The smallest minimal blocking sets of $\mathrm{Q}(2n, q)$, for small odd q

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In [2] we used results on the size of the smallest minimal blocking sets of $\mathrm{Q}(4, q)$, q even (from [1]) and projection arguments the find the following characterization of the smallest minimal blocking sets of $\mathrm{Q}(6, q)$, q even, $q \geqslant 32$:

Theorem 1 *Let $\mathcal{K}$ be a minimal blocking set of $\mathrm{Q}(6, q)$, different from an ovoid of $\mathrm{Q}(6, q)$, $|\mathcal{K}| \leqslant q^3 + q$. Then there is a point $p \in \mathrm{Q}(6, q) \setminus \mathcal{K}$ with the following property: $T_p(\mathrm{Q}(6, q)) \cap \mathrm{Q}(6, q) = p\mathrm{Q}(4, q)$and $\mathcal{K}$ consists of all the points of the lines L on p meeting $\mathrm{Q}(4, q)$ in an ovoid $\mathcal{O}$, minus the point p itself, and $|\mathcal{K}| = q^3 + q$.*

The results of [1] for $\mathrm{Q}(4, q)$, q even, could be extended to $q = 3, 5, 7$. Then using the same projection arguments we proved the above characterization for $q = 3, 5, 7$.

Using inductive arguments we can find analogous results for $\mathrm{Q}(2n, q), q = 3, 5, 7$. The situation is now dependent on q, since $\mathrm{Q}(6, 3)$ has an ovoid, but $\mathrm{Q}(6, q)$, $q = 5, 7$, not. For $q = 5, 7$, we proved the following characterization.

Theorem 2 *Let $\mathcal{K}$ be a minimal blocking set of $\mathrm{Q}(2n + 2, q)$, $n \geqslant 2$, $|\mathcal{K}| \leqslant q^{n+1} + q^{n-1}$. Then there is an $(n - 2)$-dimensional space π, $\pi \subset \mathrm{Q}(2n + 2, q)$, $\pi \cap \mathcal{K} = \emptyset$, with the following property: $T_\pi(\mathrm{Q}(2n+2, q)) \cap \mathrm{Q}(2n+2, q) = \pi\mathrm{Q}(4, q)$ and $\mathcal{K}$ is a cone with vertex π and base $\mathcal{O}$, where $\mathcal{O}$ is an ovoid of $\mathrm{Q}(4, q)$, minus the points of the vertex π, and $|\mathcal{K}| = q^{n+1} + q^{n-1}$.*

For $q = 3$ we proved a characterization using ovoids of $\mathrm{Q}(6, 3)$.

Theorem 3 *Let $\mathcal{K}$ be a minimal blocking set of $\mathrm{Q}(2n + 2, 3)$, $n \geqslant 3$, $|\mathcal{K}| \leqslant q^{n+1} + q^{n-2}$. Then there is an $(n - 3)$-dimensional space π, $\pi \subset \mathrm{Q}(2n + 2, 3)$, $\pi \cap \mathcal{K} = \emptyset$, with the following property: $T_\pi(\mathrm{Q}(2n+2, 3)) \cap \mathrm{Q}(2n+2, 3) = \pi\mathrm{Q}(6, 3)$ and $\mathcal{K}$ is a cone with vertex π and base $\mathcal{O}$, where $\mathcal{O}$ is an ovoid of $\mathrm{Q}(6, 3)$, minus the points of the vertex π, and $|\mathcal{K}| = q^{n+1} + q^{n-2}$.*

We will discuss several aspects of the theorems and the difficulties which arise for other values of q.

References

1. J. Eisfeld, L. Storme, T. Szőnyi, and P. Sziklai, *Covers and blocking sets of classical generalized quadrangles*, Discrete Math., 238(1-3):35–51, 2001.

2. J. De Beule and L. Storme, *The smallest minimal blocking sets of* $\mathrm{Q}(6, q)$, *q even*, J. Combin. Des., to appear.