```
> restart;
```

```
> with(linalg);
```

[*BlockDiagonal*, *GramSchmidt*, *JordanBlock*, *LUdecomp*, *QRdecomp*, *Wronskian*, *addcol*, *addrow*, *adj*, *adjoint*, *angle*, *augment*, *backsub*, *band*, *basis*, *bezout*, *blockmatrix*, *charmat*, *charpoly*, *cholesky*, *col*, *coldim*, *colspace*, *colspan*, *companion*, *concat*, *cond*, *copyinto*, *crossprod*, *curl*, *definite*, *delcols*, *delrows*, *det*, *diag*, *diverge*, *dotprod*, *eigenvals*, *eigenvalues*, *eigenvectors*, *eigenvects*, *entermatrix*, *equal*, *exponential*, *extend*, *ffgausselim*, *fibonacci*, *forwardsub*, *frobenius*, *gausselim*, *gaussjord*, *geneqns*, *genmatrix*, *grad*, *hadamard*, *hermite*, *hessian*, *hilbert*, *htranspose*, *ihermite*, *indexfunc*, *innerprod*, *intbasis*, *inverse*, *ismith*, *issimilar*, *iszero*, *jacobian*, *jordan*, *kernel*, *laplacian*, *leastsqrs*, *linsolve*, *matadd*, *matrix*, *minor*, *minpoly*, *mulcol*, *mulrow*, *multiply*, *norm*, *normalize*, *nullspace*, *orthog*, *permanent*, *pivot*, *potential*, *randmatrix*, *randvector*, *rank*, *ratform*, *row*, *rowdim*, *rowspace*, *rowspan*, *rref*, *scalarmul*, *singularvals*, *smith*, *stackmatrix*, *submatrix*, *subvector*, *sumbasis*, *swapcol*, *swaprow*, *sylvester*, *toeplitz*, *trace*, *transpose*, *vandermonde*, *vecpotent*, *vectdim*, *vector*, *wronskian*] **(1)**

```
> M0 := Matrix([[1, 0, 0, 0, 0], [0, 1, 0, 0, 0], [0, 0, 1, 0, 0], [0, 0, 0, 1, 0], [0, 0, 0, 0, 1]]);
```

$$M0 := \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \quad \textbf{(2)}$$

```
> M1 := Matrix([[0, 1, 0, 0, 0], [30, 1, 3, 5, 0], [0, 12, 3, 0, 9], [0, 16, 0, 5, 6], [0, 0, 24, 20, 15]]);
```

$$M1 := \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 30 & 1 & 3 & 5 & 0 \\ 0 & 12 & 3 & 0 & 9 \\ 0 & 16 & 0 & 5 & 6 \\ 0 & 0 & 24 & 20 & 15 \end{bmatrix} \quad \textbf{(3)}$$

```
> M2 := Matrix([[0, 0, 1, 0, 0], [0, 12, 3, 0, 9], [120, 12, 36, 30, 21], [0, 0, 24, 30, 18], [0, 96, 56, 60, 72]]);
```

(4)

$$M2 := \begin{bmatrix} 0 & 0 & 1 & 0 & 0 \\ 0 & 12 & 3 & 0 & 9 \\ 120 & 12 & 36 & 30 & 21 \\ 0 & 0 & 24 & 30 & 18 \\ 0 & 96 & 56 & 60 & 72 \end{bmatrix} \quad (4)$$

> $M3 := Matrix([[0, 0, 0, 1, 0], [0, 16, 0, 5, 6], [0, 0, 24, 30, 18], [96, 16, 24, 20, 12], [0, 64, 48, 40, 60]]);$

$$M3 := \begin{bmatrix} 0 & 0 & 0 & 1 & 0 \\ 0 & 16 & 0 & 5 & 6 \\ 0 & 0 & 24 & 30 & 18 \\ 96 & 16 & 24 & 20 & 12 \\ 0 & 64 & 48 & 40 & 60 \end{bmatrix} \quad (5)$$

> $M4 := Matrix([[0, 0, 0, 0, 1], [0, 0, 24, 20, 15], [0, 96, 56, 60, 72], [0, 64, 48, 40, 60], [320, 160, 192, 200, 172]]);$

$$M4 := \begin{bmatrix} 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 24 & 20 & 15 \\ 0 & 96 & 56 & 60 & 72 \\ 0 & 64 & 48 & 40 & 60 \\ 320 & 160 & 192 & 200 & 172 \end{bmatrix} \quad (6)$$

> $eigvals1 := eigenvalues(M1);$

$$eigvals1 := -15, 3, 9, 30, -3 \quad (7)$$

> $eigvals2 := eigenvalues(M2);$

$$eigvals2 := 12, 30, 120, -6, -6 \quad (8)$$

> $eigvals3 := eigenvalues(M3);$

$$eigvals3 := 0, 96, 12, -12, 24 \quad (9)$$

> $eigvals4 := eigenvalues(M4);$

$$eigvals4 := 320, -40, -16, -4, 8 \quad (10)$$

>

>

>

> $M := M1;$

$$M := \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 30 & 1 & 3 & 5 & 0 \\ 0 & 12 & 3 & 0 & 9 \\ 0 & 16 & 0 & 5 & 6 \\ 0 & 0 & 24 & 20 & 15 \end{bmatrix} \quad (11)$$

> $eigvals := eigvals1;$

$$eigvals := -15, 3, 9, 30, -3 \quad (12)$$

> $l1 := -15;$

$$l1 := -15 \quad (13)$$

> $l2 := -3;$

$$l2 := -3 \quad (14)$$

> $l3 := 3;$

$$l3 := 3 \quad (15)$$

> $l4 := 9;$

$$l4 := 9 \quad (16)$$

> $l5 := 30;$

$$l5 := 30 \quad (17)$$

> $e1 := 567;$

$$e1 := 567 \quad (18)$$

> $e2 := 0;$

$$e2 := 0 \quad (19)$$

> $e3 := expand(evalm([0, 1, 0, 0, 0].M)[1] \cdot e1);$

$$e3 := 17010 \quad (20)$$

> $e4 := expand(evalm([0, 1, 0, 0, 0].M^2)[1] \cdot e1);$

$$e4 := 17010 \quad (21)$$

> $e5 := expand(evalm([0, 1, 0, 0, 0].M^3)[1] \cdot e1);$

$$e5 := 2500470 \quad (22)$$

> $V := transpose(vandermonde([l1, l2, l3, l4, l5]));$

(23)

$$V := \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ -15 & -3 & 3 & 9 & 30 \\ 225 & 9 & 9 & 81 & 900 \\ -3375 & -27 & 27 & 729 & 27000 \\ 50625 & 81 & 81 & 6561 & 810000 \end{bmatrix} \quad (23)$$

> $W := evalm(\ inverse(V).transpose(Matrix([e1, e2, e3, e4, e5]))\ );$

$$W := \begin{bmatrix} 21 \\ 315 \\ 140 \\ 90 \\ 1 \end{bmatrix} \quad (24)$$

>

>

>

> $M := M2;$

$$M := \begin{bmatrix} 0 & 0 & 1 & 0 & 0 \\ 0 & 12 & 3 & 0 & 9 \\ 120 & 12 & 36 & 30 & 21 \\ 0 & 0 & 24 & 30 & 18 \\ 0 & 96 & 56 & 60 & 72 \end{bmatrix} \quad (25)$$

> $eigvals := eigvals2;$

$$eigvals := 12, 30, 120, -6, -6 \quad (26)$$

> $l1 := -6;$

$$l1 := -6 \quad (27)$$

> $l2 := 12;$

$$l2 := 12 \quad (28)$$

> $l3 := 30;$

$$l3 := 30 \quad (29)$$

> $l4 := 120;$

$$l4 := 120 \quad (30)$$

> $e1 := 567;$

$$e1 := 567 \quad (31)$$

```
> e2 := 0;
```

$$e2 := 0 \tag{32}$$

```
> e3 := expand(evalm([0, 0, 1, 0, 0].M)[1] · e1);
```

$$e3 := 68040 \tag{33}$$

```
> e4 := expand(evalm([0, 0, 1, 0, 0].M^2)[1]·e1);
```

$$e4 := 2449440 \tag{34}$$

```
> V := transpose(vandermonde([l1, l2, l3, l4]));
```

$$V := \begin{bmatrix} 1 & 1 & 1 & 1 \\ -6 & 12 & 30 & 120 \\ 36 & 144 & 900 & 14400 \\ -216 & 1728 & 27000 & 1728000 \end{bmatrix} \tag{35}$$

```
> W := evalm( inverse(V).transpose(Matrix([e1, e2, e3, e4])));
```

$$W := \begin{bmatrix} 405 \\ 140 \\ 21 \\ 1 \end{bmatrix} \tag{36}$$

```
>
>
>
> M := M3;
```

$$M := \begin{bmatrix} 0 & 0 & 0 & 1 & 0 \\ 0 & 16 & 0 & 5 & 6 \\ 0 & 0 & 24 & 30 & 18 \\ 96 & 16 & 24 & 20 & 12 \\ 0 & 64 & 48 & 40 & 60 \end{bmatrix} \tag{37}$$

```
> eigvals := eigvals3;
```

$$eigvals := 0, 96, 12, -12, 24 \tag{38}$$

```
> l1 := -12;
```

$$l1 := -12 \tag{39}$$

```
> l2 := 0;
```

$$l2 := 0 \tag{40}$$

```
> l3 := 12;
```

$$l3 := 12 \tag{41}$$

```
> l4 := 24;
```

$$l4 := 24 \tag{42}$$

```
> l5 := 96;
```

$$l5 := 96 \tag{43}$$

```
> e1 := 567;
```

$$e1 := 567 \tag{44}$$

```
> e2 := 0;
```

$$e2 := 0 \tag{45}$$

```
> e3 := expand(evalm([0, 0, 0, 1, 0].M)[1] · e1);
```

$$e3 := 54432 \tag{46}$$

```
> e4 := expand(evalm([0, 0, 0, 1, 0].M^2)[1]·e1);
```

$$e4 := 1088640 \tag{47}$$

```
> e5 := expand(evalm([0, 0, 0, 1, 0].M^3)[1]·e1);
```

$$e5 := 96671232 \tag{48}$$

```
> V := transpose(vandermonde([l1, l2, l3, l4, l5]));
```

$$V := \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ -12 & 0 & 12 & 24 & 96 \\ 144 & 0 & 144 & 576 & 9216 \\ -1728 & 0 & 1728 & 13824 & 884736 \\ 20736 & 0 & 20736 & 331776 & 84934656 \end{bmatrix} \tag{49}$$

```
> W := evalm( inverse(V).transpose(Matrix([e1, e2, e3, e4, e5])));
```

$$W := \begin{bmatrix} 140 \\ 315 \\ 90 \\ 21 \\ 1 \end{bmatrix} \tag{50}$$

```
>
>
>
> M := M4;
```

$$M := \begin{bmatrix} 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 24 & 20 & 15 \\ 0 & 96 & 56 & 60 & 72 \\ 0 & 64 & 48 & 40 & 60 \\ 320 & 160 & 192 & 200 & 172 \end{bmatrix} \quad (51)$$

> $eigvals := eigvals4;$

$$eigvals := 320, -40, -16, -4, 8 \quad (52)$$

> $l1 := -40;$

$$l1 := -40 \quad (53)$$

> $l2 := -16;$

$$l2 := -16 \quad (54)$$

> $l3 := -4;$

$$l3 := -4 \quad (55)$$

> $l4 := 8;$

$$l4 := 8 \quad (56)$$

> $l5 := 320;$

$$l5 := 320 \quad (57)$$

> $e1 := 567;$

$$e1 := 567 \quad (58)$$

> $e2 := 0;$

$$e2 := 0 \quad (59)$$

> $e3 := expand(evalm([0, 0, 0, 0, 1].M)[1] \cdot e1);$

$$e3 := 181440 \quad (60)$$

> $e4 := expand(evalm([0, 0, 0, 0, 1].M^2)[1] \cdot e1);$

$$e4 := 31207680 \quad (61)$$

> $e5 := expand(evalm([0, 0, 0, 0, 1].M^3)[1] \cdot e1);$

$$e5 := 10546744320 \quad (62)$$

> $V := transpose(vandermonde([l1, l2, l3, l4, l5]));$

(63)

$$V := \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ -40 & -16 & -4 & 8 & 320 \\ 1600 & 256 & 16 & 64 & 102400 \\ -64000 & -4096 & -64 & 512 & 32768000 \\ 2560000 & 65536 & 256 & 4096 & 10485760000 \end{bmatrix} \quad \textbf{(63)}$$

```
> W := evalm( inverse(V).transpose(Matrix([e1, e2, e3, e4, e5])) );
```

$$W := \begin{bmatrix} 21 \\ 90 \\ 140 \\ 315 \\ 1 \end{bmatrix} \quad \textbf{(64)}$$

```
> with(CurveFitting);
```

$$[ArrayInterpolation, BSpline, BSplineCurve, Interactive, LeastSquares, Lowess, PolynomialInterpolation, RationalInterpolation, Spline, ThieleInterpolation] \quad \textbf{(65)}$$

```
> P2a := PolynomialInterpolation([[-15, 30], [-3, -6], [3, 12], [9, -6], [30, 120]], x);
```

$$P2a := \frac{19}{14256} x^4 - \frac{127}{4752} x^3 - \frac{601}{1584} x^2 + \frac{1711}{528} x + \frac{555}{88} \quad \textbf{(66)}$$

```
> P2b := PolynomialInterpolation([[-15, 24], [-3, 0], [3, -12], [9, 12], [30, 96]], x);
```

$$P2b := -\frac{19}{23760} x^4 + \frac{127}{7920} x^3 + \frac{1129}{2640} x^2 - \frac{1887}{880} x - \frac{861}{88} \quad \textbf{(67)}$$

```
> P2 := PolynomialInterpolation([[-15, 54], [-3, -6], [3, 0], [9, 6], [30, 216]], x);
```

$$P2 := \frac{19}{35640} x^4 - \frac{127}{11880} x^3 + \frac{191}{3960} x^2 + \frac{1447}{1320} x - \frac{153}{44} \quad \textbf{(68)}$$

```
> P3 := PolynomialInterpolation([[-15, -40], [-3, 8], [3, -4], [9, -16], [30, 320]], x);
```

$$P3 := \frac{1}{7128} x^4 + \frac{35}{2376} x^3 - \frac{115}{792} x^2 - \frac{563}{264} x + \frac{145}{44} \quad \textbf{(69)}$$

```
>
```