On a special type of solutions for the higher spin Dirac operator $\mathcal{Q}_{k,l}$

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In this talk, we will determine a specific set of solutions of the higher spin Dirac Operators $Q_{k,l}$. These are operators of the form $\pi_{k,l}\underline{\partial}_x : \mathcal{C}^\infty(\mathbb{R}, \mathcal{S}_{k,l}) \to \mathcal{C}^\infty(\mathbb{R}, \mathcal{S}_{k,l})$. Here, $\pi_{k,l}$ is a projection operator, $\underline{\partial}_x$ is the Dirac Operator, and $\mathcal{S}_{k,l}$ is the space of spinor valued polynomials, homogeneous of degree k and l in 2 vector variables $\underline{u}$ and $\underline{v}$ respectively, such that they are in the kernel of $\underline{\partial}_u, \underline{\partial}_v$ and $\langle \underline{u}, \underline{\partial}_v \rangle$. The latter space is called the space of simplicial monogenics. We will determine the set of solutions of this operator, while also being in the kernel of $\underline{\partial}_x$. To that end, we will introduce the space of triple monogenics, which are spinor valued polynomials in 3 vector variables $\underline{x}, \underline{u}, \underline{v}$, killed by $\underline{\partial}_x, \underline{\partial}_u$ and $\underline{\partial}_v$, and make a connection to representation theory. More specifically, we will use irreducible representations of the Lie Algebra $\mathfrak{sl}_3\mathbb{C}$, an algebra that, when realized in Clifford Algebra, can be seen as an operator algebra, generated by $\langle \underline{x}, \underline{\partial}_u \rangle, \langle \underline{x}, \underline{\partial}_v \rangle, \langle \underline{u}, \underline{\partial}_v \rangle, \langle \underline{u}, \underline{\partial}_x \rangle, \langle \underline{v}, \underline{\partial}_x \rangle, \langle \underline{v}, \underline{\partial}_u \rangle, \mathbb{E}_x - \mathbb{E}_u, \mathbb{E}_x - \mathbb{E}_v$ and $\mathbb{E}_u - \mathbb{E}_v$, where $\mathbb{E}$ is the Euler Operator.