On the structure of the directions not determined by large affine point sets

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Many results on extendability of affine point sets not determining a given set of directions are known. The strongest results are known in the planar case, [3]. An extendability result known for general dimension is the following. Originally, it was proved in [2] for $n = 3$. A proof for general n can be found in [1].

Theorem *Let $q = p^h$, p an odd prime and $h > 1$, and let $U \subseteq \mathrm{AG}(n, q)$, $n \geq 3$, be a set of affine points of size $q^{n-1} - 2$, which does not determine a set D of at least $p + 2$ directions. Then U can be extended to a set of size q, not determining the set D of directions.*

The natural question is whether the latter theorem can be improved in the sense that extendability of sets of size $q^{n-1} - \varepsilon$ is investigated, for $\varepsilon > 2$.

We explain how we obtain information on the structure of the set of non-determined directions if we assume that U cannot be extended without determining more directions. The following theorem is discussed, and we explain further developments in general dimension.

Theorem *Let $U \subset AG(3, q) \subset PG(3, q)$, $|U| = q^2 - \varepsilon$. Let $D \subseteq H_\infty$ be the set of directions determined by U and put $N = H_\infty \setminus D$ the set of non-determined directions. Then U can be extended to a set $\bar{U} \supseteq U$, $|\bar{U}| = q^2$ determining the same directions only, or N is contained in a curve of H_∞, of degree $\varepsilon(\varepsilon - 1)^2$.*

References

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