The smallest minimal blocking set of $\mathrm{Q}(6, q)$, q even

Jan De Beule
joint work with Leo Storme

Definitions

An ovoid $\mathcal{O}$ of $\mathrm{Q}(2n, q)$ is a set of points such that every generator meets $\mathcal{O}$ in exactly one point.

A blocking set $\mathcal{K}$ of $\mathrm{Q}(2n, q)$ is a set of points such that every generator meets $\mathcal{K}$ in at least one point. $\mathcal{K}$ is minimal iff $\mathcal{K} \setminus \{p\}$ is not a blocking set $\forall p \in \mathcal{K}$.

$\mathcal{O}$ is an ovoid of $\mathrm{Q}(4, q)$ (resp. $\mathrm{Q}(6, q)$), then $|\mathcal{O}| = q^2 + 1$ (resp. $|\mathcal{O}| = q^3 + 1$)

$\mathcal{K}$ is an ovoid of $\mathrm{Q}(4, q)$ (resp. $\mathrm{Q}(6, q)$), then $|\mathcal{K}| = q^2 + 1 + r$ (resp. $|\mathcal{K}| = q^3 + 1 + r$)

Lines with many points

Theorem 1. *(Eisfeld, Storme, Szőnyi and Sziklai) A blocking set of* $\mathrm{Q}(4,q)$*,* q *even,* $q \geqslant 32$*, of size* q^2+1+r*, with* $0 < r \leq \sqrt{q}$*, contains an ovoid.*

Theorem 2. *(Bichara and Korchmáros) If* S *is a* $(q+2)$*-set in* $\mathrm{PG}(2,q)$*,* q *even, and if* S *has* r *internal nuclei,* $r > q/2$*, then every point of* Ω *is an internal nucleus.*

Towards the known blocking set

Lemma 1. *Let* $\mathcal{C}$ *be a minimal cover of* $\mathrm{Q}(4,q)$, $|\mathcal{C}| = q^2 + 1 + r$, $0 < r \leqslant q - 1$. *If each multiple point has excess at least* $\sqrt{q}$, *then the set* E *of multiple points is a sum of lines, with the sum of the weights of the lines equal to* r.

Final Theorem

Theorem 3. *Let $\mathcal{K}$ be a minimal blocking set of $\mathrm{Q}(6,q)$, q even, $|\mathcal{K}| \leqslant q^3+q$, $q \geqslant 32$. Then there is a point $p \in \mathrm{Q}(6,q) \backslash \mathcal{K}$ with the following property: $T_p(\mathrm{Q}(6,q)) \cap \mathrm{Q}(6,q) = p\mathrm{Q}(4,q)$ and $\mathcal{K}$ consists of all the points of the lines L on p meeting $\mathrm{Q}(4,q)$ in an ovoid $\mathcal{O}$, minus the point p itself, and $|\mathcal{K}| = q^3+q$.*