MINI-COURSE ON MOUFANG SETS

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A Moufang set is essentially a doubly transitive permutation group such that the point stabilizer contains a normal subgroup which is regular on the remaining points. These regular normal subgroups are called the *root groups* and they are assumed to be conjugate and to generate the Moufang set. (The root groups are *not* assumed to be nilpotent.) J. Tits introduced this notion in the context of twin buildings, but it is in fact a tool to study absolutely simple algebraic groups of relative rank one; the Moufang sets are precisely the Moufang buildings of rank one. It turns out that this notion is related to other algebraic structures as well.

Here is an outline of the topics to be covered in the mini-course.

(1) Definition of a Moufang set.

(2) Motivation and situation.
   (a) Connection with saturated split BN-pairs of rank one.
   (b) Connection with rank one groups.
   (c) Connection with higher rank groups.

(3) The construction $\mathbb{M}(U, \tau)$, the Hua maps, the Hua subgroup and the little projective group. Criterion for when $\mathbb{M}(U, \tau)$ is a Moufang set.

(4) First properties of Moufang sets.
   (a) Definition of the μ-maps.
   (b) Properties of the μ-maps.
   (c) Connection of the μ-maps with the Hua maps.

(5) Examples of Moufang sets.
   (a) The Moufang set $\mathbb{M}(K)$, K a commutative field. Connection with $\mathrm{PSL}_2(K)$.
   (b) The Moufang set $\mathbb{M}(K)$, K a skew-field or octonion division algebra. Connection with $\mathrm{PSL}_2(K)$ for some appropriate definition of $\mathrm{PSL}_2(K)$.
   (c) Definition of a quadratic Jordan division algebra. The Moufang set $\mathbb{M}(\mathcal{J})$, where $\mathcal{J}$ is a quadratic Jordan division algebra. Connection with the two previous examples.
   (d) Some interesting examples where the root groups are not abelian.
   (e) The examples in (c) and (d) that do not arise from algebraic groups.

(6) More advanced properties of Moufang sets.
   (a) Properties of the Hua maps.
   (b) Identities in Moufang sets.
   (c) Root subgroups and the fixed points of the Hua maps.

(7) Special Moufang sets.
   (a) Definition of special Moufang set.
   (b) The structure of the root groups and the action of the Hua subgroup on the root groups.
   (c) The "special implies abelian root groups conjecture" and some results.
   (d) Connection with $\mathbb{M}(\mathcal{J})$, where $\mathcal{J}$ is a quadratic Jordan division algebra. The "special + abelian root groups implies QJDA conjecture" and some results.
   (e) Finite special Moufang sets.