Developments in Phan theory

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Geometric methods in the theory of Chevalley groups and their generalizations like Kac-Moody groups have made tremendous
advances during the last few decades. Among the most noteworthy and influencial of these advances are the systematic application of the concept of amalgams, the local-to-global approach, ingenious applications of combinatorial topology and geometric group theory, the theory of abstract root groups, and the interaction of Kac-Moody groups and twin buildings.
These methods have proven fruitful over and over again in
proving, simplifying and generalizing several results in group theory and have had their impact in other areas of mathematics.

In my talk I attempt to give a report on the results and on the developments in recent years in the project called Curtis-Phan-Tits Theory (or, short, Phan Theory). This project has been initiated by Bennett, Hoffman, Shpectorov and myself with the goal to revise Phan's results in order to make them accessible for the ongoing revision of the classification of the finite simple groups. While this original project can considered finished, I would also like to point out what problems might be of interest in the cases of infinite fields and infinite dimension.