11 Flanders Mathematics Olympiad 1998-1999: First Round.

The first round consists of 30 multiple choice problems which are selected by the FMO jury. Scores are computed as follows: a correct answer yields 5 points, a blanc answer 1 point and no points are given for a wrong answer. Participants may spend 3 hours solving the problems.

11.1 The problems

1. Which one of the following expressions does not equal the other ones?

(A) $(2^4)^8$	(B) $(4^2)^8$	(C) $2^{16}.16^2$	(D) $2^{16}.2^{16}$	(E) $4^8.4^8$

2. Let a, b, c and $d \in \mathbb{R}_0$. The maximum number of different numbers between the following five equals

$$(a:b):(c:d) \quad , \quad ((a:b):c):d \quad , \quad (a:(b:c)):d$$

$$a:((b:c):d) \quad , \quad a:(b:(c:d))$$

(A) 1	(B) 2	(C) 3	(D) 4	(E) 5

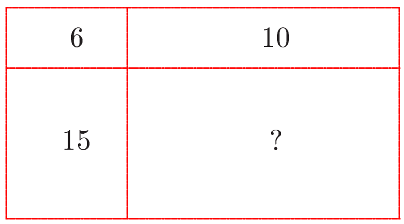
3. A rectangular (dimensions a and b, $a \neq b$) is rotated around the side with length a (resp. b), the volume of the obtained cylinder is denoted with I_a (resp. I_b). The ratio $\dfrac{\mathrm{I}_a}{\mathrm{I}_b}$ equals

(A) $\dfrac{a}{b}$	(B) $\dfrac{b}{a}$	(C) 1	(D) $\dfrac{a^2}{b^2}$	(E) $\dfrac{b^2}{a^2}$

4. The midpoints of four circles (in the plane) with radius 1 are the vertices of a square with side 1. Through how many points pass at least two circles?

(A) 4	(B) 6	(C) 8	(D) 12	(E) 16

5. A rectangular is divided into four smaller rectangulars with areas as shown in the picture. The area of the fourth rectangular equals



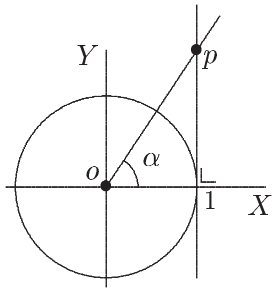
(A) 9	(B) 21	(C) 25	(D) 30	(E) 32

6. Five sympathetic students want to buy a birthday present for their teacher in mathematics.
   - Gunther thinks that she will be 38 on the 16th of March.
   - Kris thinks that she will be 40 on the 17th of April.
   - Luk thinks that she will be 38 on the 17th of April.
   - Olivier thinks that she will be 38 on the 17th of March.
   - Steven thinks that she will be 40 on the 16th of March.

   If we suppose that one of them is correct and that the other ones aren't totally wrong (i.e. the age, the day or the month is correct), who is then right?

(A) Gunther	(B) Kris	(C) Luk	(D) Olivier	(E) Steven

7. In the following figure $|op|$ equals



(A) 1	(B) $\cos\alpha$	(C) $\dfrac{1}{\cos\alpha}$	(D) $1+\operatorname{tg}\alpha$	(E) $\sqrt{1+\sin^2\alpha}$

8. If p with coordinates $(-x, y)$ lies in the third quadrant, then the coordinates of the reflection point of p with respect to the bisector of the second and fourth quadrant are

(A) (x, y)	(B) (y, x)	(C) $(y, -x)$	(D) $(x, -y)$	(E) $(-y, x)$

9. If $x^4 + 4x^3 + 6px^2 + 4qx + r$ is divisible by $x^3 + 3x^2 + 9x + 3$, then $(p+q)r$ equals

(A) -18	(B) 12	(C) 15	(D) 27	(E) 45

10. We consider an orthonormal coordinate system in which two parabolas with equation

$$y = x^2 - 4x - 5 \quad \text{and} \quad y = -\frac{1}{2}(x^2 - 4x - 5)$$

bound an area. The surface of the smallest rectangular, with sides parallel to the coordinate axes and enclosing that area, equals

(A) 18	(B) 27	(C) 54	(D) 72	(E) 81

11. The number of functions with domain $\{1, 2, 3, 4\}$ and with values belonging to $\{1, 2, 3\}$ equals

(A) 6	(B) 12	(C) 24	(D) 64	(E) 81

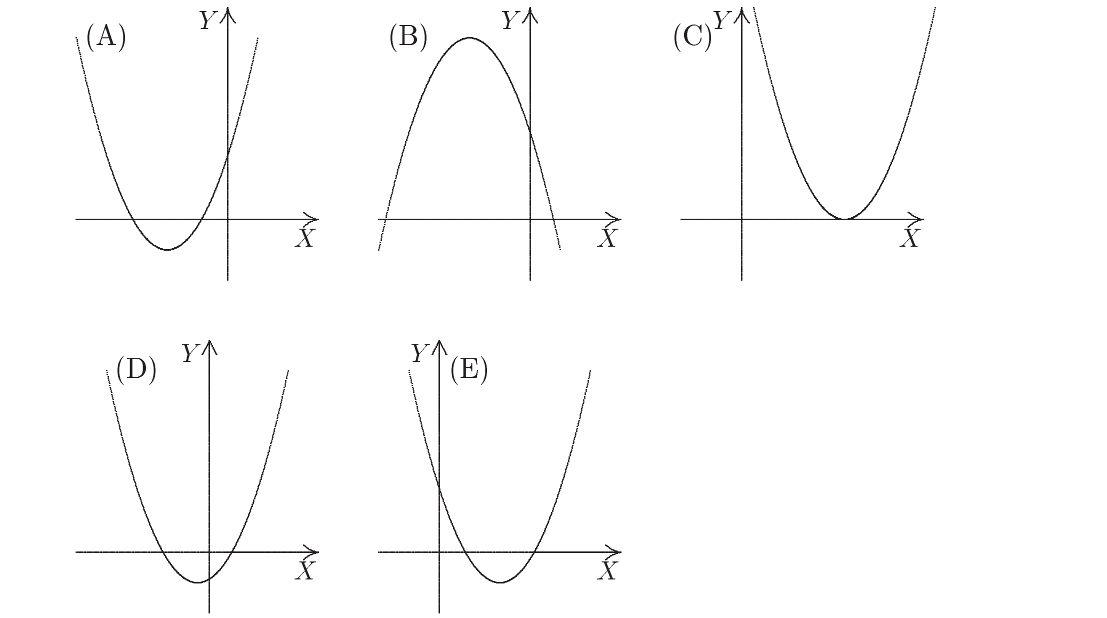
12. Consider the following three functions, resp. f, g and h, $\mathbb{R} \to \mathbb{R}$:

$$f(x) = |1 - x| \quad , \quad g(x) = 1 - |x| \quad , \quad h(x) = |1 - |x||.$$

Then, for each $x \in \mathbb{R}$:

(A) $f(x) \leq g(x) \leq h(x)$ (B) $g(x) \leq h(x) \leq f(x)$
(C) $g(x) \leq f(x) \leq h(x)$ (D) $h(x) \leq f(x) \leq g(x)$
(E) $h(x) \leq g(x) \leq f(x)$

13. Which of the following five parabolas is the graph of a quadratic function $f(x) = ax^2 + bx + c$ with a, b and c strict positive numbers?



14. If $0 < x < 2\pi$ and $\sin x \;.\; \cos x \;.\; \mathrm{tg}\, x \;.\; \mathrm{cotg}\, x < 0$, then x belongs to

(A) $]0, \pi[$ (B) $]0, \frac{\pi}{2}[\cup]\pi, \frac{3\pi}{2}[$ (C) $]\pi, 2\pi[$
(D) $]\frac{\pi}{2}, \pi[$ (E) $]\frac{\pi}{2}, \pi[\cup]\frac{3\pi}{2}, 2\pi[$

15. We consider all natural numbers consisting of three digits (every digit being different from 0), with the property that one of the digits is the sum of the other two (e.g. 154, 743, ...). How many such numbers do there exist?

(A) 20	(B) 60	(C) 96	(D) 108	(E) 120

16. If n is a natural number between 1000 and 2000, then

$$\frac{1+3+5+7+\ldots+(2n-1)}{(2n+1)+(2n+3)+\ldots+(4n-1)}$$

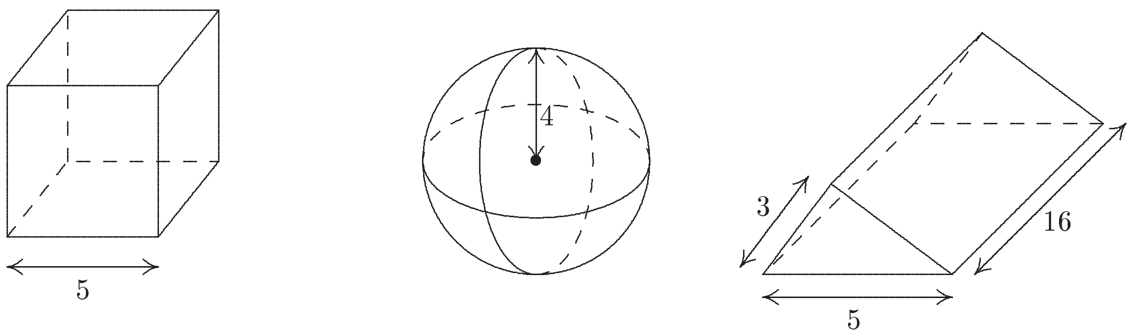
equals

(A) $\frac{1}{3}$	(B) $\frac{1}{2}$	(C) 1
(D) 2	(E) depending on the value of n	

17. Let $m = 777\ldots77$ be the number consisting of 99 digits 7 and $n = 999\ldots99$ be the number consisting of 77 digits 9. The number of different digits in the product $m.n$ equals

(A) 1	(B) 2	(C) 3	(D) 4	(E) 5

18. Let C, S and P be the volumes of resp. the cube, the sphere and the right prism with dimensions as shown in the picture.



Then, C, B and P satisfy

(A) $S < C < P$	(B) $C < S < P$	(C) $P < S < C$
(D) $P < C < S$	(E) $C < P < S$	

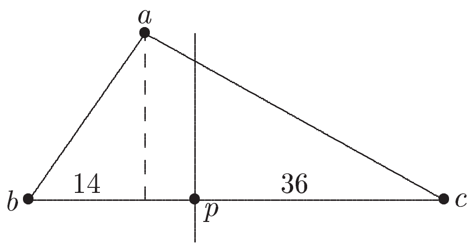
19. Let $f : \mathbb{R} \to \mathbb{R} : x \mapsto x^2 - 1$. Let A be the set of all $x \in \mathbb{R}$ such that $f(x) \in [0, 3]$. Then, A equals

(A) $[0, 8]$	(B) $[-2, 2]$	(C) $[1, 2]$
(D) $[0, 2]$	(E) $[-2, -1] \cup [1, 2]$	

20. In every triangle with sides a, b and c, $a^2 + b^2 + c^2$ is smaller than or equal to

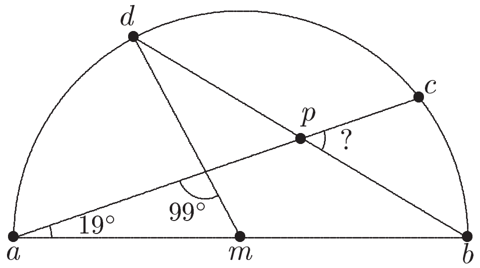
(A) $ab + ac + bc$	(B) $(a-b)^2 + (b-c)^2 + (c-a)^2$
(C) $a^3 + b^3 + c^3$	(D) $4a + 4b + 4c$
(E) $a(b+c) + b(c+a) + c(a+b)$	

21. The basis of a triangle abc is divided by the altitude from a into two parts; one part has length 14, the other one length 36 (see picture). A straight line perpendicular to bc divides the triangle abc in two parts with the same area and intersects bc at a point p. The ratio $\frac{|bp|}{|cp|}$ equals



(A) 1	(B) $\frac{2}{3}$	(C) $\frac{11}{18}$	(D) $\frac{5}{9}$	(E) $\frac{1}{2}$

22. Consider the half of a circle with diameter $[ab]$ and midpoint m. Two points c and d are lying on it as shown in the figure. The angle between ac and bd measures



(A) 50°	(B) 51°	(C) 52°	(D) 53°	(E) 54°

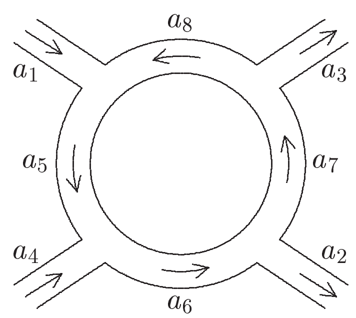
23. 40 balls are numbered from 1 to 40. They are put in boxes. If a box contains ball n, then the box cannot contain a ball with a number that is a multiple of n. How many boxes do you need at least?

(A) 5	(B) 6	(C) 12	(D) 14	(E) 20

24. Consider the following roundabout: one can drive only following the direction of the arrows. Every hour

- 1000 cars arrive through a_1,
- 800 cars leave the roundabout through a_2,
- 1500 cars drive over a_5 and
- 900 cars drive over a_7.

How many cars arrive each hour through a_4 and how many cars leave each hour the roundabout through a_3?



(A) 0 and 400	(B) 200 and 400	(C) 400 and 400
(D) 400 and 800	(E) 400 and 600	

25. If An stands on the shoulders of Bart, she can hardly look over the wall.
If Bart stands on the shoulders of Kris, he cannot look over the wall at all.
If Kris stands on the shoulders of Dirk, he can easily look over the wall.
The smallest one and the tallest one of these four people are

(A) An and Kris	(B) An and Bart	(C) Dirk and Kris
(D) Dirk and Bart	(E) impossible to determine	

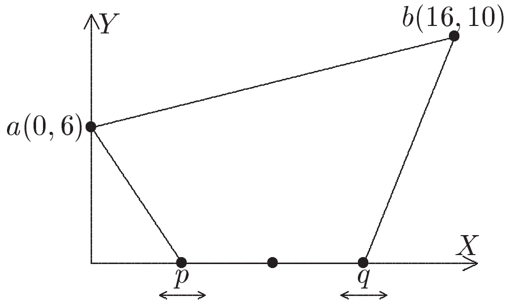
26. A rectangular is obtained by extending two of the opposite sides of a square with 10% and by shortening the other sides with 10%. In comparison with the area of the square, the area of the rectangular is

(A) the same	(B) 10% bigger	(C) 1% bigger
(D) 10% smaller	(E) 1% smaller	

27. If the numbers from 1 to 6 are written in random order, a number consisting of 6 digits is obtained. The chance that this number is divisible by 6 equals

(A) $\frac{1}{6}$	(B) $\frac{1}{3}$	(C) $\frac{1}{2}$	(D) $\frac{2}{3}$	(E) $\frac{5}{6}$

28. In an orthonormal coordinate system we consider the points $a(0,6)$ and $b(16,10)$. The length of $[pq]$ is 8 and is fixed; $[pq]$ moves along the X-axes. Determine the abscissa of the midpoint of $[pq]$ such that the perimeter of the quadrangle $abqp$ is minimal.



(A) 4	(B) 7	(C) 8	(D) 10	(E) 12

29. For which of the following equations, the graph is exactly a triangle?

(A) $x.y.(x+y-1)=0$	(B) $\sqrt{x}.\sqrt{y}.\sqrt{x+y-1}=0$
(C) $\sqrt{x}.\sqrt{y}.\sqrt{1-x-y}=0$	(D) $\lvert x\rvert+\lvert y\rvert+\lvert x+y-1\rvert=0$
(E) $\sqrt{x}+\sqrt{y}+\sqrt{x+y-1}=0$	

30. Consider the mappings $f(x)=1-x$ and $g(x)=\frac{1}{x}$ and all possible compositions of these two mappings (e.g. $f\circ f$, $f\circ g$, $g\circ f\circ g$, $\ldots$). The total number of all mappings

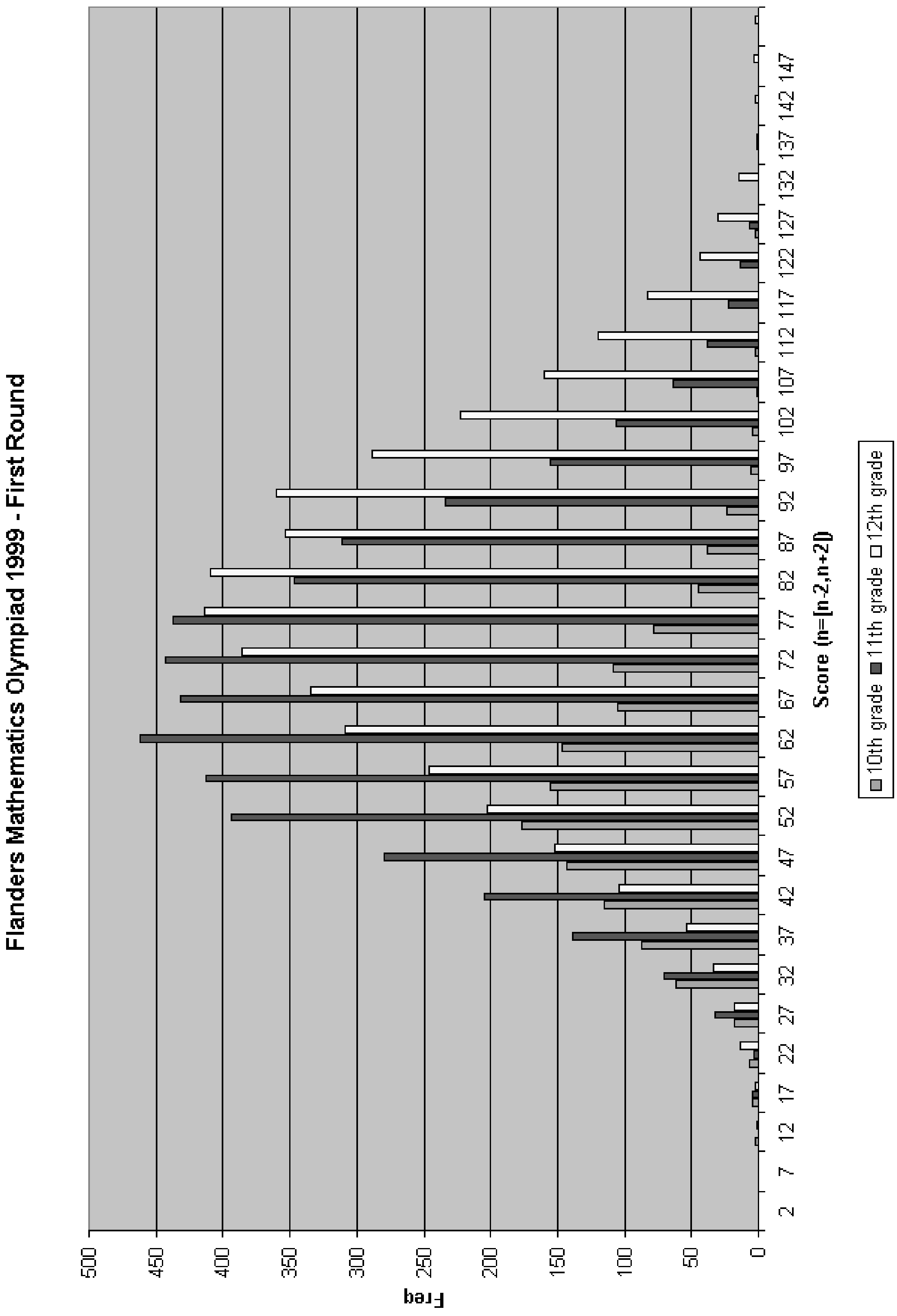
obtained in this way is

(A)	not bigger than five
(B)	bigger than five but not bigger than ten
(C)	bigger than 10 but not bigger than 100
(D)	a finite number bigger than 100
(E)	infinite

11.2 Answer patterns of the First Round

Flanders Mathematics Olympiad 1999 – First Round (all participants, i.e. 10381)									
Problem	Correct	A	B	C	D	E	Correct	Wrong	Blank
1	C	1.15	1.46	92.61*	2.36	1.42	92.61	6.43	0.96
2	D	5.46	5.64	11.35	50.69*	12.45	50.69	34.97	14.34
3	B	15.70	37.08*	15.21	5.39	3.38	37.08	39.75	23.17
4	D	34.35	5.44	7.54	43.52*	1.09	43.52	48.53	7.95
5	C	0.41	0.62	89.52*	2.56	2.98	89.52	6.66	3.82
6	D	1.19	0.77	2.05	91.99*	0.83	91.99	4.91	3.09
7	C	1.28	2.86	54.11*	11.49	17.81	54.11	33.55	12.33
8	E	6.65	5.19	8.01	44.79	28.02*	28.02	64.70	7.28
9	C	2.69	4.30	32.46*	3.64	2.26	32.46	12.91	54.63
10	E	6.25	5.89	4.90	4.15	17.72*	17.72	21.28	61.00
11	E	4.60	21.30	7.02	8.05	7.72*	7.72	41.01	51.27
12	B	1.51	78.89*	6.08	2.87	1.94	78.89	12.55	8.57
13	A	39.62*	4.46	22.32	5.57	10.24	39.62	42.68	17.70
14	E	2.68	7.58	4.33	6.37	52.33*	52.33	21.05	26.63
15	D	2.00	8.84	10.74	38.48*	16.47	38.48	38.22	23.30
16	A	22.67*	7.55	2.52	1.38	25.86	22.67	37.32	40.01
17	E	1.36	8.54	18.65	22.98	27.55*	27.55	51.60	20.84
18	D	12.63	9.71	9.28	41.54*	14.72	41.54	46.50	11.96
19	E	4.84	7.55	10.35	2.25	58.48*	58.48	25.07	16.45
20	E	7.07	6.58	35.00	3.57	24.78*	24.78	52.36	22.87
21	B	5.96	16.87*	8.51	7.48	2.02	16.87	24.01	59.11
22	A	41.67*	6.64	5.45	4.09	3.05	41.67	19.25	39.08
23	B	19.98	37.70*	4.60	3.49	15.38	37.70	43.50	18.80
24	B	1.45	73.13*	1.80	1.99	6.05	73.13	11.42	15.45
25	E	1.08	3.29	18.95	18.25	47.22*	47.22	41.71	11.07
26	E	10.52	1.31	2.03	6.46	74.79*	74.79	20.47	4.74
27	C	24.19	10.56	25.25*	3.81	2.33	25.25	40.96	33.80
28	B	13.46	8.99*	18.74	4.94	5.91	8.99	43.14	47.87
29	C	3.04	4.49	3.06*	8.14	2.90	3.06	18.58	78.36
30	B	5.65	6.10*	4.00	2.61	29.51	6.10	41.80	52.10

11.3 Histogram of the first round.



12 Flanders Mathematics Olympiad 1998-1999: Second Round.

Scores are computed in the same way as for the first round. Participants may spend 2 hours in solving the problems.

12.1 The problems

The following approximations can be of help for solving some of these problems.
$\sqrt{2} \approx 1{,}4$; $\sqrt{3} \approx 1{,}7$; $\sqrt{5} \approx 2{,}2$; $\sqrt{6} \approx 2{,}4$; $\sqrt{7} \approx 2{,}6$; $\sqrt{8} \approx 2{,}8$; $\sqrt{10} \approx 3{,}16$

1. Simplify

$$\frac{(x^{-1}+y^{-1})^{-1}-(x^{-1}-y^{-1})^{-1}}{(y^{-1}-x^{-1})^{-1}-(y^{-1}+x^{-1})^{-1}} \text{ with } x, y \neq 0,\ x \neq y \text{ and } x \neq -y.$$

(A) $\frac{x}{y}$	(B) $\frac{y}{x}$	(C) $\frac{2y}{x+y}$	(D) $\frac{y-x}{y}$	(E) $\frac{y-x}{y+x}$

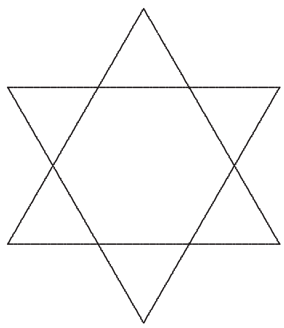
2. Which one of the following inequalities is not correct?

(A) $\sin 300° < \sin 350°$	(B) $\cos 300° < \cos 350°$
(C) $\sin 300° < \cos 350°$	(D) $\operatorname{tg} 300° < \operatorname{tg} 350°$
(E) $\operatorname{cotg} 300° < \operatorname{cotg} 350°$	

3. If $\frac{1}{7000}$ is written in its decimal form, the 1999th digit after the decimal point is

(A) 2	(B) 4	(C) 5	(D) 7	(E) 8

4. A Star of David is formed by putting two equilateral triangles above each other. The figure consists of a regular hexagon surrounded by six smaller equilateral triangles. Compute the ratio of the surface of the whole star and the surface of the inner hexagon.

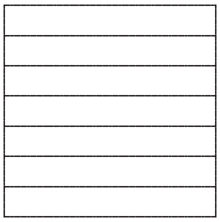


(A) $\frac{3}{2}$	(B) $\sqrt{2}$	(C) $\sqrt{3}$	(D) 2	(E) 3

5. The diagonal of a rectangle is twice as long as one of the sides with length, say a. The length of the other side equals

(A) a	(B) $a\sqrt{3}$	(C) $a\sqrt{5}$	(D) $\frac{5}{4}a$	(E) $\frac{3}{2}a$

6. A square is divided into 7 congruent rectangles. The perimeter of each rectangle is 32. Then, the perimeter of the square equals



(A) 56	(B) 98	(C) 112	(D) 196	(E) 224

7. A car with 5 tyres (one of them being a spare tyre) drives 10000 kilometers such that every tyre is equally used. How many kilometers has been droven with each tyre?

(A) 2000	(B) 2500	(C) 7500	(D) 8000	(E) 10000

8. $M(x)$ means: "x is a man"; $A(x)$ means: "Rosa loves x".
The meaning of

$$\forall x : M(x) \Rightarrow A(x)$$

is

(A) Only men love Rosa.
(B) Every man loves Rosa.
(C) Rosa loves only men.
(D) Rosa does not love women.
(E) Rosa loves every man.

9. Let x_1 and x_2 be the two roots of the quadratic equation $ax^2 + bx + c = 0$. Let $\frac{x_1}{x_2}$ and $\frac{x_2}{x_1}$ be the roots of the equation $\alpha x^2 + \beta x + 1 = 0$, then β equals

(A) $2 - \frac{b^2}{ac}$	(B) $\frac{b^2}{ac} - 2$	(C) $2 - \frac{ab^2}{c}$	(D) $2 - \frac{b^2}{c}$	(E) $\frac{b^2}{c} - 2$

10. The set of all real numbers smaller than or equal to their reciprocal, is

(A) $]0, 1]$	(B) $[-1, 1]$	(C) $]-\infty, -1]$
(D) $[-1, 0[\cup]0, 1]$	(E) $]-\infty, -1] \cup]0, 1]$	

11. The number of solutions in $\mathbb{R}^2$ of the following system equals

$$\begin{cases} |x| + y = 11 \\ x + |y| = 99 \end{cases}$$

(A) 0	(B) 1	(C) 2	(D) 3	(E) 4

12. Assume f and g are functions with domain $\mathbb{R}$ and with the property $f(0) = g(0) = 0$. Consider the following statements:

- $f(a) = 0 \Rightarrow g(f(a)) = 0$
- $g(f(a)) = 0 \Rightarrow f(a) = 0$
- $g(a) = 0 \Rightarrow g(f(a)) = 0$
- $g(f(a)) = 0 \Rightarrow g(a) = 0$

How many statements are correct?

(A) 0	(B) 1	(C) 2	(D) 3	(E) 4

13. Assume that the system

$$\begin{cases} x_1 + x_2 = a_1 \\ x_2 + x_3 = a_2 \\ x_3 + x_4 = a_3 \\ x_4 + x_1 = a_4 \end{cases}$$

with the unknowns x_1, x_2, x_3, x_4 has got a solution, then

(A) $a_1 + a_2 + a_3 + a_4 = 0$ (B) $a_1 + a_2 = a_3 + a_4$
(C) $a_1 + a_3 = a_2 + a_4$ (D) $a_1 + a_4 = a_2 + a_3$
(E) The assumption is wrong; the system doesn't have a solution.

14. $\lfloor x \rfloor$ stands for the greatest integer not exceeding x. For example: $\lfloor \pi \rfloor = 3$ and $\lfloor -\pi \rfloor = -4$. The set of solutions in $\mathbb{R}$ of $\lfloor x + 1 \rfloor = \lfloor \frac{x+2}{2} \rfloor$ is

(A) an open interval (B) a half-open interval
(C) a closed interval (D) a singleton
(E) empty

15. Let $m(x_1, y_1)$, $n(x_2, y_2)$ and $p(x_3, y_3)$ be three points in the plane. The points a, b and c (also in this plane) are such that m is the middle of $[ab]$, n is the middle of $[bc]$ and p is the middle of $[ca]$. The x-coordinate of c is

(A) $x_1 + x_2 + x_3$ (B) $x_1 + x_2 - x_3$ (C) $x_2 + x_3 - x_1$
(D) $x_3 + x_1 - x_2$ (E) $x_3 - x_1 - x_2$

16. The number of roots of the equation $\cos x + \sin x = \frac{1}{2}$ with $x \in [0, \frac{\pi}{2}]$ equals

(A) 0	(B) 1	(C) 2
(D) 3	(E) none of the previous	

17. The symbol $\bullet$ is a digit. If $\bullet 3 \times 6528 = 3\bullet \times 8256$, than the symbol $\bullet$ denotes

(A) 2	(B) 4	(C) 6	(D) 7	(E) 9

18. Let x be the smallest number consisting of three digits, every digit different from 0, such that the last digit of the product of these three digits equals 0. The last digit of the sum of these digits equals

(A) 0	(B) 2	(C) 4	(D) 6	(E) 8

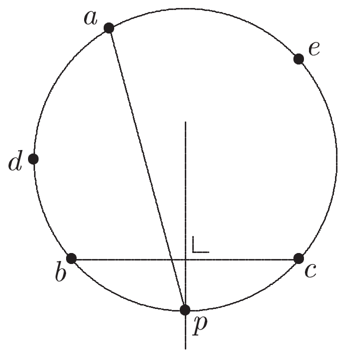
19. The decimal form of 2^{1999} consists of m digits and the decimal form of 5^{1999} of n digits. The sum $m + n$ is

(A) smaller than 1998	(B) 1998	(C) 1999
(D) 2000	(E) greater than 2000	

20. We are given two squares. The length of the sides of each square is an integer. The difference between the area of the squares is 77 (units). The sum s of the length of one side of each square satisfies

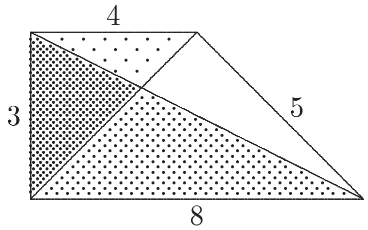
(A) $s \in [1, 10]$	(B) $s \in [11, 20]$	(C) $s \in [21, 40]$
(D) $s \in [41, 80]$	(E) there is more than one solution	

21. We consider the arc $\overset{\frown}{adb}$ (130°) and the arc $\overset{\frown}{aec}$ (150°) on a circle (see picture). The point p is one of the intersection points of the perpendicular bisector of $[bc]$ and the circle (see picture). The acute angle formed by that perpendicular bisector and pa measures



(A) 5°	(B) 10°	(C) 15°	(D) 20°	(E) 25°

22. A right-angled trapezium has sides with lengths as shown in the figure. The two diagonals divide the trapezium into four triangles (see figure). The areas of these four triangles are having the same ratios as the numbers

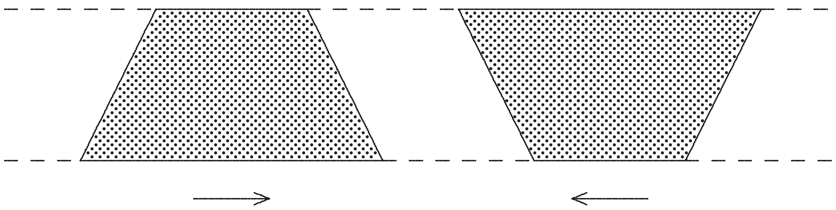


(A) 1, 1, 2, 4	(B) 1, 1, 3, 4	(C) 1, 2, 3, 4	(D) 1, 2, 2, 4	(E) 2, 2, 3, 4

23. One draws an equilateral triangle abc in the plane. Where do we have to choose the origin such that $\vec{c} = \vec{a} - \vec{b}$?

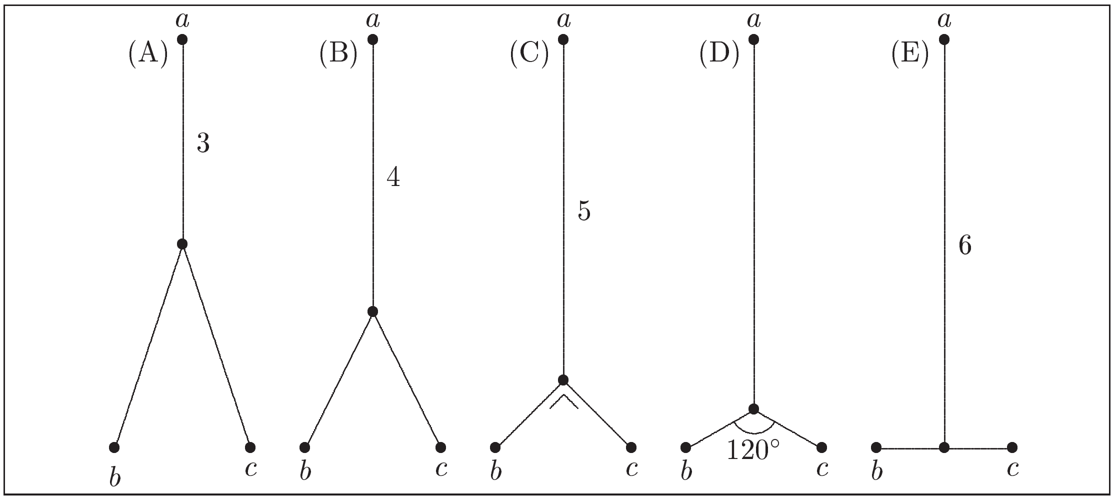
(A) In one of the vertices of the triangle.
(B) On the circumscribed circle of the triangle.
(C) In the reflection point of a with respect to bc.
(D) In the reflection point of b with respect to ac.
(E) In the reflection point of c with respect to ab.

24. Two congruent isosceles trapezia having a small base of length a, raised sides of the same length a and a great base which is twice as long, move along two axes, which are extending the bases of the trapezia (see figure). In a certain position, the area of the overlapping part is maximal. Compute this maximal area.



(A) a^2	(B) $\frac{a^2\sqrt{3}}{2}$	(C) $\frac{3a^2\sqrt{3}}{4}$	(D) $\frac{5a^2\sqrt{3}}{4}$	(E) $\frac{5a^2\sqrt{3}}{8}$

25. Villages a, b and c will be connected with straight roads. The distance between b and c is 2 km; a is on the perpendicular bisector of $[bc]$, 6 km away from $[bc]$. One decides to choose a point on this bisector and to connect the villages with that chosen point. In which one of the following choices (see figure) is the total length of the roads minimal?



26. In an orthonormal coordinate system the slope of the bisector of the acute angle between the straight lines $y = x$ and $y = 2x$ equals

(A) $1+\sqrt{2}$	(B) $\frac{4}{3}$	(C) $\frac{\sqrt{10}+1}{3}$	(D) $\sqrt{2}$	(E) $\frac{3}{2}$

27. Given: $f_1(x) = 1 - |x|$. Define: $\forall n > 1 : f_n(x) = 1 - |f_{n-1}(x)|$.
Compute $f_{1999}(1999)$.

(A) 0	(B) 1	(C) -1	(D) 1999	(E) -1998

28. Let x and y be real numbers with sum 1; $A = x^2 + y$ and $B = x + y^2$. Consider the following statements:
I. $A = B$ for all x and y.
II. $A \neq B$ for some x and y.
III. $A \leq 1$ for all x and y.
IV. $A > 1$ for some x and y.
The correct statements are

(A) I	(B) II	(C) I and III	(D) I and IV	(E) II and III

29. The points of gravity of the faces of a tetrahedron *abcd* form another tetrahedron V_z. The midpoints of the edges of the tetrahedron *abcd* are the vertices of a convex body L_m. The quotient of the volumes of L_m and V_z equals

(A) 4	(B) 6	(C) 9	(D) 13,5	(E) 18

30. Five people (A, B, C, D and E) are either judge or lawyer. It is known that judges always tell the truth, while lawyers always lie. We also know that

- A is a judge;
- B says that he is a judge;
- C says that D is a judge;
- D says that B and E can't be both a judge;
- E says that A and B are judges.

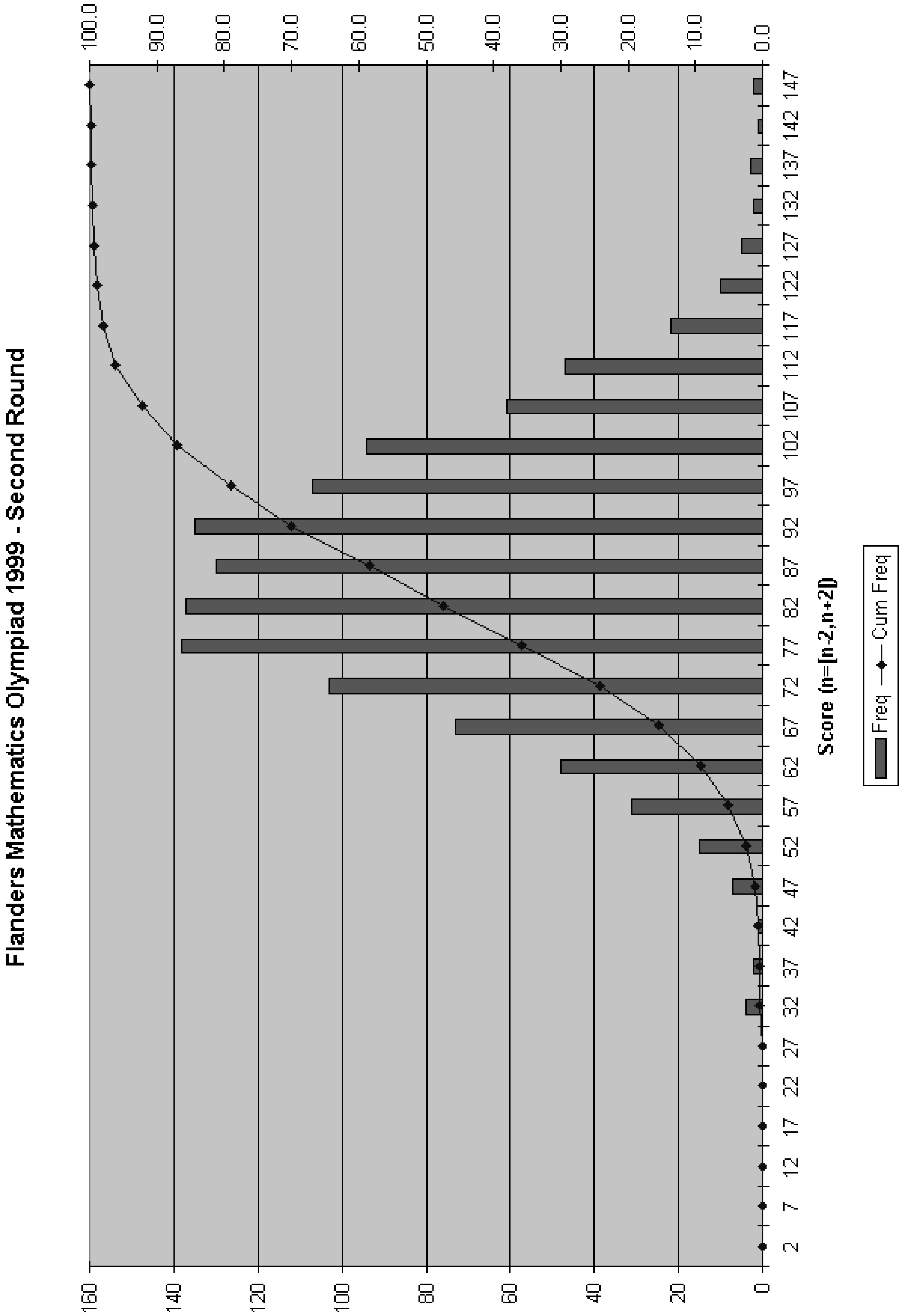
How many judges are there among A, B, C and D?

(A) 2	(B) 3	(C) 4
(D) 5	(E) cannot be determined	

12.2 Answer patterns of the second round.

Flanders Mathematics Olympiad 1999 – Second Round Answer patterns									
Problem	Correct	A	B	C	D	E	Correct	Wrong	Blank
1	A	69.19*	10.02	1.61	1.53	1.27	69.19	14.43	16.38
2	E	4.07	1.70	4.24	8.15	77.93*	77.93	18.17	3.90
3	E	7.22	10.44	8.06	5.77	45.50*	45.50	31.49	23.01
4	D	2.97	1.02	1.10	88.37*	1.27	88.37	6.37	5.26
5	B	1.78	93.63*	2.97	0.08	0.51	93.63	5.35	1.02
6	A	71.48*	1.10	2.46	16.04	3.99	71.48	23.68	4.84
7	D	4.07	1.10	2.63	87.52*	0.68	87.52	8.49	3.99
8	E	0.25	2.21	9.00	0.68	85.14*	85.14	12.14	2.72
9	A	13.67*	8.23	2.63	3.48	1.95	13.67	16.30	70.03
10	E	5.77	3.65	2.04	5.77	80.90*	80.90	17.23	1.87
11	B	11.12	27.76*	30.22	3.48	6.28	27.76	51.10	21.14
12	B	3.31	57.81*	12.56	3.23	11.12	57.81	30.22	11.97
13	C	1.02	1.02	80.65*	2.04	3.06	80.65	7.30	12.05
14	B	10.36	37.35*	11.63	11.63	4.33	37.35	37.95	24.70
15	C	1.70	4.50	62.05*	3.31	1.53	62.05	11.04	26.91
16	A	43.21*	13.58	10.78	0.93	3.65	43.21	29.03	27.76
17	B	0.25	86.50*	1.61	1.02	2.55	86.50	5.43	8.06
18	E	2.38	1.36	0.68	1.27	79.37*	79.37	5.69	14.94
19	D	13.24	2.72	5.35	10.78*	13.67	10.78	34.97	54.24
20	E	4.50	30.90	3.48	4.41	16.64*	16.64	43.29	40.07
21	A	31.92*	13.33	6.79	4.75	1.95	31.92	26.91	41.17
22	D	1.87	1.10	6.28	53.82*	2.29	53.82	11.54	34.63
23	C	3.90	4.84	57.39*	3.06	5.26	57.39	17.06	25.55
24	E	8.15	14.94	11.38	10.70	19.61*	19.61	45.16	35.23
25	D	6.62	3.74	27.42	45.50*	5.43	45.50	43.21	11.29
26	C	1.95	7.64	7.81*	7.89	32.68	7.81	50.17	42.02
27	A	28.95*	6.03	3.40	3.23	15.28	28.95	27.93	43.12
28	D	5.69	10.61	13.07	41.17*	11.63	41.17	41.00	17.83
29	D	4.75	2.89	4.67	3.31*	0.93	3.31	13.24	83.45
30	B	0.34	69.78*	3.31	0.68	16.38	69.78	20.71	9.51

12.3 Histogram of the second round.



13 The 1999 Final Round

1. Determine all natural numbers, having 6 digits, say $abcdef$, with $a \neq 0$ and $d \neq 0$ and such that $abcdef = (def)^2$.

2. Let $[mn]$ be a diameter of a circle with a given radius and $[ab]$ a chord with a given (fixed) length on this circle. Assume that $[ab]$ neither coincides with $[mn]$ nor is perpendicular to $[mn]$. Let p be the midpoint of $[ab]$ and let c and d be the orthogonal projections of a and b on $[mn]$. Proof that the angle $\widehat{cpd}$ is independent of the position of the chord $[ab]$.

3. Determine all possible functions $f : \mathbb{R} \to \mathbb{R}$ and $g : \mathbb{R} \to \mathbb{R}$ which satisfy the following conditions:

   (1) For all s and $t \in \mathbb{R}$: $2f(s) - g(s) = f(t) - t$;
   (2) For all $s \in \mathbb{R}$: $f(s)\,g(s) \geq s + 1$.

4. Consider a, b, n, $m \in \mathbb{N} \setminus \{0, 1\}$ such that $a^n - 1$ and $b^m + 1$ are prime. Give as much information as possible concerning the integers a, b, n and m.