

The Hermitian variety $H(5, 4)$ has no ovoid

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We consider the Hermitian varieties $H(2n + 1, q^2)$. An ovoid \mathcal{O} is a set of points of $H(2n + 1, q^2)$ such that every generator of $H(2n + 1, q^2)$ meets the set \mathcal{O} in exactly one point.

In general, a lot of research has been done to prove the existence or non-existence of ovoids of classical polar spaces. One of the open cases are the Hermitian varieties $H(2n + 1, q^2)$. For $n = 1$, every Hermitian curve $H(2, q^2)$ contained in $H(3, q^2)$ constitutes an ovoid of $H(3, q^2)$, and even different examples can be found. On the other hand, no Hermitian variety $H(2n + 1, q^2)$, $n \geq 2$ having ovoids is known. Furthermore, it is known ([1]) that $H(2n + 1, q^2)$, $q = p^h$, p prime has no ovoid when

$$p^{2n+1} > \left(\frac{2n+p}{2n+1}\right)^2 - \left(\frac{2n+p-1}{2n+1}\right)^2.$$

From this it follows that for each prime p there exists an integer n_p such that $H(2n + 1, q^2)$ with $n \geq n_p$ has no ovoid. All obtained integers n_p are larger than two. Thus, for no polar space $H(5, q^2)$ the existence of an ovoid has been decided. We introduce a combinatorial approach that shows that $H(5, 4)$ has no ovoid.

References

- [1] G. Eric Moorhouse. Some p -ranks related to Hermitian varieties. *J. Statist. Plann. Inference*, 56(2):229–241, 1996. Special issue on orthogonal arrays and affine designs, Part II.