

On the smallest minimal blocking sets of $Q(2n, q)$

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Using results on the size of the smallest minimal blocking sets of $Q(4, q)$, q even, of Eisfeld, Storme, Szőnyi and Sziklai [2], and results concerning the number of internal nuclei of $(q+2)$ -sets in $PG(2, q)$, q even, of Bichara and Korchmáros [1], together with projection arguments, we obtain the following characterization of the smallest minimal blocking sets of $Q(6, q)$, q even and $q \geq 32$:

Theorem 1 *Let \mathcal{K} be a minimal blocking set of $Q(6, q)$, q even, $|\mathcal{K}| \leq q^3 + q$, $q \geq 32$. Then there is a point $p \in Q(6, q) \setminus \mathcal{K}$ with the following property: $T_p(Q(6, q)) \cap Q(6, q) = pQ(4, q)$ and \mathcal{K} consists of all the points of the lines L on p meeting $Q(4, q)$ in an ovoid \mathcal{O} , minus the point p itself, and $|\mathcal{K}| = q^3 + q$.*

Replacing the used results on $Q(4, q)$ by a computer result on $Q(4, 5)$, we can apply the projection arguments to prove the theorem for the case $q = 5$. Generalizing the proof for $Q(6, 5)$ we find the characterization of the smallest minimal blocking sets of $Q(8, 5)$:

Theorem 2 *Let \mathcal{K} be a minimal blocking set of $Q(8, 5)$, $|\mathcal{K}| \leq q^4 + q^2$. Then there is a line $L \subset Q(8, 5)$, $L \cap \mathcal{K} = \emptyset$, with the following property: $T_L(Q(8, 5)) \cap Q(8, 5) = LQ(4, 5)$ and \mathcal{K} consists of all the points of the lines M on p_i , $p_i \in L$, meeting $Q(4, q)$ in an ovoid \mathcal{O} , minus the points p_i themselves, and $|\mathcal{K}| = q^4 + q^2$.*

We will discuss the use of inductive arguments to obtain the characterization of the smallest minimal blocking sets of $Q(2n, 5)$.

References

1. A. Bichara and G. Korchmáros, *Note on $(q+2)$ -sets in a Galois plane of order q* , Combinatorial and geometric structures and their applications (Trento, 1980), pages 117–121. North-Holland, Amsterdam, 1982.
2. J. Eisfeld, L. Storme, T. Szőnyi, and P. Sziklai, *Covers and blocking sets of classical generalized quadrangles*, Discrete Math., 238(1-3):35–51, 2001.