```
> restart;
> with(linalg);
```

[BlockDiagonal, GramSchmidt, JordanBlock, LUdecomp, QRdecomp, Wronskian, addcol, addrow, adj, adjoint, angle, augment, backsub, band, basis, bezout, blockmatrix, charmat, charpoly, cholesky, col, coldim, colspace, colspan, companion, concat, cond, copyinto, crossprod, curl, definite, delcols, delrows, det, diag, diverge, dotprod, eigenvals, eigenvalues, eigenvectors, eigenvects, entermatrix, equal, exponential, extend, ffgausselim, fibonacci, forwardsub, frobenius, gausselim, gaussjord, geneqns, genmatrix, grad, hadamard, hermite, hessian, hilbert, htranspose, ihermite, indexfunc, innerprod, intbasis, inverse, ismith, issimilar, iszero, jacobian, jordan, kernel, laplacian, leastsqrs, linsolve, matadd, matrix, minor, minpoly, mulcol, mulrow, multiply, norm, normalize, nullspace, orthog, permanent, pivot, potential, randmatrix, randvector, rank, ratform, row, rowdim, rowspace, rowspan, rref, scalarmul, singularvals, smith, stackmatrix, submatrix, subvector, sumbasis, swapcol, swaprow, sylvester, toeplitz, trace, transpose, vandermonde, vecpotent, vectdim, vector, wronskian] **(1)**

```
> s := s; ta := ta; tb := tb;
```

$$s := s$$
$$ta := ta$$
$$tb := tb \qquad \textbf{(2)}$$

```
>
> M0 := Matrix([[1, 0, 0, 0, 0, 0, 0, 0], [0, 1, 0, 0, 0, 0, 0, 0], [0, 0, 1, 0, 0, 0, 0, 0], [0, 0, 0, 1, 0,
  0, 0, 0], [0, 0, 0, 0, 1, 0, 0, 0], [0, 0, 0, 0, 0, 1, 0, 0], [0, 0, 0, 0, 0, 0, 1, 0], [0, 0, 0, 0, 0, 0,
  0, 1]]);
```

$$M0 := \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \qquad \textbf{(3)}$$

```
> M1 := Matrix([[0, 1, 1, 1, 0, 0, 0, 0], [s·ta, s − 1, 0, 0, ta, 0, 1, 0], [s·tb, 0, s − 1, 0, 0, tb, 1,
  0], [s, 0, 0, s − 1, 1, 1, 0, 0], [0, s·ta, 0, s·ta, (s − 1)·(ta + 1), 0, 0, 1], [0, 0, s·tb, s·tb, 0,
  (s − 1)·(tb + 1) , 0, 1], [0, s·tb, s·ta, 0, 0, 0, 2·(s − 1), ta + tb − 1], [0, 0, 0, 0, s·tb, s
```

$\cdot ta, s\cdot(ta+tb-1), (ta+tb+1)\cdot(s-1)]]);$

$$M1 := [[0, 1, 1, 1, 0, 0, 0, 0], [s\,ta, s-1, 0, 0, ta, 0, 1, 0], [s\,tb, 0, s-1, 0, 0, tb, 1, 0], [s, 0, 0, s-1, 1, 1, 0, 0], [0, s\,ta, 0, s\,ta, (s-1)\,(ta+1), 0, 0, 1], [0, 0, s\,tb, s\,tb, 0, (s-1)\,(tb+1), 0, 1], [0, s\,tb, s\,ta, 0, 0, 0, 2\,s-2, ta+tb-1], [0, 0, 0, 0, s\,tb, s\,ta, s\,(ta+tb-1), (ta+tb+1)\,(s-1)]] \quad (4)$$

> $eigvals1 := eigenvalues(M1);$

$$eigvals1 := -ta-tb-1, s-2, 2\,s-1, s\,tb+s-1, -tb-1+s, s\,ta+s-1, -ta-1+s, s\,ta+s\,tb+s \quad (5)$$

> $l1 := -ta-tb-1;$

$$l1 := -ta-tb-1 \quad (6)$$

> $l2 := -tb-1+s;$

$$l2 := -tb-1+s \quad (7)$$

> $l3 := -ta-1+s;$

$$l3 := -ta-1+s \quad (8)$$

> $l4 := s-2;$

$$l4 := s-2 \quad (9)$$

> $l5 := 2\cdot s-1;$

$$l5 := 2\,s-1 \quad (10)$$

> $l6 := s\cdot ta+s-1;$

$$l6 := s\,ta+s-1 \quad (11)$$

> $l7 := s\cdot tb+s-1;$

$$l7 := s\,tb+s-1 \quad (12)$$

> $l8 := s\cdot ta+s\cdot tb+s;$

$$l8 := s\,ta+s\,tb+s \quad (13)$$

> $e1 := (s+1)\cdot(s\cdot ta+1)\cdot(s\cdot tb+1);$

$$e1 := (1+s)\,(s\,ta+1)\,(s\,tb+1) \quad (14)$$

> $e2 := 0;$

$$e2 := 0 \quad (15)$$

> $e3 := expand(evalm([0, 1, 1, 1, 0, 0, 0, 0].M1)[1]\cdot e1);$

(16)

$$e3 := s^4\, ta^2\, tb + s^4\, ta\, tb^2 + s^4\, ta\, tb + s^3\, ta^2\, tb + s^3\, ta\, tb^2 + s^3\, ta^2 + 3\, s^3\, ta\, tb + s^3\, tb^2 + s^3\, ta + s^3\, tb + s^2\, ta^2 + 2\, s^2\, ta\, tb + s^2\, tb^2 + 2\, s^2\, ta + 2\, s^2\, tb + s^2 + s\, ta + s\, tb + s \quad \textbf{(16)}$$

> $e4 := expand(evalm([0, 1, 1, 1, 0, 0, 0, 0].M1^2)[1] \cdot e1);$

$$e4 := s^5\, ta^2\, tb + s^5\, ta\, tb^2 + s^5\, ta\, tb + s^4\, ta^2 + 2\, s^4\, ta\, tb + s^4\, tb^2 - s^3\, ta^2\, tb - s^3\, ta\, tb^2 + s^4\, ta + s^4\, tb - s^3\, ta\, tb + s^3\, ta + s^3\, tb - s^2\, ta^2 - 2\, s^2\, ta\, tb - s^2\, tb^2 + s^3 - s^2\, ta - s^2\, tb - s\, ta - s\, tb - s \quad \textbf{(17)}$$

> $e5 := expand(evalm([0, 1, 1, 1, 0, 0, 0, 0].M1^3)[1] \cdot e1);$

$$e5 := s^5\, ta^4\, tb + s^5\, ta\, tb^4 + s^6\, ta^2\, tb + s^6\, ta\, tb^2 + 3\, s^5\, ta^3\, tb + 6\, s^5\, ta^2\, tb^2 + 3\, s^5\, ta\, tb^3 + s^4\, ta^4\, tb + s^4\, ta\, tb^4 + s^6\, ta\, tb + 2\, s^5\, ta^2\, tb + 2\, s^5\, ta\, tb^2 + s^4\, ta^4 + 4\, s^4\, ta^3\, tb + 6\, s^4\, ta^2\, tb^2 + 4\, s^4\, ta\, tb^3 + s^4\, tb^4 + s^5\, ta^2 + 2\, s^5\, ta\, tb + s^5\, tb^2 + 3\, s^4\, ta^3 + 11\, s^4\, ta^2\, tb + 11\, s^4\, ta\, tb^2 + 3\, s^4\, tb^3 + s^3\, ta^4 + s^3\, ta^3\, tb + s^3\, ta\, tb^3 + s^3\, tb^4 + s^5\, ta + s^5\, tb + 2\, s^4\, ta^2 + 4\, s^4\, ta\, tb + 2\, s^4\, tb^2 + 4\, s^3\, ta^3 + 10\, s^3\, ta^2\, tb + 10\, s^3\, ta\, tb^2 + 4\, s^3\, tb^3 + s^4\, ta + s^4\, tb + 5\, s^3\, ta^2 + 11\, s^3\, ta\, tb + 5\, s^3\, tb^2 + s^2\, ta^3 + s^2\, tb^3 + s^4 + 2\, s^3\, ta + 2\, s^3\, tb + 4\, s^2\, ta^2 + 8\, s^2\, ta\, tb + 4\, s^2\, tb^2 + 3\, s^2\, ta + 3\, s^2\, tb + s\, ta + s\, tb + s \quad \textbf{(18)}$$

> $e6 := expand(evalm([0, 1, 1, 1, 0, 0, 0, 0].M1^4)[1] \cdot e1);$

$$e6 := s^6\, ta^5\, tb + s^6\, ta\, tb^5 + 5\, s^6\, ta^4\, tb + 5\, s^6\, ta\, tb^4 + s^7\, ta^2\, tb + s^7\, ta\, tb^2 + 9\, s^6\, ta^3\, tb + 20\, s^6\, ta^2\, tb^2 + 9\, s^6\, ta\, tb^3 + s^5\, ta^5 + s^5\, ta^4\, tb + s^5\, ta\, tb^4 + s^5\, tb^5 - s^4\, ta^5\, tb - s^4\, ta\, tb^5 + s^7\, ta\, tb + 5\, s^6\, ta^2\, tb + 5\, s^6\, ta\, tb^2 + 5\, s^5\, ta^4 + 5\, s^5\, ta^3\, tb + 5\, s^5\, ta\, tb^3 + 5\, s^5\, tb^4 - 5\, s^4\, ta^4\, tb - 5\, s^4\, ta\, tb^4 + s^6\, ta^2 + 2\, s^6\, ta\, tb + s^6\, tb^2 + 9\, s^5\, ta^3 + 29\, s^5\, ta^2\, tb + 29\, s^5\, ta\, tb^2 + 9\, s^5\, tb^3 + s^4\, ta^4 - 9\, s^4\, ta^3\, tb - 20\, s^4\, ta^2\, tb^2 - 9\, s^4\, ta\, tb^3 + s^4\, tb^4 - s^3\, ta^5 - s^3\, ta^4\, tb - s^3\, ta\, tb^4 - s^3\, tb^5 + s^6\, ta + s^6\, tb + 5\, s^5\, ta^2 + 10\, s^5\, ta\, tb + 5\, s^5\, tb^2 + 5\, s^4\, ta^3 - 5\, s^4\, ta^2\, tb - 5\, s^4\, ta\, tb^2 + 5\, s^4\, tb^3 - 5\, s^3\, ta^4 - 5\, s^3\, ta^3\, tb - 5\, s^3\, ta\, tb^3 - 5\, s^3\, tb^4 + s^5\, ta + s^5\, tb + 9\, s^4\, ta^2 + 20\, s^4\, ta\, tb + 9\, s^4\, tb^2 - 9\, s^3\, ta^3 - 30\, s^3\, ta^2\, tb - 30\, s^3\, ta\, tb^2 - 9\, s^3\, tb^3 - s^2\, ta^4 - s^2\, tb^4 + s^5 + 5\, s^4\, ta + 5\, s^4\, tb - 5\, s^3\, ta^2 - 11\, s^3\, ta\, tb - 5\, s^3\, tb^2 - 5\, s^2\, ta^3 - 5\, s^2\, tb^3 - 10\, s^2\, ta^2 - 22\, s^2\, ta\, tb - 10\, s^2\, tb^2 - 6\, s^2\, ta - 6\, s^2\, tb - s\, ta - s\, tb - s \quad \textbf{(19)}$$

> $e7 := expand(evalm([0, 1, 1, 1, 0, 0, 0, 0].M1^5)[1] \cdot e1);$

$$e7 := s^7\, ta^6\, tb + s^7\, ta\, tb^6 + 6\, s^7\, ta^5\, tb + 6\, s^7\, ta\, tb^5 + 15\, s^7\, ta^4\, tb + 15\, s^7\, ta\, tb^4 + s^6\, ta^6 + 15\, s^6\, ta^4\, tb^2 + 20\, s^6\, ta^3\, tb^3 + 15\, s^6\, ta^2\, tb^4 + s^6\, tb^6 + s^8\, ta^2\, tb + s^8\, ta\, tb^2 + 19\, s^7\, ta^3\, tb + 50\, s^7\, ta^2\, tb^2 + 19\, s^7\, ta\, tb^3 + 6\, s^6\, ta^5 + s^6\, ta^4\, tb + 25\, s^6\, ta^3\, tb^2 + 25\, s^6\, ta^2\, tb^3 + s^6\, ta\, tb^4 + 6\, s^6\, tb^5 - s^5\, ta^5\, tb + 15\, s^5\, ta^4\, tb^2 + 20\, s^5\, ta^3\, tb^3 \quad \textbf{(20)}$$

$$+ 15 s^5 ta^2 tb^4 - s^5 ta\, tb^5 + s^4 ta^6 tb + s^4 ta\, tb^6 + s^8 ta\, tb + 9 s^7 ta^2 tb + 9 s^7 ta\, tb^2$$
$$+ 15 s^6 ta^4 + 6 s^6 ta^3 tb - 30 s^6 ta^2 tb^2 + 6 s^6 ta\, tb^3 + 15 s^6 tb^4 + 9 s^5 ta^4 tb$$
$$+ 60 s^5 ta^3 tb^2 + 60 s^5 ta^2 tb^3 + 9 s^5 ta\, tb^4 + 6 s^4 ta^5 tb + 6 s^4 ta\, tb^5 + s^7 ta^2 + 2 s^7 ta\, tb$$
$$+ s^7 tb^2 + 19 s^6 ta^3 + 64 s^6 ta^2 tb + 64 s^6 ta\, tb^2 + 19 s^6 tb^3 + s^5 ta^4 + 11 s^5 ta^3 tb$$
$$+ 20 s^5 ta^2 tb^2 + 11 s^5 ta\, tb^3 + s^5 tb^4 - s^4 ta^5 + 29 s^4 ta^4 tb + 35 s^4 ta^3 tb^2$$
$$+ 35 s^4 ta^2 tb^3 + 29 s^4 ta\, tb^4 - s^4 tb^5 + s^3 ta^6 + s^3 ta^5 tb + s^3 ta\, tb^5 + s^3 tb^6 + s^7 ta$$
$$+ s^7 tb + 9 s^6 ta^2 + 18 s^6 ta\, tb + 9 s^6 tb^2 + 6 s^5 ta^3 - 44 s^5 ta^2 tb - 44 s^5 ta\, tb^2$$
$$+ 6 s^5 tb^3 - 6 s^4 ta^4 + 54 s^4 ta^3 tb + 120 s^4 ta^2 tb^2 + 54 s^4 ta\, tb^3 - 6 s^4 tb^4 + 6 s^3 ta^5$$
$$+ 6 s^3 ta^4 tb + 6 s^3 ta\, tb^4 + 6 s^3 tb^5 + s^6 ta + s^6 tb + 14 s^5 ta^2 + 40 s^5 ta\, tb + 14 s^5 tb^2$$
$$- 14 s^4 ta^3 - 5 s^4 ta^2 tb - 5 s^4 ta\, tb^2 - 14 s^4 tb^3 + 14 s^3 ta^4 + 30 s^3 ta^3 tb$$
$$+ 20 s^3 ta^2 tb^2 + 30 s^3 ta\, tb^3 + 14 s^3 tb^4 + s^2 ta^5 + s^2 tb^5 + s^6 + 9 s^5 ta + 9 s^5 tb$$
$$- 14 s^4 ta^2 - 40 s^4 ta\, tb - 14 s^4 tb^2 + 14 s^3 ta^3 + 95 s^3 ta^2 tb + 95 s^3 ta\, tb^2 + 14 s^3 tb^3$$
$$+ 6 s^2 ta^4 + 6 s^2 tb^4 - 5 s^4 ta - 5 s^4 tb - 11 s^3 ta\, tb + 15 s^2 ta^3 + 15 s^2 tb^3 - 5 s^3 ta$$
$$- 5 s^3 tb + 20 s^2 ta^2 + 52 s^2 ta\, tb + 20 s^2 tb^2 + 10 s^2 ta + 10 s^2 tb + s\, ta + s\, tb + s$$

> $e8 := expand(evalm([0, 1, 1, 1, 0, 0, 0, 0].M1^6)[1] \cdot e1);$

$$e8 := s^8 ta^7 tb + s^8 ta\, tb^7 + 7 s^8 ta^6 tb + 7 s^8 ta\, tb^6 + 21 s^8 ta^5 tb + 21 s^8 ta\, tb^5 + s^7 ta^7 \quad \textbf{(21)}$$
$$+ 21 s^7 ta^5 tb^2 + 35 s^7 ta^4 tb^3 + 35 s^7 ta^3 tb^4 + 21 s^7 ta^2 tb^5 + s^7 tb^7 + 35 s^8 ta^4 tb$$
$$+ 35 s^8 ta\, tb^4 + 7 s^7 ta^6 + 105 s^7 ta^4 tb^2 + 140 s^7 ta^3 tb^3 + 105 s^7 ta^2 tb^4 + 7 s^7 tb^6$$
$$+ s^9 ta^2 tb + s^9 ta\, tb^2 + 34 s^8 ta^3 tb + 112 s^8 ta^2 tb^2 + 34 s^8 ta\, tb^3 + 21 s^7 ta^5 + s^7 ta^4 tb$$
$$+ 119 s^7 ta^3 tb^2 + 119 s^7 ta^2 tb^3 + s^7 ta\, tb^4 + 21 s^7 tb^5 + 20 s^6 ta^5 tb + 56 s^6 ta^4 tb^2$$
$$+ 70 s^6 ta^3 tb^3 + 56 s^6 ta^2 tb^4 + 20 s^6 ta\, tb^5 + s^5 ta^6 tb - 21 s^5 ta^5 tb^2 - 35 s^5 ta^4 tb^3$$
$$- 35 s^5 ta^3 tb^4 - 21 s^5 ta^2 tb^5 + s^5 ta\, tb^6 - s^4 ta^7 tb - s^4 ta\, tb^7 + s^9 ta\, tb + 14 s^8 ta^2 tb$$
$$+ 14 s^8 ta\, tb^2 + 35 s^7 ta^4 + 7 s^7 ta^3 tb - 140 s^7 ta^2 tb^2 + 7 s^7 ta\, tb^3 + 35 s^7 tb^4$$
$$+ 98 s^6 ta^4 tb + 245 s^6 ta^3 tb^2 + 245 s^6 ta^2 tb^3 + 98 s^6 ta\, tb^4 + 7 s^5 ta^5 tb$$
$$- 105 s^5 ta^4 tb^2 - 140 s^5 ta^3 tb^3 - 105 s^5 ta^2 tb^4 + 7 s^5 ta\, tb^5 - 7 s^4 ta^6 tb - 7 s^4 ta\, tb^6$$
$$+ s^8 ta^2 + 2 s^8 ta\, tb + s^8 tb^2 + 34 s^7 ta^3 + 132 s^7 ta^2 tb + 132 s^7 ta\, tb^2 + 34 s^7 tb^3$$
$$+ s^6 ta^4 + 99 s^6 ta^3 tb + 238 s^6 ta^2 tb^2 + 99 s^6 ta\, tb^3 + s^6 tb^4 - s^5 ta^5 + 41 s^5 ta^4 tb$$
$$- 84 s^5 ta^3 tb^2 - 84 s^5 ta^2 tb^3 + 41 s^5 ta\, tb^4 - s^5 tb^5 + s^4 ta^6 - 41 s^4 ta^5 tb$$
$$- 56 s^4 ta^4 tb^2 - 70 s^4 ta^3 tb^3 - 56 s^4 ta^2 tb^4 - 41 s^4 ta\, tb^5 + s^4 tb^6 - s^3 ta^7 - s^3 ta^6 tb$$
$$- s^3 ta\, tb^6 - s^3 tb^7 + s^8 ta + s^8 tb + 14 s^7 ta^2 + 28 s^7 ta\, tb + 14 s^7 tb^2 + 7 s^6 ta^3$$
$$- 168 s^6 ta^2 tb - 168 s^6 ta\, tb^2 + 7 s^6 tb^3 - 7 s^5 ta^4 + 133 s^5 ta^3 tb + 280 s^5 ta^2 tb^2$$
$$+ 133 s^5 ta\, tb^3 - 7 s^5 tb^4 + 7 s^4 ta^5 - 133 s^4 ta^4 tb - 245 s^4 ta^3 tb^2 - 245 s^4 ta^2 tb^3$$

$$- 133\, s^4\, ta\, tb^4 + 7\, s^4\, tb^5 - 7\, s^3\, ta^6 - 7\, s^3\, ta^5\, tb - 7\, s^3\, ta\, tb^5 - 7\, s^3\, tb^6 + s^7\, ta + s^7\, tb$$
$$+ 20\, s^6\, ta^2 + 84\, s^6\, ta\, tb + 20\, s^6\, tb^2 - 20\, s^5\, ta^3 + 133\, s^5\, ta^2\, tb + 133\, s^5\, ta\, tb^2$$
$$- 20\, s^5\, tb^3 + 20\, s^4\, ta^4 - 133\, s^4\, ta^3\, tb - 350\, s^4\, ta^2\, tb^2 - 133\, s^4\, ta\, tb^3 + 20\, s^4\, tb^4$$
$$- 20\, s^3\, ta^5 - 42\, s^3\, ta^4\, tb - 35\, s^3\, ta^3\, tb^2 - 35\, s^3\, ta^2\, tb^3 - 42\, s^3\, ta\, tb^4 - 20\, s^3\, tb^5$$
$$- s^2\, ta^6 - s^2\, tb^6 + s^7 + 14\, s^6\, ta + 14\, s^6\, tb - 28\, s^5\, ta^2 - 140\, s^5\, ta\, tb - 28\, s^5\, tb^2$$
$$+ 28\, s^4\, ta^3 + 154\, s^4\, ta^2\, tb + 154\, s^4\, ta\, tb^2 + 28\, s^4\, tb^3 - 28\, s^3\, ta^4 - 140\, s^3\, ta^3\, tb$$
$$- 140\, s^3\, ta^2\, tb^2 - 140\, s^3\, ta\, tb^3 - 28\, s^3\, tb^4 - 7\, s^2\, ta^5 - 7\, s^2\, tb^5 - 14\, s^5\, ta - 14\, s^5\, tb$$
$$+ 14\, s^4\, ta^2 + 28\, s^4\, ta\, tb + 14\, s^4\, tb^2 - 14\, s^3\, ta^3 - 266\, s^3\, ta^2\, tb - 266\, s^3\, ta\, tb^2$$
$$- 14\, s^3\, tb^3 - 21\, s^2\, ta^4 - 21\, s^2\, tb^4 + 14\, s^3\, ta^2 + 111\, s^3\, ta\, tb + 14\, s^3\, tb^2 - 35\, s^2\, ta^3$$
$$- 35\, s^2\, tb^3 + 14\, s^3\, ta + 14\, s^3\, tb - 35\, s^2\, ta^2 - 114\, s^2\, ta\, tb - 35\, s^2\, tb^2 - 15\, s^2\, ta$$
$$- 15\, s^2\, tb - s\, ta - s\, tb - s$$

```
> V := transpose(vandermonde([l1, l2, l3, l4, l5, l6, l7, l8]));
```

$$V := [[1, 1, 1, 1, 1, 1, 1, 1],$$
$$[-ta - tb - 1, -tb - 1 + s, -ta - 1 + s, s - 2, 2\, s - 1, s\, ta + s - 1, s\, tb + s - 1, s\, ta + s\, tb + s],$$
$$[(-ta - tb - 1)^2, (-tb - 1 + s)^2, (-ta - 1 + s)^2, (s - 2)^2, (2\, s - 1)^2, (s\, ta + s - 1)^2, (s\, tb + s - 1)^2, (s\, ta + s\, tb + s)^2],$$
$$[(-ta - tb - 1)^3, (-tb - 1 + s)^3, (-ta - 1 + s)^3, (s - 2)^3, (2\, s - 1)^3, (s\, ta + s - 1)^3, (s\, tb + s - 1)^3, (s\, ta + s\, tb + s)^3],$$
$$[(-ta - tb - 1)^4, (-tb - 1 + s)^4, (-ta - 1 + s)^4, (s - 2)^4, (2\, s - 1)^4, (s\, ta + s - 1)^4, (s\, tb + s - 1)^4, (s\, ta + s\, tb + s)^4],$$
$$[(-ta - tb - 1)^5, (-tb - 1 + s)^5, (-ta - 1 + s)^5, (s - 2)^5, (2\, s - 1)^5, (s\, ta + s - 1)^5, (s\, tb + s - 1)^5, (s\, ta + s\, tb + s)^5],$$
$$[(-ta - tb - 1)^6, (-tb - 1 + s)^6, (-ta - 1 + s)^6, (s - 2)^6, (2\, s - 1)^6, (s\, ta + s - 1)^6, (s\, tb + s - 1)^6, (s\, ta + s\, tb + s)^6],$$
$$[(-ta - tb - 1)^7, (-tb - 1 + s)^7, (-ta - 1 + s)^7, (s - 2)^7, (2\, s - 1)^7, (s\, ta + s - 1)^7, (s\, tb + s - 1)^7, (s\, ta + s\, tb + s)^7]] \qquad \textbf{(22)}$$

```
> W := simplify(evalm( inverse(V).transpose(Matrix([e1, e2, e3, e4, e5, e6, e7, e8]))));
```

$$W := \begin{bmatrix} \frac{s^3 (s^2\, ta\, tb + s\, ta + s\, tb + 1)}{s^2 + s\, ta + s\, tb + ta\, tb} \\ \frac{(s^2\, ta\, tb + s\, ta + s\, tb + 1)\, s^2\, ta}{s^2 + s\, ta + s\, tb + ta\, tb} \\ \frac{(s^2\, ta\, tb + s\, ta + s\, tb + 1)\, s^2\, tb}{s^2 + s\, ta + s\, tb + ta\, tb} \\ s^2\, ta\, tb \\ \frac{(s^2\, ta\, tb + s\, ta + s\, tb + 1)\, s\, tb\, ta}{s^2 + s\, ta + s\, tb + ta\, tb} \\ s\, tb \\ s\, ta \\ 1 \end{bmatrix} \tag{23}$$

>

>

>

> $M1 := Matrix([[0, 1, 1, 1, 0, 0, 0, 0], [s \cdot ta, s - 1, 0, 0, ta, 0, 1, 0], [s \cdot ta, 0, s - 1, 0, 0, ta, 1, 0], [s, 0, 0, s - 1, 1, 1, 0, 0], [0, s \cdot ta, 0, s \cdot ta, (s - 1) \cdot (ta + 1), 0, 0, 1], [0, 0, s \cdot ta, s \cdot ta, 0, (s - 1) \cdot (ta + 1)\ , 0, 1], [0, s \cdot ta, s \cdot ta, 0, 0, 0, 2 \cdot (s - 1), ta + ta - 1], [0, 0, 0, 0, s \cdot ta, s \cdot ta, s \cdot (ta + ta - 1), (ta + ta + 1) \cdot (s - 1)]]);$

$$\begin{aligned} M1 := [&[0, 1, 1, 1, 0, 0, 0, 0], \\ &[s\, ta, s - 1, 0, 0, ta, 0, 1, 0], \\ &[s\, ta, 0, s - 1, 0, 0, ta, 1, 0], \\ &[s, 0, 0, s - 1, 1, 1, 0, 0], \\ &[0, s\, ta, 0, s\, ta, (s - 1)\, (ta + 1), 0, 0, 1], \\ &[0, 0, s\, ta, s\, ta, 0, (s - 1)\, (ta + 1), 0, 1], \\ &[0, s\, ta, s\, ta, 0, 0, 0, 2\, s - 2, 2\, ta - 1], \\ &[0, 0, 0, 0, s\, ta, s\, ta, s\, (2\, ta - 1), (2\, ta + 1)\, (s - 1)]] \end{aligned} \tag{24}$$

> $eigvals1 := eigenvalues(M1);$

$$eigvals1 := -2\, ta - 1, s - 2, 2\, s - 1, 2\, s\, ta + s, s\, ta + s - 1, -ta - 1 + s, s\, ta + s - 1, -ta - 1 + s \tag{25}$$

> $l1 := -2 \cdot ta - 1;$

$$l1 := -2\, ta - 1 \tag{26}$$

> $l2 := -ta - 1 + s;$

$$l2 := -ta - 1 + s \tag{27}$$

> $l3 := s - 2;$

$$l3 := s - 2 \tag{28}$$

> $l4 := 2 \cdot s - 1;$

$$l4 := 2\, s - 1 \tag{29}$$

> $l5 := s \cdot ta + s - 1;$

$$l5 := s\, ta + s - 1 \tag{30}$$

> $l6 := 2 \cdot s \cdot ta + s;$

$$l6 := 2\, s\, ta + s \tag{31}$$

> $e1 := (s + 1) \cdot (s \cdot ta + 1)^2;$

$$e1 := (1 + s)\, (s\, ta + 1)^2 \tag{32}$$

> $e2 := 0;$

$$e2 := 0 \tag{33}$$

> $e3 := expand(evalm([0, 1, 1, 1, 0, 0, 0, 0].M1)[1] \cdot e1);$

$$e3 := 2\, s^4\, ta^3 + s^4\, ta^2 + 2\, s^3\, ta^3 + 5\, s^3\, ta^2 + 2\, s^3\, ta + 4\, s^2\, ta^2 + 4\, s^2\, ta + s^2 + 2\, s\, ta + s \tag{34}$$

> $e4 := expand\left(evalm\left([0, 1, 1, 1, 0, 0, 0, 0].M1^2\right)[1] \cdot e1\right);$

$$e4 := 2\, s^5\, ta^3 + s^5\, ta^2 + 4\, s^4\, ta^2 - 2\, s^3\, ta^3 + 2\, s^4\, ta - s^3\, ta^2 + 2\, s^3\, ta - 4\, s^2\, ta^2 + s^3 - 2\, s^2\, ta - 2\, s\, ta - s \tag{35}$$

> $e5 := expand\left(evalm\left([0, 1, 1, 1, 0, 0, 0, 0].M1^3\right)[1] \cdot e1\right);$

$$e5 := 2\, s^5\, ta^5 + 2\, s^6\, ta^3 + 12\, s^5\, ta^4 + 2\, s^4\, ta^5 + s^6\, ta^2 + 4\, s^5\, ta^3 + 16\, s^4\, ta^4 + 4\, s^5\, ta^2 + 28\, s^4\, ta^3 + 4\, s^3\, ta^4 + 2\, s^5\, ta + 8\, s^4\, ta^2 + 28\, s^3\, ta^3 + 2\, s^4\, ta + 21\, s^3\, ta^2 + 2\, s^2\, ta^3 + s^4 + 4\, s^3\, ta + 16\, s^2\, ta^2 + 6\, s^2\, ta + 2\, s\, ta + s \tag{36}$$

> $e6 := expand\left(evalm\left([0, 1, 1, 1, 0, 0, 0, 0].M1^4\right)[1] \cdot e1\right);$

$$e6 := 2\, s^6\, ta^6 + 10\, s^6\, ta^5 + 2\, s^7\, ta^3 + 38\, s^6\, ta^4 + 4\, s^5\, ta^5 - 2\, s^4\, ta^6 + s^7\, ta^2 + 10\, s^6\, ta^3 + 20\, s^5\, ta^4 - 10\, s^4\, ta^5 + 4\, s^6\, ta^2 + 76\, s^5\, ta^3 - 36\, s^4\, ta^4 - 4\, s^3\, ta^5 + 2\, s^6\, ta + 20\, s^5\, ta^2 - 20\, s^3\, ta^4 + 2\, s^5\, ta + 38\, s^4\, ta^2 - 78\, s^3\, ta^3 - 2\, s^2\, ta^4 + s^5 + 10\, s^4\, ta - 21\, s^3\, ta^2 - 10\, s^2\, ta^3 - 42\, s^2\, ta^2 - 12\, s^2\, ta - 2\, s\, ta - s \tag{37}$$

> $V := transpose(vandermonde([l1, l2, l3, l4, l5, l6]));$

(38)

$$V := \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ -2\,ta-1 & -ta-1+s & s-2 & 2\,s-1 & s\,ta+s-1 & 2\,s\,ta+s \\ (-2\,ta-1)^2 & (-ta-1+s)^2 & (s-2)^2 & (2\,s-1)^2 & (s\,ta+s-1)^2 & (2\,s\,ta+s)^2 \\ (-2\,ta-1)^3 & (-ta-1+s)^3 & (s-2)^3 & (2\,s-1)^3 & (s\,ta+s-1)^3 & (2\,s\,ta+s)^3 \\ (-2\,ta-1)^4 & (-ta-1+s)^4 & (s-2)^4 & (2\,s-1)^4 & (s\,ta+s-1)^4 & (2\,s\,ta+s)^4 \\ (-2\,ta-1)^5 & (-ta-1+s)^5 & (s-2)^5 & (2\,s-1)^5 & (s\,ta+s-1)^5 & (2\,s\,ta+s)^5 \end{bmatrix} \quad \textbf{(38)}$$

> *W* := *simplify*(*evalm*(*inverse*(*V*).*transpose*(*Matrix*([*e1*, *e2*, *e3*, *e4*, *e5*, *e6*]))));

$$W := \begin{bmatrix} \frac{s^3\,(s^2\,ta^2+2\,s\,ta+1)}{s^2+2\,s\,ta+ta^2} \\ \frac{2\,(s^2\,ta^2+2\,s\,ta+1)\,ta\,s^2}{s^2+2\,s\,ta+ta^2} \\ ta^2\,s^2 \\ \frac{(s^2\,ta^2+2\,s\,ta+1)\,ta^2\,s}{s^2+2\,s\,ta+ta^2} \\ 2\,s\,ta \\ 1 \end{bmatrix} \quad \textbf{(39)}$$

>

>

>

> *M1* := *Matrix*([[0, 1, 1, 0, 0, 0], [2·*s*·*ta*, *s* − 1, 0, *ta*, 2, 0], [*s*, 0, *s* − 1, 1, 0, 0], [0, *s*·*ta*, 2·*s*·*ta*, (*s* − 1)·(*ta* + 1), 0, 2], [0, *s*·*ta*, 0, 0, 2·(*s* − 1), 2·*ta* − 1], [0, 0, 0, *s*·*ta*, *s*·(2· *ta* − 1), (*s* − 1)·(2· *ta* + 1)]]);

$$M1 := \begin{bmatrix} 0 & 1 & 1 & 0 & 0 & 0 \\ 2\,s\,ta & s-1 & 0 & ta & 2 & 0 \\ s & 0 & s-1 & 1 & 0 & 0 \\ 0 & s\,ta & 2\,s\,ta & (s-1)\,(ta+1) & 0 & 2 \\ 0 & s\,ta & 0 & 0 & 2\,s-2 & 2\,ta-1 \\ 0 & 0 & 0 & s\,ta & s\,(2\,ta-1) & (2\,ta+1)\,(s-1) \end{bmatrix} \quad \textbf{(40)}$$

> *eigenvalues*(*M1*);

$$-2\,ta-1,\ s-2,\ 2\,s-1,\ s\,ta+s-1,\ -ta-1+s,\ 2\,s\,ta+s \quad \textbf{(41)}$$

>