Common characterizations of the finite Moufang polygons

H. Van Maldeghem
State University of Ghent

1. Introduction and notation

The notion of a generalised polygon was introduced by Tits (1959). The main idea was to give a geometric interpretation of the Chevalley groups and the twisted groups of rank 2. A *generalised polygon* is a generalised n-gon for some positive integer $n \neq 1$. A *generalised n-gon of order* (s, t), $s, t, \in \mathbb{N}^* \cup \{\infty\}$ is a point-line incidence geometry $S = (P, B, I)$ satisfying (*GP1*) up to (*GP4*).

(*GP1*) *There are $s + 1$ points incident with each line.*
(*GP2*) *There are $t + 1$ lines incident with each point.*
(*GP3*) *Every two incident pairs of varieties (a variety is a point or a line) lie in a common circuit consisting of $2n$ varieties.*
(*GP4*) *Every (proper) circuit in S contains at least $2n$ varieties.*

In this paper, we will deal mainly with *thick* generalised polygons, ie. $s, t, > 1$. If both s and t are finite, then S is also finite (by a simple counting argument). Thick generalised n-gons exist for every $n > 1$ (eg. via free construction). A generalised 2-gon is a trivial geometry: every point is incident with every line. Although this structure is important in the theory of diagrams (see Buekenhout (1979)), we will not consider it in this paper. A thick generalised 3-gon is an ordinary projective plane. Generalised 4-gons, 6-gons and 8-gons are also called respectively *generalised quadrangles, hexagons* and *octagons*. An important result of Feit and Higman (1964) says that finite thick generalised n-gons exist only if $n = 2, 3, 4, 6$, or 8 and there are examples in each of these cases. An important subclass of the class of generalised polygons are the so-called *Moufang polygons*. They satisfy the *Moufang condition* (condition (M) below). Before stating (M), we need some definitions. Let $S = (P, B, I)$ be a generalised n-gon, $n > 2$. A (proper) circuit consisting of $2n$ varieties is called an *apartment*. Let A be an apartment and x a variety of S. We denote the set of all varieties incident with x but distinct from the $2n$ varieties of A by $A^*(x)$. A chain of $n + 1$ distinct consecutively incident varieties is called a *root*. If n is even, then every root has a unique middle element. If this is a point, then we call the root *short*, otherwise *long*. Let $\mathfrak{R} = (x_0 I x_1 I \dots I x_n)$ be a root. If α is an automorphism of S fixing all elements of $A^*(x_1)$, $A^*(x_2)$,

$A^*(x_{n-1})$, then we call α an $\mathfrak{R}$-*elation* or, in general, a *root-elation*. If the group of $\mathfrak{R}$-elations act transitively on the set of apartments containing $\mathfrak{R}$ (for fixed $\mathfrak{R}$), then we call $\mathfrak{R}$ a *transitive root*. In that case the action just mentioned is *regular*.

We now state the Moufang Condition:

(*M*) *Every root in S is transitive.*

It is easily seen that the condition (M) is equivalent to asking that every root in a certain fixed apartment of S is transitive. A celebrated result of Tits (1976) and Weiss (1979) states that infinite thick Moufang n-gons can only exist for $n = 3, 4, 6, 8$ (disregarding the generalised 2-gons) and there are finite and infinite examples in each case. A theorem of Fong and Seitz (1973) implies that a finite thick generalised polygon S is Moufang if and only if S or its point-line dual arises naturally from the Chevalley groups or twisted groups of rank 2. Moreover it was proved by Tits (1979, personal communication, to appear) that also all infinite Moufang generalised n-gons are known. So a characterization of a class of Moufang polygons is very often at the same time a characterization of a class of geometries associated with Chevalley groups or twisted groups, partly motivating the results in this paper.

In §2 - 4 we collect characterization theorems about big classes of (mostly finite) Moufang polygons. By *big class* we understand, as a general rule, a class of Moufang polygons containing all finite n-gons for at least one $n \in \{4, 6, 8\}$, thus leaving out the numerous characterizations of classical projective planes and all the characterizations of certain classical quadrangles gathered in Payne and Thas (1984). Also, almost all characterizations we will mention include projective planes and they mostly follow from Wagner's Theorem (1965). In that case, we will not mention references.

2. Characterizations by elations

2.1 Definitions

Let $S = (P, B, I)$ be a generalised n-gon. A *flag* in S is an incident point-line pair. Let $\{p, L\}$ be a flag of S, $p \in P$, $L \in B$. If an automorphism α of S fixes every point incident with L and every line incident with p, then we call α an *elation (with source* $\{p,L\}$*)*. For instance, a root-elation is an elation. A *central root-elation (with centre p)* is an automorphism of S fixing all varieties at distance $\leq$

$n/2$ from p. If n is even, it is a root-elation for every short root having p as middle element. Dually, one defines an *axial root-elation*. Now let $C = (x_0 I x_1 I \ldots I x_k)$ be a chain of length k, $3 \le k \le n$. Then a *C-elation* is an elation with sources (x_{i-1}, x_i), $i = 2, 3, \ldots, k - 1$. If every chain C of fixed length k, $3 \le k \le n$, the group of C-elations acts transitively (and hence regularly) on the set of apartments containing C, then we call S *k-Moufang*. Finally, we call S *half-Moufang* provided n is even and every short root of S is transitive, or every long root of S is transitive.

2.2 Characterizations

Theorem 1 (Van Maldeghem et al. to appear, $n = 4$; Van Maldeghem 1991a, $n = 6, 8$). *Let S be a finite thick generalised n-gon, n = 3, 4, 6, or 8 and suppose $3 \le k \le n$. Then S is Moufang if and only if it is k-Moufang.*

Theorem 2 (Walker). *Let S be a finite thick generalised n-gon, n = 3, 6, or 8. Then S is Moufang if and only if it admits an automorphism group acting on the set of ordered pairs (F, A), where F is a flag contained in an appartment A, and for every root $\mathfrak{R}$ in S, there exists a non-trivial $\mathfrak{R}$-elation.*

Theorem 3 (Walker). *Let S be a finite thick generalised n-gon, n = 6, or 8. Then S is Moufang if and only if every point of S is the centre of a non-trivial central root-elation, or every line of S is the axis of a non-trivial axial root-elation.*

Theorem 4 (Thas et al. to appear). *A finite thick generalised quadrangle is Moufang if and only if it is half-Moufang .*

2.3 Remarks

(a) Theorem 1 is trivial for $k = n$ and it is almost trivial for $k > 3$.

(b) Suppose a generalised polygon S admits an automorphism group G acting transitively on the ordered pairs (F, A) as in Theorem 2. Then G is a group with a (saturated) (B, N)-pair and it is conjectured by Tits (1974) that every such finite thick generalised polygon is Moufang.

(c) Similarly, as one can reduce the Moufang condition to the condition that every root in a fixed appartment is transitive, one can reduce the hypotheses of the above theorems (but not always that far).

(d) Theorem 3 has a partial analogue for $n = 4$, namely the result of Ealy (thesis) characterizing the finite Moufang quadrangles of *even characteristic*.

3. Characterizations by homologies

3.1 Definitions

Let S again be a generalised n-gon. There is a natural distance map d inherited from the incidence graph of S. We call the two varieties in S *opposite* if they are at a distance n (maximal distance) from each other. Let x, y be two opposite varieties in S and suppose σ is an automorphism of S fixing every variety incident with x or y. Then we call σ a *generalised homology*, or an *(x, y)-homology*. Now, let A be an apartment containing both x and y and let z be incident with x and lie in A. If the group of all (x, y)-homologies acts transitively on the set $A^*(z)$, then we call S *(x, y)-transitive*. This definition does not depend on the choice of A. If u is in A and if it is incident with z (so u is at a distance 2 from x), then we call S *(x, y)-quasi-transitive* provided the group of all (x, y)-homologies acts transitively on the set $A^*(u)$. Note that S is (x, y)-(quasi)-transitive if and only if S is (y, x)-(quasi)-(transitive).

3.2 Characterizations

Theorem 5. *Let S be a finite thick generalised n-gon of order (s, t), $n =$ 3, 4, 6, or 8 and if $n = 8$, then $s, t > 2$. Then S is Moufang if and only if at least one of the following occurs:*

(a) *$n = 3$ or 6 and S is (x,y)-transitive for every pair (x, y) of opposite varieties.*

(b) *$n = 4$ and S is (x, y)-transitive for every pair (x,y) of opposite points, or S is (X, Y)-transitive for every pair (X, Y) of opposite lines.*

(c) *$n = 8$ and S, or its point-line dual, is (X, Y)-transitive for every pair (X,Y) of opposite lines and (x, y)-quasi-transitive for every pair (x, y) of opposite points.*

Proof: For the case $n = 4$, see Thas (1986); for $n = 6$, see Van Maldeghem (1990). Let $n = 8$. Note first that all finite thick Moufang octagons have the desired property by the commutator relations in Tits (1983) and the calculations at the end of Tits (1983). We will show the converse in detail in §5 below.

4. Geometric characterizations

4.1 Definitions

Let S be a thick generalised polygon (not necessarily finite). Suppose $d(x, y) = l < n$ for two varieties x, y in S, then for every k, $1 \leq k \leq l$, there exists a unique variety x_k at a distance k from x and at a distance $l - k$ from y. We denote $x_k = \Pi_x^k(y)$. Now let $C = (x_1 \mathrm{I} x_2 \mathrm{I} \ldots \mathrm{I} x_{k-1})$ be a chain of length $k - 2$, $3 \leq k \leq n$. Let $A = (y_1 \mathrm{I} y_2 \mathrm{I} \ldots \mathrm{I} y_{2n} \mathrm{I} y_1)$ and $A' = (y'_1 \mathrm{I} y'_2 \mathrm{I} \ldots \mathrm{I} y'_{2n} \mathrm{I} y'_1)$ be two ordered apartments in S. We call A and A' *in perspective from* C if for every $i \in \{1, \ldots, k - 1\}$ and every $j \in \{1, \ldots, 2n\}$, we have $d(x_i, y_j) = d(x_i, y'_j)$ and $\Pi_{x_i}^1(y_j) = \Pi_{x_i}^1(y'_j)$ if $d(x_i, y_j) < n$. The configuration (C, A, A') is called a *generalised k-Desargues configuration.* For $n = 3$, we have $k = 3$ and we get a usual (little) Desargues configuration, (see Hughes and Piper (1973)). Now S is called *C-Desarguesian* if for every ordered apartment $A = (y_1 \mathrm{I} y_2 \mathrm{I} \ldots \mathrm{I} y_{2n} \mathrm{I} y_1)$ with $d(x_1, y_1) < n$, and every variety $y \neq x_1$, $y \mathrm{I} \Pi_{x_1}^1(y_1)$, there exists an ordered apartment $A' = (y'_1 \mathrm{I} y'_2 \mathrm{I} \ldots \mathrm{I} y'_{2n} \mathrm{I} y'_1)$ in perspective with A from C such that $\Pi_{x_i}^2(y'_1) = y$. In that case C is called a *Desarguesian chain.* If every chain of length k is Desarguesian, then S is called *k-Desarguesian* and if $k = 3$, then we simply say that S is *Desarguesian.*

4.2 Characterizations

Theorem 6 (Hughes and Piper 1973, $n = 3$), (Thas and Van Maldeghem to appear, $n = 4$) (Van Maldeghem 1991a, $n > 4$). *A thick generalised n-gon S, $n > 2$, is Moufang if and only if S is n-Desarguesian.*

Theorem 7 (Hughes and Piper 1973, $n = 3$), (Van Maldeghem et al. to appear, $n = 4$). *A finite thick generalised n-gon S, $n = 3$ or 4, is Moufang if and only if S is Desarguesian.*

4.3 Remarks

(a) For projective planes, the definition above of *Desarguesian* is in classical terms in fact *little Desarguesian.*

(b) Theorem 7 is probably also true for $n > 4$, but it seems that one will not be able to avoid a very long, tiresome, dull and messy proof.

(c) There are some other beautiful geometric characterizations of Moufang n-gons due to Thas (1983) $n = 4$ and Ronan (1980, 1981) $n = 6$

not included here, because the conditions in these theorems depend strongly on n.
(d) As a general concluding remark, we could say that the generalization of all theorems above, which do not include all Moufang polygons to all Moufang polygons is an open question. It is also an open question as to which characterizations of finite Moufang polygons above also hold in the infinite case.

5. Proof of Theorem 5, case $n = 8$

Throughout this section we suppose that S is a finite generalised octogon of order (s, t), $s, t > 2$ and S is (X, Y)-transitive for every pair of opposite lines X, Y and S is (x, y)-quasi-transitive for every pair of mutually opposite points x, y. All there is left to show is that in this case S is Moufang.

5.1 Step 1

We will use the following property frequently. Suppose an automorphism σ fixes an apartment A, all varieties incident with a certain element x of A (suppose eg. without loss of generality that x is a line) and also at least three lines through a certain point of A. Then σ fixes a suboctogon S' of order (s, t'), $t' > 1$ (cf. Walker thesis) and hence $t' = t$ by Thas (1979), (cf. also Van Maldeghem (1991b)). So σ is the identity. Also, if $s > t$, then S does not contain suboctogons of order $(s, 1)$, see Thas (1979).

5.2 Step 2

Now suppose $A = p_1 I L_1 I p_2 I L_3 I p_4 I L_5 I p_6 I L_7 I p_8 I L_8 I p_7 I L_6 I p_5 I L_4 I p_3 I L_2 I p_1)$ is an apartment in S. We denote the group of automorphisms of S fixing every variety incident with at least one of the varieties belonging to a certain set D and fixing in addition all varieties in E by $\mathcal{H}_E(D)$, (omitting the curl brackets of D and E for short). Now by Step 1, the group $\mathcal{H}(p_1, p_8)$ acts semi-regularly on $A^*(L_1)$ and since S is (p_1, p_8)-quasi-transitive, we have $s > t$.

Suppose $\sigma \in \mathcal{H}(L_1, L_8)$ fixes an element $p \in A^*(L_3)$. Then σ fixes the flag complex S' of a generalised quadrangle of order (s, s') with $1 < s' < s$ (indeed, $s' = s$ implies that S' is a suboctogon of order $(s, 1)$, contradicting $s > t$). Let $L \in A^*(p_2)$ and put $L' = L^{\sigma}$. Let $\sigma' \in \mathcal{H}(L_3, L_6)$ be such that $L'^{\sigma'} = L$; then $\sigma\sigma'$ fixes a suboctogon S'' of order (s', t') with $1 < t' < t$. But $S' \cap S''$ is a suboctogon of order $(s', 1)$ of S'', hence $s' < t'$ and so $s' < t$. Now consider $\alpha \in \mathcal{H}(p_2, p_7)$ not preserving S'' (α exists since $t > s'$ and $\mathcal{H}(p_2, p_7)$ acts semi-regularly on $A^*(L_1)$). Then $S'' \cap S''^{\alpha}$ is a suboctogon of order (s'', t') of S'' with $s'' < s$. Hence $s'' = 1$ and $t' < s'$, a contradiction. So $\mathcal{H}(L_1, L_8)$ acts semi-regularly on $A^*(L_3)$.

5.3 Step 3

Choose a fixed $L \in A^*(p_2)$ and let $L' \in A^*(p_2)$ and $p \in A^*(L_3)$. Denote by $\sigma(p, L')$ the unique (L_1, L'_8)-homology mapping L to L', where L'_8 is at distance 2 from L_6 and at distance 5 from p. If $p \neq p' \in A^*(L_3)$, then by Step 1, $p_4{}^{\sigma(p, L')} \neq p_4{}^{\sigma(p', L')}$. Hence, by a simple counting argument, for every point $p'_4 \mathrm{I} L_3$, $p'_4 \neq p_2$, there exists a unique $p \,\mathrm{I} L_3$, $p \neq p_2$, such that $p_4{}^{\sigma(p, L')} = p'_4$. Varying L', we find $t - 2$ distinct non-trivial generalised homologies of the form $\sigma(p, L')$ mapping p_4 to p'_4. In particular, it is now easy to see that the automorphism group of S has a (B, N)-pair, (cf. Van Maldeghem (1991b), Lemma 2).

5.4 Step 4

In this step we show that, if an automorphism β of S fixes A, all lines through p_1 and at least three lines through p_3, then it must fix all lines through p_3. Suppose the contrary. Then β fixes the flag complex S of a generalised quadrangle of order (t', t) with $1 < t' < t$. But by transitivity, there exists a (p_1, p_8)-homology σ not preserving S', hence $S' \cap S'^{\sigma}$ is the flag complex of a generalised subquadrangle of S' of order (t'', t) with $t'' < t'$, contradicting the result of Thas (1982). Hence, $t' = t$ and β fixes all lines through p_3.

5.5 Step 5

The following results are proved in Van Maldeghem (1991c) for generalised hexagons, but it is easy to adjust the proof for generalised octogons.

Lemma 8. *Let* $X \subseteq \{p_1, p_3, p_5, L_2, L_4, L_6\}$. If $\mathcal{H}_{(p_2, p_7)}(X) \neq 1$, *then* $|\mathcal{H}_{(p_2, p_7)}(X)| = t$.

Lemma 9. (1) *We have* $\mathcal{H}_{p_7}(p_1, L_1) \neq 1$.

(2) *Let* $X \subseteq \{p_3, p_5, L_2, L_4, L_6\}$ *and suppose for every* $x \in X$, $\mathcal{H}(x, y)$ acts non-trivially on $A^*(p_2)$, *where* y *is opposite to* x *in* A. *Then,* $\mathcal{H}_{p_7}(X, p_1, L_1) \neq 1$

These two lemmas, together with Step 1, imply readily that

$$|\mathcal{H}(L_1, p_1, L_2, L_4, p_5, L_6)| = t.$$

Now, let $\sigma \in \mathcal{H}(L_1, p_1, L_2, L_4, p_4, L_6)$. Suppose σ does not fix all lines through p_3 and let $L \mathrm{I} p_3$ be such that $L^\sigma = L' \neq L$. Set $L_3{}^\sigma = L'_3$ and let $M, M' \in A^*(p_2)$ with $M \neq M' \neq L'_3 \neq M$. Let p'_8 resp. p''_8 be at distance two from p_7 and at distance five from M resp. M' and consider the (p_1, p'_8)-homology α_M, resp. (p_1, p''_8)-homology $\alpha_{M'}$, mapping L_3 to L'_3. Then by Step 4, $L^{\alpha_M} \neq L^{\alpha_{M'}}$. Since there are $t-2$ choices for M, we can choose M such that it maps L to L'. Considering $\sigma\alpha_M{}^{-1}$ we see, by Step 4, that this is impossible, hence $L = L'$ and $\sigma \in \mathcal{H}(L_1, p_1, L_2, p_3, L_4, p_5, L_6)$, and consequently all short roots are transitive.

5.6 Step 6

In view of Theorem 2, all we need to show is that there exists a non-trivial "long-root-elation". So suppose $\mathcal{H}(p_5, L_4, p_3, L_2, p_1, L_1, p_2)$ is trivial. Then the commutator

$$\left[\ \mathcal{H}(L_6, p_5, L_4, p_3, L_2, p_1, L_1),\ \mathcal{H}(L_4, p_8, L_2, p_1, L_1 p_2,, L_3)\ \right]$$

is also trivial and this means geometrically that every element of $\mathcal{H}(L_6, p_5, L_4, p_3, L_2, p_1, L_1)$ fixes every every point collinear with p_1 or p_5. Now let $q_4 \in A^*(L_3)$ and suppose M_8 is at a distance two from L_6 and at a distance five from q_4. Let $L \in A^*(p_2)$ and denote by σ any non-trivial (L_1, L_8)-homology. Let σ' be the unique (L_1, M_8)-homology mapping L^σ back to L. Since $\sigma\sigma'$ is non-trivial, it must act semi-regularly on the set of points incident with L_3 and distinct from p_2 (by Step 1). Similarly, as in Step 5, using the results of Step 3, we see that $\sigma\sigma'$ fixes every line incident with p_1, p_2, p_3 or p_5. Note that it also fixes at least three points incident with L_4. Now, by varying q_4, one can see that, in fact, $|\mathcal{H}(p_2, L_1, p_1, p_3, p_5)| = s$. So, by symmetry, there exists

$\alpha \in \mathcal{H}(p_2,p_1, p_3,L_4,p_5)$ such that $\alpha\sigma\sigma'$ fixes A. By Step 1, this must be the identity, hence $|\mathcal{H}(p_2, L_1, p_1, p_3,L_4,p_5)| = s$.

5.7 Step 7

Let $1 \neq \sigma \in \mathcal{H}(p_5, L_4, p_3, p_1,L_1, p_2)$ and $1 \neq \tau \in \mathcal{H}(L_2, p_1, L_1, p_2, L_3,p_4,L_5)$. Then $\sigma\tau\sigma^{-1}\tau^{-1} \in \mathcal{H}(L_4, p_3, L_2, p_1,L_1, p_2, L_3)$, otherwise $\mathcal{H}(p_3, L_2,p_1,L_1, p_2, L_3, p_4) \neq 0$. Geometrically, this implies that σ fixes every point collinear with p_3 or p_1. Now define the chain $p_1 \mathrm{I} M_2 \mathrm{I} q_3 \mathrm{I} M_4 \mathrm{I} q_5 \mathrm{I} M_6 \mathrm{I} q_7 \mathrm{I} M_8 \mathrm{I} p_8$; then clearly, $\sigma \in \mathcal{H}(M_2, p_1, L_1, p_2)$. We can now choose elations $\alpha'' \in \mathcal{H}(M_2, p_1, L_1, p_2, L_3, p_4, L_5)$, $\alpha' \in \mathcal{H}(q_3,M_2, p_1, p_2, L_3, p_4)$, $\alpha'' \in \mathcal{H}(M_4, q_3,M_2, p_1, L_1, p_2, L_3)$, and $\alpha''' \in \mathcal{H}(q_5,M_4, q_3,M_2, p_1, L_1, p_2)$ such that $\sigma' = \sigma\alpha\alpha'\alpha''\alpha'''$ fixes the apartment $p_1 \mathrm{I} M_2 \mathrm{I} q_3 \mathrm{I} M_4 \mathrm{I} q_5 \mathrm{I} M_6 \mathrm{I} q_7 \mathrm{I} M_8 \mathrm{I} p_8 \mathrm{I} L_7 \mathrm{I} p_6 \mathrm{I} L_5 \mathrm{I} p_4 \mathrm{I} L_3 \mathrm{I} p_2 \mathrm{I} L_1 \mathrm{I} p_1$. But σ' fixes all lines through p_1 and at least three points incident with M_2, (because the order of α''' divides s and α''' fixes already two points on M_2, namely p_1 and q_3). By Step 1, $\sigma' = 1$. Hence $\alpha''' = (\sigma\alpha\alpha'\alpha'')^{-1}$. But $\sigma\alpha\alpha'\alpha''$ fixes every point incident with M_2, hence so does α'''. But since $\sigma \neq 1$, we have $1 \neq \alpha''' \in \mathcal{H}(q_5,M_4, q_3,M_2, p_1, L_1, p_2)$. This completes the proof of the Theorem.

Acknowledgement. The author is a Research Associate at the National Fund for Scientific Research (Belgium).

References

F. Buekenhout (1979), Diagrams for geometries and groups, *J. Combin. Theory Ser. A* **27**, 121-151.
W. Feit and G. Higman (1964), The nonexistence of certain generalised polygons, *J. Algebra* **1**, 114-131.
P. Fong and G. Seitz (1973), Groups with a (B,N)-pair of rank 2, I and II, *Invent. Math.* **21**, 1-57 and *Invent. Math.* **24** (1974), 191-239.
D.R. Hughes and F.C. Piper (1973), *Projective Planes* , Springer-Verlag, Berlin.
M.A. Ronan (1980), A geometric characterization of the Moufang hexagons, *Invent. Math.*, **57**, 227-262.
M.A. Ronan (1981), A combinatorial characterization of the dual Moufang hexagons, *Geom. Dedicata* **11**, 61-67.
C.E. Ealy, Jr., Generalised quadrangles and odd transpositions, *Thesis* .
S.E. Payne and J.A. Thas (1984), *Finite Generalized Quadrangles*, Pitman, London.

J.A. Thas (1972), 4-gonal subconfigurations of a given 4-gonal configuration, *Atti Accad. Naz. Lincei Rend.* **53**, 520-530.

J.A. Thas (1979), A restriction on the parameters of a suboctagon, *J. Combin. Theory Ser. A*, **40**, 385-387.

J.A. Thas (1981), New combinatorial characterizations of generalized quadrangles, *European J. Combin.* **2**, 299-303.

J.A. Thas (1986), The classification of all (x, y)-transitive generalized quadrangles, *J. Combin. Theory Ser. A* **42**, 154-157.

J. Tits (1959), Sur la trialité et certain groupes qui s'en déduisant, *Inst. Hautes Etudes Sci. Publ. Math.* **2**, 14-60.

J. Tits (1974), *Buildings of spherical type and finite BN-pairs*, Lecture Notes in Mathematics 386, Springer-Verlag, Berlin.

J. Tits (1976), Non-existence de certains polygons généralisès, I and II, *Invent. Math.* **36**, 275-284 and *Invent. Math.* **51** (1979) 267-269.

J. Tits (1979), Classification of buildings of spherical type and Moufang polygons: a survey, *Teorie Combinatorie, Volume* 1, 229-246, Accademia Nazionale dei Lincei, Rome.

J. Tits (1983), Moufang octagons and the Ree groups of type $^2F_4$, *Amer. J. Math.* **105**, 539-594.

J. Tits (1990), Quadrangles de Moufang I, *preprint*.

J.A. Thas, S.E. Payne and H. Van Maldeghem (1991), Half Moufang implies Moufang for finite generalised quadrangles, *submitted*.

J.A. Thas and H. Van Maldeghem (to appear). Generalised Desargues configurations in generalised quadrangles. *Bull. Soc. Math. Belg.*.

H. Van Maldeghem (1991a), A configurational characterization of the Moufang generalised polygons, *European. J. Combin.*, to appear.

H. Van Maldeghem (1991b), On finite Moufang polygons, *submitted*.

H. Van Maldeghem (1991c), A characterization of the finite Moufang hexagons by generalized homologies, *submitted*.

H. Van Maldeghem, S.E. Payne and J.A. Thas (1991), Desarguesian generalized quadrangles are classical or dual classical, *submitted*.

A. Wagner (1965), On finite affine line transitive planes, *Math. Z.* **87**, 1-11.

M. Walker, On central root automorphisms of finite generalised polygons, *Ph.D. Thesis*, University of London.

R. Weiss (1979), The nonexistence of certain Moufang polygons, *Invent. Math.* **51**, 261-266.