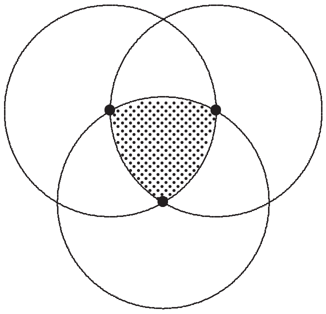
11. The number of natural numbers $n$ for which the statement $2^n > n^3$ is incorrect equals

(A) 2 (B) 4 (C) 6 (D) 8 (E) more than 8

12. A cyclist rides up a hill with a velocity of 16 km/h. He returns immediately, following the same route, with a velocity of 48 km/h. The average velocity (expressed in km/h) during the whole route is

(A) 12 (B) 24
(C) 30 (D) 32
(E) not defined because the covered route is unknown

13. Three circles of radius 1, intersect such that each circle passes through the midpoints of the other two circles. What is the area of the shaded region (see picture)?

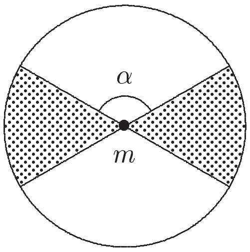


| | | |
|---|---|---|
| (A) $\frac{3\pi}{4} - \sqrt{3}$ | (B) $\frac{\pi - \sqrt{3}}{2}$ | (C) $\frac{\pi - 1}{2}$ |
| (D) $\frac{\pi + 1}{2}$ | (E) none of the previous | |

14. The equation $\sqrt{x-p} = x$ has got two different real roots if and only if $p$ belongs to

| | | |
|---|---|---|
| (A) $]-\infty, 0]$ | (B) $]-\infty, \frac{1}{4}[$ | (C) $]-\infty, \frac{1}{4}]$ |
| (D) $[0, \frac{1}{4}[$ | (E) $]\frac{1}{4}, +\infty[$ | |

15. The shaded region has got the same perimeter as the given circle with midpoint $m$. The angle $\alpha$ (measured in radians) equals




| | | | | |
|---|---|---|---|---|
| (A) 2 | (B) 3 | (C) $\frac{\pi}{2}$ | (D) $\frac{2\pi}{3}$ | (E) $\frac{3\pi}{4}$ |

16. The number of different real solutions of the equation $4 - x^2 = \frac{1}{x^3}$ equals

| | | | | |
|---|---|---|---|---|
| (A) 1 | (B) 2 | (C) 3 | (D) 4 | (E) 5 |

17. If $f(x) = x - 1$ and $(g \circ f)(x) = x^2 - 1$, then $g(3)$ equals

| | | | | |
|---|---|---|---|---|
| (A) 3 | (B) 4 | (C) 8 | (D) 9 | (E) 15 |

18. A man has got in total 2800 children, grandchildren, great-grandchildren and great-great-grandchildren. Each one of them is still alive. The great-great-grandchildren don't have children of their own. Each one of the children, the grandchildren and the

great-grandchildren have got the same number of children as the man himself. How many children does he have?

| (A) 5 | (B) 6 | (C) 7 | (D) 8 | (E) 9 |
|---|---|---|---|---|

19. A fragile stick of length 70 cm breaks into two parts. The stick can not break at both ends (10 cm). The probability of breaking at a given point is the same for all points except for those at the ends (of length 10 cm at each side of the stick). Compute the probability that the difference between the longest and the shortest part of the stick is at least 10 cm.

| (A) 40% | (B) 50% | (C) 60% | (D) 70% | (E) 80% |
|---|---|---|---|---|

20. If $x$ and $y$ satisfy

$$\begin{cases} 83249\,x + 16751\,y & = 108249 \\ 16751\,x + 83249\,y & = 41751 \end{cases}$$

then $\dfrac{x}{y}$ equals

| (A) 1 | (B) 2 | (C) 3 | (D) 4 | (E) 5 |
|---|---|---|---|---|

21.

$$\frac{2}{0,1998\,1998\,1998\,\ldots} + \frac{2}{0,01998\,1998\,1998\,\ldots} + \frac{2}{0,001998\,1998\,1998\,\ldots}$$

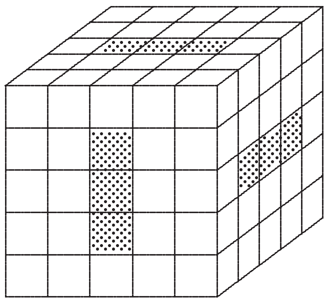
is a natural number. Which one of the following numbers is a prime factor of that number?

| (A) 2 | (B) 3 | (C) 5 | (D) 7 | (E) 11 |
|---|---|---|---|---|

22. The length of the shortest diagonal of a regular polygon with $n$ vertices inscribed in a circle of radius 1, equals

| (A) $\sin\dfrac{\pi}{n}$ | (B) $2\sin\dfrac{\pi}{n}$ | (C) $\sin\dfrac{2\pi}{n}$ | (D) $2\sin\dfrac{2\pi}{n}$ | (E) $2\,\mathrm{tg}\,\dfrac{\pi}{n}$ |
|---|---|---|---|---|

23. A cube, consisting of 125 congruent cubes, is drilled through in three places. The holes are rectangular-shaped (see picture). How many cubes are remaining after the drilling?



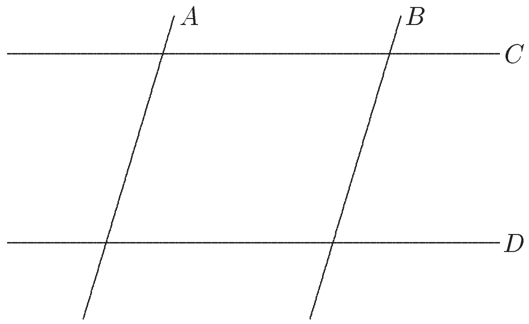
| (A) 80 | (B) 86 | (C) 88 | (D) 89 | (E) 116 |
|---|---|---|---|---|

24. Four people are accused in a trial. It is known that
- if A is guilty, then B is guilty;
- if B is guilty, then C is guilty or A is not guilty;
- if D is not guilty, then A is guilty and C is not guilty;
- if D is guilty, then A is guilty.

How many of the accused are guilty?

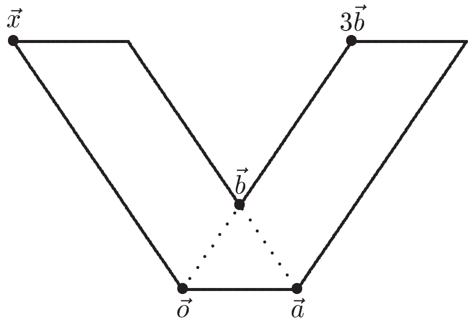
| | | |
|---|---|---|
| (A) 1 | (B) 2 | (C) 3 |
| (D) 4 | (E) not defined from these data. | |

25. A parallelogram is formed by $A, B, C$ and $D$ (see picture). $S_A$ stands for the perpendicular reflexion through $A$.
$S_A \circ S_B \circ S_C \circ S_D$ is



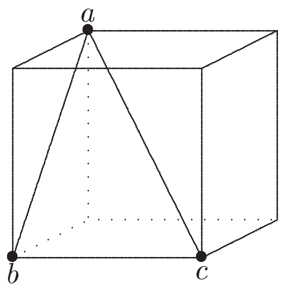

| | |
|---|---|
| (A) a translation | (B) a perpendicular reflexion |
| (C) a reflection through a point | (D) homothety |
| (E) a rotation over an angle different from 180°. | |

26. Two congruent parallelograms have one side in common. They form the letter $V$, as shown in the picture. The vertices are named as in the picture. The vector $\vec{x}$ equals




| | | | | |
|---|---|---|---|---|
| (A) $-3\vec{b}$ | (B) $2\vec{a} - 3\vec{b}$ | (C) $3\vec{b} - 2\vec{a}$ | (D) $3\vec{b} - 3\vec{a}$ | (E) $\vec{a} - 3\vec{b}$ |

27. Consider a cube. Which statement about the triangle $abc$ (see picture) is correct?




(A) The triangle is isosceles and obtuse.
(B) The triangle is isosceles and acute.
(C) The triangle is isosceles and right-angled
(D) The triangle is right-angled and has got three different sides.
(E) The triangle is not isosceles, not right-angled and has got three different sides.

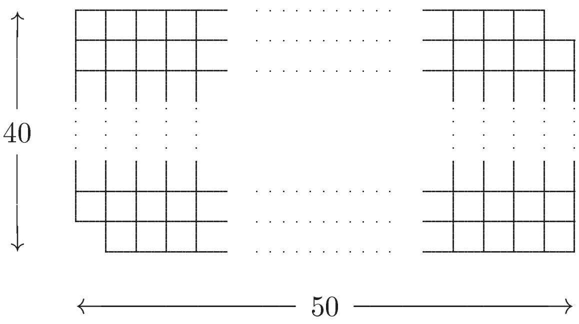
28. Consider a right-angled triangle with perpendicular sides of length respectively 3 and 4. One draws the median from the right angle (foot $z$) and the altitude (foot $h$). Compute the distance from $z$ to $h$.

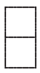
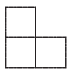
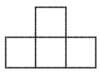
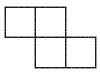
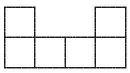
(A) $\frac{1}{2}$ (B) $\frac{7}{10}$ (C) $\frac{\sqrt{2}}{2}$ (D) $\frac{3}{4}$ (E) 1

29. The number of natural numbers $n$ that satisfy $(n^2-2n)^{n^2+47}=(n^2-2n)^{16n-16}$ equals

(A) 1 (B) 2 (C) 3 (D) 4 (E) 5

30. We take a sheet of squared paper of 40 by 50 squares. One square is cut away in two opposite corners. Which form of snippings of paper can be used to cover the remaining 1998 squares without overlap?

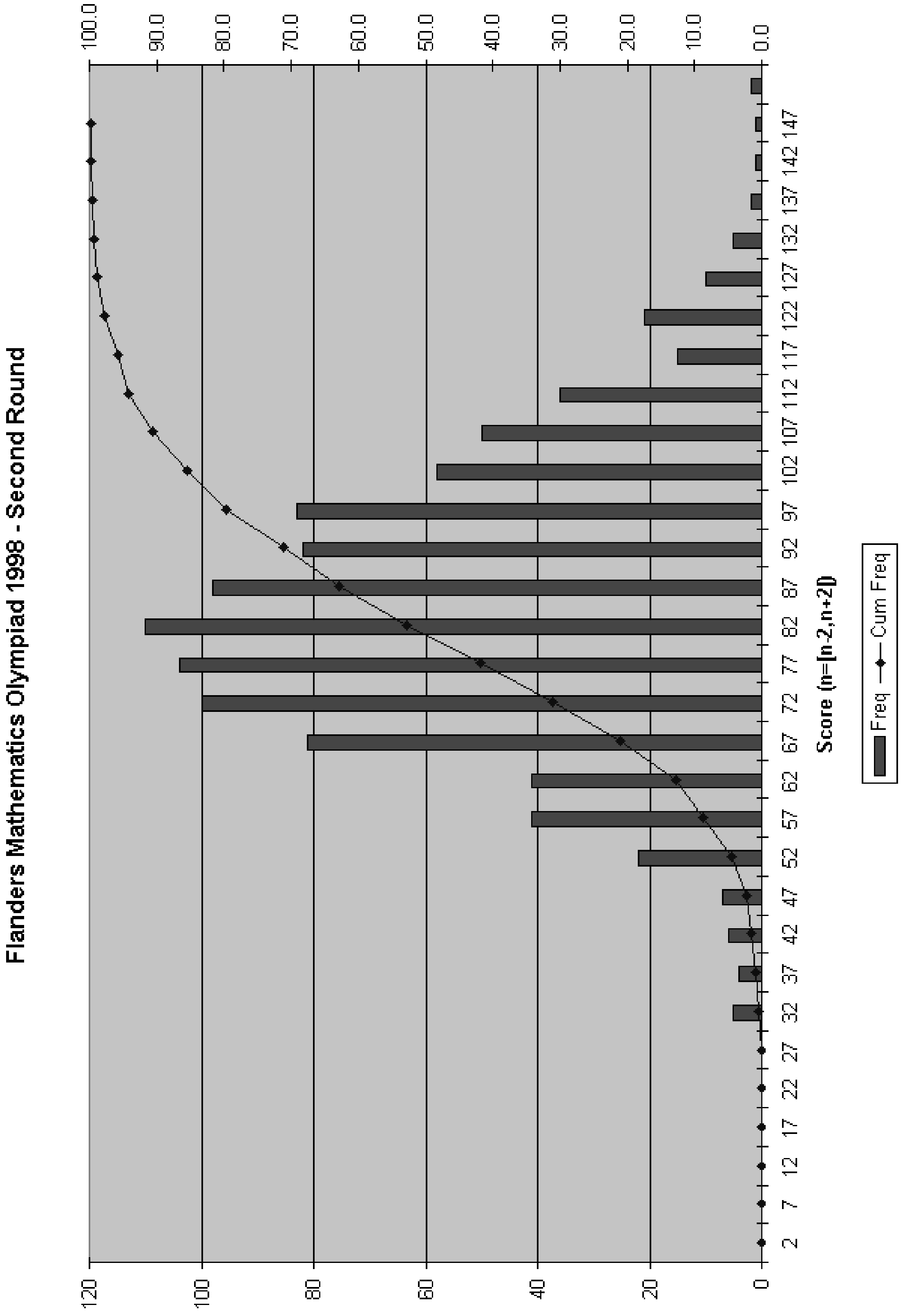



(A)  (B)  (C)  (D)  (E) 

## 9.2 Answer patterns of the Second Round.

| Flanders Mathematics Olympiad 1998 – Second Round<br>Answer Patterns (all participants, i.e. 985) | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| Problem | Correct | A | B | C | D | E | Correct | Wrong | Blank |
| 1 | C | 2.84 | 5.28 | 90.86* | 0.51 | 0.10 | 90.86 | 8.83 | 0.30 |
| 2 | C | 35.03 | 14.21 | 40.81* | 1.52 | 4.87 | 40.81 | 55.63 | 3.55 |
| 3 | B | 7.51 | 88.63* | 0.51 | 0.71 | 0.10 | 88.63 | 8.83 | 2.54 |
| 4 | D | 4.16 | 27.41 | 11.27 | 24.47* | 1.93 | 24.47 | 44.87 | 30.66 |
| 5 | E | 0.81 | 12.99 | 0.81 | 0.71 | 82.44* | 82.44 | 15.33 | 2.23 |
| 6 | A | 63.76* | 1.22 | 0.51 | 4.26 | 15.74 | 63.76 | 21.83 | 14.42 |
| 7 | E | 21.62 | 1.32 | 10.76 | 1.83 | 40.51* | 40.51 | 35.63 | 23.86 |
| 8 | E | 2.64 | 1.83 | 0.61 | 3.45 | 76.04* | 76.04 | 8.73 | 15.23 |
| 9 | D | 8.93 | 14.01 | 3.96 | 59.5*9 | 4.77 | 59.59 | 31.68 | 8.73 |
| 10 | D | 2.13 | 8.93 | 6.29 | 42.94* | 3.86 | 42.94 | 21.32 | 35.74 |
| 11 | D | 14.01 | 0.81 | 1.93 | 41.02* | 36.55 | 41.02 | 53.30 | 5.69 |
| 12 | B | 2.64 | 60.51* | 0.71 | 23.65 | 7.21 | 60.51 | 34.31 | 5.18 |
| 13 | B | 7.11 | 29.95* | 4.26 | 1.32 | 10.56 | 29.95 | 23.35 | 46.70 |
| 14 | D | 9.44 | 51.47 | 8.22 | 7.92* | 5.08 | 7.92 | 74.42 | 17.66 |
| 15 | A | 45.89* | 1.22 | 2.74 | 16.55 | 4.77 | 45.89 | 25.48 | 28.63 |
| 16 | C | 3.05 | 7.41 | 12.18* | 2.03 | 22.03 | 12.18 | 34.52 | 53.30 |
| 17 | E | 3.76 | 23.65 | 3.55 | 6.40 | 47.41* | 47.41 | 37.66 | 14.92 |
| 18 | C | 0.91 | 0.51 | 89.9*5 | 1.02 | 1.02 | 89.95 | 3.55 | 6.50 |
| 19 | E | 4.77 | 1.62 | 8.12 | 2.13 | 70.56* | 70.56 | 17.06 | 12.39 |
| 20 | E | 4.77 | 11.07 | 6.50 | 3.86 | 15.53* | 15.53 | 26.19 | 58.27 |
| 21 | E | 19.70 | 11.88 | 1.73 | 1.83 | 20.41* | 20.41 | 35.23 | 44.37 |
| 22 | D | 1.62 | 11.07 | 5.38 | 39.09* | 6.40 | 39.09 | 24.57 | 36.35 |
| 23 | C | 2.03 | 7.61 | 82.23* | 4.97 | 0.10 | 82.23 | 14.92 | 2.84 |
| 24 | D | 0.91 | 2.03 | 5.28 | 66.40* | 12.69 | 66.40 | 21.02 | 12.59 |
| 25 | A | 60.61* | 3.15 | 4.77 | 4.67 | 6.60 | 60.61 | 19.29 | 20.10 |
| 26 | D | 3.76 | 1.52 | 2.94 | 71.68* | 0.81 | 71.68 | 9.04 | 19.29 |
| 27 | D | 0.10 | 0.61 | 1.12 | 84.26* | 10.56 | 84.26 | 13.50 | 2.23 |
| 28 | B | 4.77 | 52.18* | 11.88 | 3.15 | 1.93 | 52.18 | 21.73 | 26.09 |
| 29 | D | 5.99 | 38.68 | 13.30 | 11.17* | 5.79 | 11.17 | 63.76 | 25.08 |
| 30 | B | 14.31 | 43.76* | 10.66 | 6.29 | 1.62 | 43.76 | 32.89 | 23.35 |

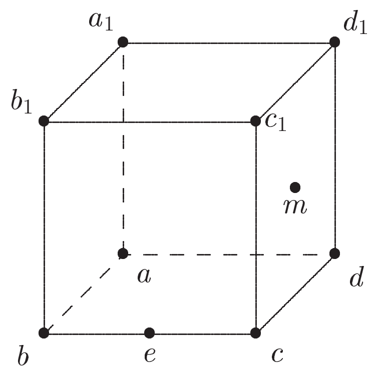
### 9.3 Histogram of the Second Round



## 10 The 1998 Final Round

1. Proof that there exist integers $a$, $b$ and $c$ satisfying $0 < a < b \leq c < 2a$ and $a + b + c = 1998$, and such that $\gcd(a, b, c)$ is maximal. Determine such a triple. Is this triple unique?

2. Given a cube with edges of length 1; given the midpoint $e$ of the edge $[bc]$ and the midpoint $m$ of the face $cdd_1c_1$ (see figure). Determine the area of the intersection of the cube with the plane determined by $a$, $e$ and $m$.




3. Determine all $3 \times 3$ magic squares.

   Definition: *A $n \times n$ magic square is a $n \times n$ matrix formed with all the integers 1, 2, ..., $n^2$ and such that the sums of the elements on each row, on each column and on both diagonals are equal.*

   An example of a $4 \times 4$ magic square is

| | | | |
|---|---|---|---|
| 1 | 15 | 14 | 4 |
| 12 | 6 | 7 | 9 |
| 8 | 10 | 11 | 5 |
| 13 | 3 | 2 | 16 |

4. The figure represents three sides of a billiard table. A white ball is lying at position $p_1$ and the red ball is lying at position $p_2$. The dimensions given are expressed in dm. The white ball is played towards the red bal in such a way that it first touches all three sides of the table (see figure). Determine the minimal distance that this white ball covers.