Extra computations for the $\mathbb{G}_3$ near hexagon

The near hexagon $\mathbb{G}_3$ has order $(s,t)=(2,11)$ and contains $v=405$ points. Let $\mathcal{B}$ denote the smallest algebra that contains A and is closed under Hadamard multiplication of matrices. Then

$$I,\ A=A_{1a}+A_{1b}\in\mathcal{B}. \tag{1}$$

Since $\mathcal{B}$ is closed under Hadamard multiplication, the fact that $a_1I+a_2A_{1a}+a_3A_{2a}+\cdots+a_{10}A_{3c}\in\mathcal{B}$ implies that $a_1^iI+a_2^iA_{1a}+a_3^iA_{2a}+\cdots+a_{10}^iA_{3c}\in\mathcal{B}$ for every $i\in\mathbb{N}\setminus\{0\}$. Since $A^2=24I+A+2A_{2a}+3A_{2b}+5A_{2c}+5A_{2d}$, Property (1) implies that

$$2A_{2a}+3A_{2b}+5(A_{2c}+A_{2d})\in\mathcal{B} \tag{2}$$

and hence also that

$$4A_{2a}+9A_{2b}+25(A_{2c}+A_{2d})\in\mathcal{B}, \tag{3}$$

$$8A_{2a}+27A_{2b}+125(A_{2c}+A_{2d})\in\mathcal{B}. \tag{4}$$

The properties (2), (3) and (4) imply that

$$A_{2a},\ A_{2b},\ A_{2c}+A_{2d}\in\mathcal{B}. \tag{5}$$

Since $AA_{2a}=6A_{1a}+2A_{1b}+2A_{2a}+9A_{3a}+3(A_{3b}+A_{3c})$ and $A_{2a}\in\mathcal{B}$, we have $6A_{1a}+2A_{1b}+9A_{3a}+3(A_{3b}+A_{3c})\in\mathcal{B}$ and hence

$$6^iA_{1a}+2^iA_{1b}+9^iA_{3a}+3^i(A_{3b}+A_{3c})\in\mathcal{B} \tag{6}$$

for every $i\in\mathbb{N}\setminus\{0\}$. Property (6) implies that

$$A_{1a},A_{1b},A_{3a},A_{3b}+A_{3c}\in\mathcal{B}. \tag{7}$$

Since $AA_{3a}=2A_{2a}+2A_{2c}+3A_{3a}+A_{3b}$ and $A_{2a},A_{3a}\in\mathcal{B}$, we have $2A_{2c}+A_{3b}\in\mathcal{B}$ and hence also that $4A_{2c}+A_{3b}\in\mathcal{B}$. The latter properties imply that

$$A_{2c},A_{3b}\in\mathcal{B}. \tag{8}$$

As $A_{2c}+A_{2d}$, $A_{3b}+A_{3c}$, A_{2c} and A_{3b} belong to $\mathcal{B}$, we also have

$$A_{2d},\ A_{3c}\in\mathcal{B}. \tag{9}$$

We conclude that all A_*'s belong to $\mathcal{B}$.