> *restart*;

> *with*(*linalg*);

[*BlockDiagonal, GramSchmidt, JordanBlock, LUdecomp, QRdecomp, Wronskian, addcol,*   **(1)**
    addrow, adj, adjoint, angle, augment, backsub, band, basis, bezout, blockmatrix, charmat,
    charpoly, cholesky, col, coldim, colspace, colspan, companion, concat, cond, copyinto,
    crossprod, curl, definite, delcols, delrows, det, diag, diverge, dotprod, eigenvals,
    eigenvalues, eigenvectors, eigenvects, entermatrix, equal, exponential, extend, ffgausselim,
    fibonacci, forwardsub, frobenius, gausselim, gaussjord, geneqns, genmatrix, grad,
    hadamard, hermite, hessian, hilbert, htranspose, ihermite, indexfunc, innerprod, intbasis,
    inverse, ismith, issimilar, iszero, jacobian, jordan, kernel, laplacian, leastsqrs, linsolve,
    matadd, matrix, minor, minpoly, mulcol, mulrow, multiply, norm, normalize, nullspace,
    orthog, permanent, pivot, potential, randmatrix, randvector, rank, ratform, row, rowdim,
    rowspace, rowspan, rref, scalarmul, singularvals, smith, stackmatrix, submatrix, subvector,
    sumbasis, swapcol, swaprow, sylvester, toeplitz, trace, transpose, vandermonde, vecpotent,
    vectdim, vector, wronskian]

> $M0 := Matrix([[1, 0, 0, 0, 0, 0, 0, 0, 0, 0], [0, 1, 0, 0, 0, 0, 0, 0, 0, 0], [0, 0, 1, 0, 0, 0, 0, 0, 0, 0], [0, 0, 0, 1, 0, 0, 0, 0, 0, 0], [0, 0, 0, 0, 1, 0, 0, 0, 0, 0], [0, 0, 0, 0, 0, 1, 0, 0, 0, 0], [0, 0, 0, 0, 0, 0, 1, 0, 0, 0], [0, 0, 0, 0, 0, 0, 0, 1, 0, 0], [0, 0, 0, 0, 0, 0, 0, 0, 1, 0], [0, 0, 0, 0, 0, 0, 0, 0, 0, 1]]);$

$$M0 := \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$
    (2)

> $M1 := Matrix([[0, 1, 0, 0, 0, 0, 0, 0, 0, 0], [6, 1, 0, 0, 0, 2, 0, 0, 0, 0],$
    $[0, 0, 0, 1, 0, 0, 2, 0, 0, 0], [0, 0, 2, 1, 0, 0, 0, 0, 2, 0],$
    $[0, 0, 0, 0, 0, 0, 0, 0, 0, 3], [0, 4, 0, 0, 0, 2, 0, 3, 0, 0],$
    $[0, 0, 4, 0, 0, 0, 2, 0, 1, 0], [0, 0, 0, 0, 0, 2, 0, 3, 0, 0],$
    $[0, 0, 0, 4, 0, 0, 2, 0, 3, 0], [0, 0, 0, 0, 6, 0, 0, 0, 0, 3]]);$

$$M1 := \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 6 & 1 & 0 & 0 & 0 & 2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 2 & 0 & 0 & 0 \\ 0 & 0 & 2 & 1 & 0 & 0 & 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 3 \\ 0 & 4 & 0 & 0 & 0 & 2 & 0 & 3 & 0 & 0 \\ 0 & 0 & 4 & 0 & 0 & 0 & 2 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 2 & 0 & 3 & 0 & 0 \\ 0 & 0 & 0 & 4 & 0 & 0 & 2 & 0 & 3 & 0 \\ 0 & 0 & 0 & 0 & 6 & 0 & 0 & 0 & 0 & 3 \end{bmatrix}$$ (3)

> $M2 := Matrix([[0, 0, 1, 0, 0, 0, 0, 0, 0, 0], [0, 0, 0, 1, 0, 0, 2, 0, 0, 0],$
  $[18, 0, 1, 0, 3, 3, 1, 0, 0, 0], [0, 6, 0, 1, 0, 0, 0, 9, 1, 3],$
  $[0, 0, 12, 0, 3, 0, 0, 0, 6, 3], [0, 0, 2, 0, 0, 0, 1, 0, 2, 0],$
  $[0, 12, 2, 0, 0, 3, 2, 0, 0, 3], [0, 0, 0, 2, 0, 0, 0, 0, 1, 0],$
  $[0, 0, 0, 2, 6, 12, 0, 9, 2, 3], [0, 0, 0, 12, 6, 0, 12, 0, 6, 6]]);$

$$M2 := \begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 2 & 0 & 0 & 0 \\ 18 & 0 & 1 & 0 & 3 & 3 & 1 & 0 & 0 & 0 \\ 0 & 6 & 0 & 1 & 0 & 0 & 0 & 9 & 1 & 3 \\ 0 & 0 & 12 & 0 & 3 & 0 & 0 & 0 & 6 & 3 \\ 0 & 0 & 2 & 0 & 0 & 0 & 1 & 0 & 2 & 0 \\ 0 & 12 & 2 & 0 & 0 & 3 & 2 & 0 & 0 & 3 \\ 0 & 0 & 0 & 2 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 2 & 6 & 12 & 0 & 9 & 2 & 3 \\ 0 & 0 & 0 & 12 & 6 & 0 & 12 & 0 & 6 & 6 \end{bmatrix}$$ (4)

> $M3 := Matrix([[0, 0, 0, 1, 0, 0, 0, 0, 0, 0], [0, 0, 2, 1, 0, 0, 0, 0, 2, 0],$
  $[0, 6, 0, 1, 0, 0, 0, 9, 1, 3], [36, 6, 2, 1, 6, 6, 2, 9, 1, 3],$
  $[0, 0, 0, 12, 6, 0, 12, 0, 6, 6], [0, 0, 0, 2, 0, 0, 4, 0, 3, 0],$
  $[0, 0, 0, 2, 6, 12, 0, 9, 2, 3], [0, 0, 4, 2, 0, 0, 2, 0, 1, 0],$
  $[0, 24, 4, 2, 6, 18, 4, 9, 2, 9], [0, 0, 24, 12, 12, 0, 12, 0, 18, 12]]);$

(5)

$$M3 := \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 2 & 1 & 0 & 0 & 0 & 0 & 2 & 0 \\ 0 & 6 & 0 & 1 & 0 & 0 & 0 & 9 & 1 & 3 \\ 36 & 6 & 2 & 1 & 6 & 6 & 2 & 9 & 1 & 3 \\ 0 & 0 & 0 & 12 & 6 & 0 & 12 & 0 & 6 & 6 \\ 0 & 0 & 0 & 2 & 0 & 0 & 4 & 0 & 3 & 0 \\ 0 & 0 & 0 & 2 & 6 & 12 & 0 & 9 & 2 & 3 \\ 0 & 0 & 4 & 2 & 0 & 0 & 2 & 0 & 1 & 0 \\ 0 & 24 & 4 & 2 & 6 & 18 & 4 & 9 & 2 & 9 \\ 0 & 0 & 24 & 12 & 12 & 0 & 12 & 0 & 18 & 12 \end{bmatrix}$$ (5)

> $M4 := Matrix([[0, 0, 0, 0, 1, 0, 0, 0, 0, 0], [0, 0, 0, 0, 0, 0, 0, 0, 0, 3],$
  $[0, 0, 12, 0, 3, 0, 0, 0, 6, 3], [0, 0, 0, 12, 6, 0, 12, 0, 6, 6],$
  $[72, 0, 12, 12, 18, 36, 12, 18, 12, 9], [0, 0, 0, 0, 6, 0, 0, 0, 0, 3],$
  $[0, 0, 0, 12, 6, 0, 12, 0, 6, 6], [0, 0, 0, 0, 2, 0, 0, 0, 0, 3],$
  $[0, 0, 24, 12, 12, 0, 12, 0, 18, 12],$
  $[0, 72, 24, 24, 18, 36, 24, 54, 24, 27]]);$

$$M4 := \begin{bmatrix} 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 3 \\ 0 & 0 & 12 & 0 & 3 & 0 & 0 & 0 & 6 & 3 \\ 0 & 0 & 0 & 12 & 6 & 0 & 12 & 0 & 6 & 6 \\ 72 & 0 & 12 & 12 & 18 & 36 & 12 & 18 & 12 & 9 \\ 0 & 0 & 0 & 0 & 6 & 0 & 0 & 0 & 0 & 3 \\ 0 & 0 & 0 & 12 & 6 & 0 & 12 & 0 & 6 & 6 \\ 0 & 0 & 0 & 0 & 2 & 0 & 0 & 0 & 0 & 3 \\ 0 & 0 & 24 & 12 & 12 & 0 & 12 & 0 & 18 & 12 \\ 0 & 72 & 24 & 24 & 18 & 36 & 24 & 54 & 24 & 27 \end{bmatrix}$$ (6)

> $M5 := Matrix([[0, 0, 0, 0, 0, 1, 0, 0, 0, 0], [0, 4, 0, 0, 0, 2, 0, 3, 0, 0],$
  $[0, 0, 2, 0, 0, 0, 1, 0, 2, 0], [0, 0, 0, 2, 0, 0, 4, 0, 3, 0],$
  $[0, 0, 0, 0, 6, 0, 0, 0, 0, 3], [12, 4, 0, 0, 0, 5, 0, 6, 0, 0],$
  $[0, 0, 2, 4, 0, 0, 3, 0, 2, 0], [0, 4, 0, 0, 0, 4, 0, 3, 0, 0],$
  $[0, 0, 8, 6, 0, 0, 4, 0, 5, 0], [0, 0, 0, 0, 6, 0, 0, 0, 0, 9]]);$

$$M5 := \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 4 & 0 & 0 & 0 & 2 & 0 & 3 & 0 & 0 \\ 0 & 0 & 2 & 0 & 0 & 0 & 1 & 0 & 2 & 0 \\ 0 & 0 & 0 & 2 & 0 & 0 & 4 & 0 & 3 & 0 \\ 0 & 0 & 0 & 0 & 6 & 0 & 0 & 0 & 0 & 3 \\ 12 & 4 & 0 & 0 & 0 & 5 & 0 & 6 & 0 & 0 \\ 0 & 0 & 2 & 4 & 0 & 0 & 3 & 0 & 2 & 0 \\ 0 & 4 & 0 & 0 & 0 & 4 & 0 & 3 & 0 & 0 \\ 0 & 0 & 8 & 6 & 0 & 0 & 4 & 0 & 5 & 0 \\ 0 & 0 & 0 & 0 & 6 & 0 & 0 & 0 & 0 & 9 \end{bmatrix}$$ (7)

> $M6 := Matrix([[0, 0, 0, 0, 0, 0, 1, 0, 0, 0], [0, 0, 4, 0, 0, 0, 2, 0, 1, 0],$
  $[0, 12, 2, 0, 0, 3, 2, 0, 0, 3], [0, 0, 0, 2, 6, 12, 0, 9, 2, 3],$
  $[0, 0, 0, 12, 6, 0, 12, 0, 6, 6], [0, 0, 2, 4, 0, 0, 3, 0, 2, 0],$
  $[36, 12, 4, 0, 6, 9, 4, 0, 0, 3], [0, 0, 0, 2, 0, 0, 0, 0, 3, 0],$
  $[0, 12, 0, 4, 6, 12, 0, 27, 4, 9], [0, 0, 24, 12, 12, 0, 12, 0, 18, 12]]);$

$$M6 := \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 4 & 0 & 0 & 0 & 2 & 0 & 1 & 0 \\ 0 & 12 & 2 & 0 & 0 & 3 & 2 & 0 & 0 & 3 \\ 0 & 0 & 0 & 2 & 6 & 12 & 0 & 9 & 2 & 3 \\ 0 & 0 & 0 & 12 & 6 & 0 & 12 & 0 & 6 & 6 \\ 0 & 0 & 2 & 4 & 0 & 0 & 3 & 0 & 2 & 0 \\ 36 & 12 & 4 & 0 & 6 & 9 & 4 & 0 & 0 & 3 \\ 0 & 0 & 0 & 2 & 0 & 0 & 0 & 0 & 3 & 0 \\ 0 & 12 & 0 & 4 & 6 & 12 & 0 & 27 & 4 & 9 \\ 0 & 0 & 24 & 12 & 12 & 0 & 12 & 0 & 18 & 12 \end{bmatrix}$$ (8)

> $M7 := Matrix([[0, 0, 0, 0, 0, 0, 0, 1, 0, 0], [0, 0, 0, 0, 0, 2, 0, 3, 0, 0],$
  $[0, 0, 0, 2, 0, 0, 0, 0, 1, 0], [0, 0, 4, 2, 0, 0, 2, 0, 1, 0],$
  $[0, 0, 0, 0, 2, 0, 0, 0, 0, 3], [0, 4, 0, 0, 0, 4, 0, 3, 0, 0],$
  $[0, 0, 0, 2, 0, 0, 0, 0, 3, 0], [8, 4, 0, 0, 0, 2, 0, 1, 0, 0],$
  $[0, 0, 4, 2, 0, 0, 6, 0, 3, 0], [0, 0, 0, 0, 6, 0, 0, 0, 0, 5]]);$

(9)

$$M7 := \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 2 & 0 & 3 & 0 & 0 \\ 0 & 0 & 0 & 2 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 4 & 2 & 0 & 0 & 2 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 2 & 0 & 0 & 0 & 0 & 3 \\ 0 & 4 & 0 & 0 & 0 & 4 & 0 & 3 & 0 & 0 \\ 0 & 0 & 0 & 2 & 0 & 0 & 0 & 0 & 3 & 0 \\ 8 & 4 & 0 & 0 & 0 & 2 & 0 & 1 & 0 & 0 \\ 0 & 0 & 4 & 2 & 0 & 0 & 6 & 0 & 3 & 0 \\ 0 & 0 & 0 & 0 & 6 & 0 & 0 & 0 & 0 & 5 \end{bmatrix}$$

(9)

> $M8 := Matrix([[0, 0, 0, 0, 0, 0, 0, 0, 1, 0], [0, 0, 0, 4, 0, 0, 2, 0, 3, 0],$
$[0, 0, 0, 2, 6, 12, 0, 9, 2, 3], [0, 24, 4, 2, 6, 18, 4, 9, 2, 9],$
$[0, 0, 24, 12, 12, 0, 12, 0, 18, 12], [0, 0, 8, 6, 0, 0, 4, 0, 5, 0],$
$[0, 12, 0, 4, 6, 12, 0, 27, 4, 9], [0, 0, 4, 2, 0, 0, 6, 0, 3, 0],$
$[72, 36, 8, 4, 18, 30, 8, 27, 4, 15],$
$[0, 0, 24, 36, 24, 0, 36, 0, 30, 24]]);$

$$M8 := \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 4 & 0 & 0 & 2 & 0 & 3 & 0 \\ 0 & 0 & 0 & 2 & 6 & 12 & 0 & 9 & 2 & 3 \\ 0 & 24 & 4 & 2 & 6 & 18 & 4 & 9 & 2 & 9 \\ 0 & 0 & 24 & 12 & 12 & 0 & 12 & 0 & 18 & 12 \\ 0 & 0 & 8 & 6 & 0 & 0 & 4 & 0 & 5 & 0 \\ 0 & 12 & 0 & 4 & 6 & 12 & 0 & 27 & 4 & 9 \\ 0 & 0 & 4 & 2 & 0 & 0 & 6 & 0 & 3 & 0 \\ 72 & 36 & 8 & 4 & 18 & 30 & 8 & 27 & 4 & 15 \\ 0 & 0 & 24 & 36 & 24 & 0 & 36 & 0 & 30 & 24 \end{bmatrix}$$

(10)

> $M9 := Matrix([[0, 0, 0, 0, 0, 0, 0, 0, 0, 1], [0, 0, 0, 0, 6, 0, 0, 0, 0, 3],$
$[0, 0, 0, 12, 6, 0, 12, 0, 6, 6], [0, 0, 24, 12, 12, 0, 12, 0, 18, 12],$
$[0, 72, 24, 24, 18, 36, 24, 54, 24, 27], [0, 0, 0, 0, 6, 0, 0, 0, 0, 9],$
$[0, 0, 24, 12, 12, 0, 12, 0, 18, 12], [0, 0, 0, 0, 6, 0, 0, 0, 0, 5],$
$[0, 0, 24, 36, 24, 0, 36, 0, 30, 24],$
$[144, 72, 48, 48, 54, 108, 48, 90, 48, 45]]);$

$$M9 := \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 6 & 0 & 0 & 0 & 0 & 3 \\ 0 & 0 & 0 & 12 & 6 & 0 & 12 & 0 & 6 & 6 \\ 0 & 0 & 24 & 12 & 12 & 0 & 12 & 0 & 18 & 12 \\ 0 & 72 & 24 & 24 & 18 & 36 & 24 & 54 & 24 & 27 \\ 0 & 0 & 0 & 0 & 6 & 0 & 0 & 0 & 0 & 9 \\ 0 & 0 & 24 & 12 & 12 & 0 & 12 & 0 & 18 & 12 \\ 0 & 0 & 0 & 0 & 6 & 0 & 0 & 0 & 0 & 5 \\ 0 & 0 & 24 & 36 & 24 & 0 & 36 & 0 & 30 & 24 \\ 144 & 72 & 48 & 48 & 54 & 108 & 48 & 90 & 48 & 45 \end{bmatrix}$$ **(11)**

> $eigvals1 := eigenvalues(M1);$
$$eigvals1 := 3, 3, 0, 0, 6, -3, 6, -3, 6, -3$$ **(12)**

> $eigvals2 := eigenvalues(M2);$
$$eigvals2 := 0, 6, 18, 9, -9, -3, 3, -9, -3, 3$$ **(13)**

> $eigvals3 := eigenvalues(M3);$
$$eigvals3 := 0, 36, 3, -9, 9, -18, 6, -6, 6, -6$$ **(14)**

> $eigvals4 := eigenvalues(M4);$
$$eigvals4 := -9, -18, 72, 18, 18, 18, 0, 0, 0, 0$$ **(15)**

> $eigvals5 := eigenvalues(M5);$
$$eigvals5 := -3, -3, 0, 0, 12, 3, 12, 3, 12, 3$$ **(16)**

> $eigvals6 := eigenvalues(M6);$
$$eigvals6 := 0, 36, 3, 6, -9, 9, -6, 12, -3, -18$$ **(17)**

> $eigvals7 := eigenvalues(M7);$
$$eigvals7 := 2, -4, 2, -4, 8, -1, 8, -1, 8, -1$$ **(18)**

> $eigvals8 := eigenvalues(M8);$
$$eigvals8 := 0, 12, -9, -6, 72, -36, 9, -12, 6, 6$$ **(19)**

> $eigvals9 := eigenvalues(M9);$
$$eigvals9 := 144, 36, -36, 9, -18, -18, 0, 0, 0, 0$$ **(20)**

>

>

>

> $M := M1 + M2;$

$$M := \begin{bmatrix} 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 6 & 1 & 0 & 1 & 0 & 2 & 2 & 0 & 0 & 0 \\ 18 & 0 & 1 & 1 & 3 & 3 & 3 & 0 & 0 & 0 \\ 0 & 6 & 2 & 2 & 0 & 0 & 0 & 9 & 3 & 3 \\ 0 & 0 & 12 & 0 & 3 & 0 & 0 & 0 & 6 & 6 \\ 0 & 4 & 2 & 0 & 0 & 2 & 1 & 3 & 2 & 0 \\ 0 & 12 & 6 & 0 & 0 & 3 & 4 & 0 & 1 & 3 \\ 0 & 0 & 0 & 2 & 0 & 2 & 0 & 3 & 1 & 0 \\ 0 & 0 & 0 & 6 & 6 & 12 & 2 & 9 & 5 & 3 \\ 0 & 0 & 0 & 12 & 12 & 0 & 12 & 0 & 6 & 9 \end{bmatrix}$$ (21)

> *eigvals* := *eigenvalues*(*M*);

$$eigvals := 0, 24, 3, 6, -12, 9, 9, -3, -3, -3$$ (22)

> *l1* := -12;

$$l1 := -12$$ (23)

> *l2* := -3;

$$l2 := -3$$ (24)

> *l3* := 0;

$$l3 := 0$$ (25)

> *l4* := 3;

$$l4 := 3$$ (26)

> *l5* := 6;

$$l5 := 6$$ (27)

> *l6* := 9;

$$l6 := 9$$ (28)

> *l7* := 24;

$$l7 := 24$$ (29)

> *e1* := 405;

$$e1 := 405$$ (30)

> *e2* := 0;

$$e2 := 0$$ (31)

> *e3* := *expand*(*evalm*([0, 1, 1, 0, 0, 0, 0, 0, 0, 0].*M*)[1] · *e1*);

$$e3 := 9720$$ (32)

> $e4 := expand(evalm([0, 1, 1, 0, 0, 0, 0, 0, 0, 0].M^2)[1] \cdot e1);$

$$e4 := 9720 \tag{33}$$

> $e5 := expand(evalm([0, 1, 1, 0, 0, 0, 0, 0, 0, 0].M^3)[1] \cdot e1);$

$$e5 := 1049760 \tag{34}$$

> $e6 := expand(evalm([0, 1, 1, 0, 0, 0, 0, 0, 0, 0].M^4)[1] \cdot e1);$

$$e6 := 5423760 \tag{35}$$

> $e7 := expand(evalm([0, 1, 1, 0, 0, 0, 0, 0, 0, 0].M^5)[1] \cdot e1);$

$$e7 := 272675160 \tag{36}$$

> $V := transpose(vandermonde([l1, l2, l3, l4, l5, l6, l7]));$

$$V := \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ -12 & -3 & 0 & 3 & 6 & 9 & 24 \\ 144 & 9 & 0 & 9 & 36 & 81 & 576 \\ -1728 & -27 & 0 & 27 & 216 & 729 & 13824 \\ 20736 & 81 & 0 & 81 & 1296 & 6561 & 331776 \\ -248832 & -243 & 0 & 243 & 7776 & 59049 & 7962624 \\ 2985984 & 729 & 0 & 729 & 46656 & 531441 & 191102976 \end{bmatrix} \tag{37}$$

> $W := evalm(inverse(V).transpose(Matrix([e1, e2, e3, e4, e5, e6, e7])));$

$$W := \begin{bmatrix} 20 \\ 175 \\ 60 \\ 90 \\ 20 \\ 39 \\ 1 \end{bmatrix} \tag{38}$$

>

>

>

> $M := M3 + M4 + M5 + M6;$

$$M := \begin{bmatrix} 0 & 0 & 0 & 1 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 4 & 6 & 1 & 0 & 2 & 2 & 3 & 3 & 3 \\ 0 & 18 & 16 & 1 & 3 & 3 & 3 & 9 & 9 & 9 \\ 36 & 6 & 2 & 17 & 18 & 18 & 18 & 18 & 12 & 12 \\ 72 & 0 & 12 & 36 & 36 & 36 & 36 & 18 & 24 & 24 \\ 12 & 4 & 2 & 6 & 6 & 5 & 7 & 6 & 5 & 3 \\ 36 & 12 & 6 & 18 & 18 & 21 & 19 & 9 & 10 & 12 \\ 0 & 4 & 4 & 4 & 2 & 4 & 2 & 3 & 4 & 3 \\ 0 & 36 & 36 & 24 & 24 & 30 & 20 & 36 & 29 & 30 \\ 0 & 72 & 72 & 48 & 48 & 36 & 48 & 54 & 60 & 60 \end{bmatrix} \tag{39}$$

> $eigvals := eigenvalues(M);$

$$eigvals := 0, 156, -3, 3, 39, 6, 6, -6, -6, -6 \tag{40}$$

> $l1 := -6;$

$$l1 := -6 \tag{41}$$

> $l2 := -3;$

$$l2 := -3 \tag{42}$$

> $l3 := 0;$

$$l3 := 0 \tag{43}$$

> $l4 := 3;$

$$l4 := 3 \tag{44}$$

> $l5 := 6;$

$$l5 := 6 \tag{45}$$

> $l6 := 39;$

$$l6 := 39 \tag{46}$$

> $l7 := 156;$

$$l7 := 156 \tag{47}$$

> $e1 := 405;$

$$e1 := 405 \tag{48}$$

> $e2 := 0;$

$$e2 := 0 \tag{49}$$

> $e3 := expand(evalm([0, 0, 0, 1, 1, 1, 1, 0, 0, 0].M)[1] \cdot e1);$

$$e3 := 63180 \tag{50}$$

> $e4 := expand(evalm([0, 0, 0, 1, 1, 1, 1, 0, 0, 0].M^2)[1] \cdot e1);$
$$e4 := 4952340 \tag{51}$$

> $e5 := expand(evalm([0, 0, 0, 1, 1, 1, 1, 0, 0, 0].M^3)[1] \cdot e1);$
$$e5 := 638793540 \tag{52}$$

> $e6 := expand(evalm([0, 0, 0, 1, 1, 1, 1, 0, 0, 0].M^4)[1] \cdot e1);$
$$e6 := 94192996500 \tag{53}$$

> $e7 := expand(evalm([0, 0, 0, 1, 1, 1, 1, 0, 0, 0].M^5)[1] \cdot e1);$
$$e7 := 14483159362980 \tag{54}$$

> $V := transpose(vandermonde([l1, l2, l3, l4, l5, l6, l7]));$

$$V := \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ -6 & -3 & 0 & 3 & 6 & 39 & 156 \\ 36 & 9 & 0 & 9 & 36 & 1521 & 24336 \\ -216 & -27 & 0 & 27 & 216 & 59319 & 3796416 \\ 1296 & 81 & 0 & 81 & 1296 & 2313441 & 592240896 \\ -7776 & -243 & 0 & 243 & 7776 & 90224199 & 92389579776 \\ 46656 & 729 & 0 & 729 & 46656 & 3518743761 & 14412774445056 \end{bmatrix} \tag{55}$$

> $W := evalm(inverse(V).transpose(Matrix([e1, e2, e3, e4, e5, e6, e7])));$

$$W := \begin{bmatrix} 175 \\ 60 \\ 90 \\ 20 \\ 39 \\ 20 \\ 1 \end{bmatrix} \tag{56}$$

> $with(CurveFitting);$

$$[ArrayInterpolation, BSpline, BSplineCurve, Interactive, LeastSquares, Lowess, \tag{57}$$
$$PolynomialInterpolation, RationalInterpolation, Spline, ThieleInterpolation]$$

> $M := M1 + M2;$

$$M := \begin{bmatrix} 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 6 & 1 & 0 & 1 & 0 & 2 & 2 & 0 & 0 & 0 \\ 18 & 0 & 1 & 1 & 3 & 3 & 3 & 0 & 0 & 0 \\ 0 & 6 & 2 & 2 & 0 & 0 & 0 & 9 & 3 & 3 \\ 0 & 0 & 12 & 0 & 3 & 0 & 0 & 0 & 6 & 6 \\ 0 & 4 & 2 & 0 & 0 & 2 & 1 & 3 & 2 & 0 \\ 0 & 12 & 6 & 0 & 0 & 3 & 4 & 0 & 1 & 3 \\ 0 & 0 & 0 & 2 & 0 & 2 & 0 & 3 & 1 & 0 \\ 0 & 0 & 0 & 6 & 6 & 12 & 2 & 9 & 5 & 3 \\ 0 & 0 & 0 & 12 & 12 & 0 & 12 & 0 & 6 & 9 \end{bmatrix}$$ **(58)**

> $PolynomialInterpolation\left(\left[\,[-12, 39], [-3, -6], [0, -3], [3, 0], [6, 3], [9, 6], [24, 156]\,\right], x\right);$

$$\frac{17}{4898880} x^6 - \frac{23}{233280} x^5 + \frac{31}{36288} x^4 - \frac{59}{36288} x^3 - \frac{241}{30240} x^2 + \frac{859}{840} x - 3$$ **(59)**

> $\dfrac{17}{4898880} M^6 - \dfrac{23}{233280} M^5 + \dfrac{31}{36288} M^4 - \dfrac{59}{36288} M^3 - \dfrac{241}{30240} M^2 + \dfrac{859}{840} M - 3$
> $\quad - M3 - M4 - M5 - M6$

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$ **(60)**

> $PolynomialInterpolation\left(\left[\,[-12, 3], [-3, -6], [0, -3], [3, 0], [6, 39], [9, 6], [24, 156]\,\right], x\right);$

$$\frac{1129}{4898880} x^6 - \frac{7897}{1632960} x^5 - \frac{223}{5184} x^4 + \frac{23069}{36288} x^3 + \frac{11143}{30240} x^2 - \frac{3637}{840} x - 3$$ **(61)**

> $\dfrac{1129}{4898880} M^6 - \dfrac{7897}{1632960} M^5 - \dfrac{223}{5184} M^4 + \dfrac{23069}{36288} M^3 + \dfrac{11143}{30240} M^2 - \dfrac{3637}{840} M - 3$
> $\quad - M3 - M4 - M5 - M6$

$$\begin{bmatrix} 0 & 0 & \dfrac{16}{9} & -\dfrac{8}{9} & 0 & 0 & -\dfrac{8}{9} & 0 & \dfrac{4}{9} & 0 \\[2mm] 0 & 0 & -\dfrac{16}{3} & \dfrac{8}{3} & 0 & 0 & \dfrac{8}{3} & 0 & -\dfrac{4}{3} & 0 \\[2mm] 32 & -16 & 0 & 0 & 8 & 8 & 0 & -4 & 0 & -4 \\[2mm] -32 & 16 & 0 & 0 & -8 & -8 & 0 & 4 & 0 & 4 \\[2mm] 0 & 0 & 32 & -16 & 0 & 0 & -16 & 0 & 8 & 0 \\[2mm] 0 & 0 & \dfrac{16}{3} & -\dfrac{8}{3} & 0 & 0 & -\dfrac{8}{3} & 0 & \dfrac{4}{3} & 0 \\[2mm] -32 & 16 & 0 & 0 & -8 & -8 & 0 & 4 & 0 & 4 \\[2mm] 0 & 0 & -\dfrac{16}{9} & \dfrac{8}{9} & 0 & 0 & \dfrac{8}{9} & 0 & -\dfrac{4}{9} & 0 \\[2mm] 32 & -16 & 0 & 0 & 8 & 8 & 0 & -4 & 0 & -4 \\[2mm] 0 & 0 & -32 & 16 & 0 & 0 & 16 & 0 & -8 & 0 \end{bmatrix} \tag{62}$$