```
> restart;
```

```
> with(linalg);
```

[*BlockDiagonal*, *GramSchmidt*, *JordanBlock*, *LUdecomp*, *QRdecomp*, *Wronskian*, *addcol*, *addrow*, *adj*, *adjoint*, *angle*, *augment*, *backsub*, *band*, *basis*, *bezout*, *blockmatrix*, *charmat*, *charpoly*, *cholesky*, *col*, *coldim*, *colspace*, *colspan*, *companion*, *concat*, *cond*, *copyinto*, *crossprod*, *curl*, *definite*, *delcols*, *delrows*, *det*, *diag*, *diverge*, *dotprod*, *eigenvals*, *eigenvalues*, *eigenvectors*, *eigenvects*, *entermatrix*, *equal*, *exponential*, *extend*, *ffgausselim*, *fibonacci*, *forwardsub*, *frobenius*, *gausselim*, *gaussjord*, *geneqns*, *genmatrix*, *grad*, *hadamard*, *hermite*, *hessian*, *hilbert*, *htranspose*, *ihermite*, *indexfunc*, *innerprod*, *intbasis*, *inverse*, *ismith*, *issimilar*, *iszero*, *jacobian*, *jordan*, *kernel*, *laplacian*, *leastsqrs*, *linsolve*, *matadd*, *matrix*, *minor*, *minpoly*, *mulcol*, *mulrow*, *multiply*, *norm*, *normalize*, *nullspace*, *orthog*, *permanent*, *pivot*, *potential*, *randmatrix*, *randvector*, *rank*, *ratform*, *row*, *rowdim*, *rowspace*, *rowspan*, *rref*, *scalarmul*, *singularvals*, *smith*, *stackmatrix*, *submatrix*, *subvector*, *sumbasis*, *swapcol*, *swaprow*, *sylvester*, *toeplitz*, *trace*, *transpose*, *vandermonde*, *vecpotent*, *vectdim*, *vector*, *wronskian*] **(1)**

```
> M0 := Matrix([[1, 0, 0, 0, 0], [0, 1, 0, 0, 0], [0, 0, 1, 0, 0], [0, 0, 0, 1, 0], [0, 0, 0, 0, 1]]);
```

$$M0 := \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \quad \textbf{(2)}$$

```
> M1 := Matrix([[0, 1, 0, 0, 0], [12, 1, 2, 3, 0], [0, 2, 2, 0, 2], [0, 8, 0, 3, 4], [0, 0, 8, 6, 6]]);
```

$$M1 := \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 12 & 1 & 2 & 3 & 0 \\ 0 & 2 & 2 & 0 & 2 \\ 0 & 8 & 0 & 3 & 4 \\ 0 & 0 & 8 & 6 & 6 \end{bmatrix} \quad \textbf{(3)}$$

```
> M2 := Matrix([[0, 0, 1, 0, 0], [0, 2, 2, 0, 2], [12, 2, 5, 0, 1], [0, 0, 0, 6, 4], [0, 8, 4, 6, 5]]);
```

$$M2 := \begin{bmatrix} 0 & 0 & 1 & 0 & 0 \\ 0 & 2 & 2 & 0 & 2 \\ 12 & 2 & 5 & 0 & 1 \\ 0 & 0 & 0 & 6 & 4 \\ 0 & 8 & 4 & 6 & 5 \end{bmatrix} \quad \textbf{(4)}$$

```
> M3 := Matrix([[0, 0, 0, 1, 0], [0, 8, 0, 3, 4], [0, 0, 0, 6, 4], [32, 8, 16, 10, 8], [0, 16, 16, 12,
      16]]);
```

$$M3 := \begin{bmatrix} 0 & 0 & 0 & 1 & 0 \\ 0 & 8 & 0 & 3 & 4 \\ 0 & 0 & 0 & 6 & 4 \\ 32 & 8 & 16 & 10 & 8 \\ 0 & 16 & 16 & 12 & 16 \end{bmatrix} \quad \textbf{(5)}$$

```
> M4 := Matrix([[0, 0, 0, 0, 1], [0, 0, 8, 6, 6], [0, 8, 4, 6, 5], [0, 16, 16, 12, 16], [48, 24, 20, 24,
      20]]);
```

$$M4 := \begin{bmatrix} 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 8 & 6 & 6 \\ 0 & 8 & 4 & 6 & 5 \\ 0 & 16 & 16 & 12 & 16 \\ 48 & 24 & 20 & 24 & 20 \end{bmatrix} \quad \textbf{(6)}$$

```
> eigvals1 := eigenvalues(M1);
```

$$eigvals1 := 2, 5, -6, 12, -1 \quad \textbf{(7)}$$

```
> eigvals2 := eigenvalues(M2);
```

$$eigvals2 := 12, 3, 7, -2, -2 \quad \textbf{(8)}$$

```
> eigvals3 := eigenvalues(M3);
```

$$eigvals3 := -8, 4, 8, -2, 32 \quad \textbf{(9)}$$

```
> eigvals4 := eigenvalues(M4);
```

$$eigvals4 := 4, -8, -6, 48, -2 \quad \textbf{(10)}$$

```
>
>
>
> M := M1;
```

$$M := \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 12 & 1 & 2 & 3 & 0 \\ 0 & 2 & 2 & 0 & 2 \\ 0 & 8 & 0 & 3 & 4 \\ 0 & 0 & 8 & 6 & 6 \end{bmatrix} \quad \textbf{(11)}$$

```
> eigvals := eigvals1;
```

(12)

$$eigvals := 2, 5, -6, 12, -1 \quad (12)$$

```
> l1 := -6;
```

$$l1 := -6 \quad (13)$$

```
> l2 := -1;
```

$$l2 := -1 \quad (14)$$

```
> l3 := 2;
```

$$l3 := 2 \quad (15)$$

```
> l4 := 5;
```

$$l4 := 5 \quad (16)$$

```
> l5 := 12;
```

$$l5 := 12 \quad (17)$$

```
> e1 := 105;
```

$$e1 := 105 \quad (18)$$

```
> e2 := 0;
```

$$e2 := 0 \quad (19)$$

```
> e3 := expand(evalm([0, 1, 0, 0, 0].M)[1] · e1);
```

$$e3 := 1260 \quad (20)$$

```
> e4 := expand(evalm([0, 1, 0, 0, 0].M^2)[1]·e1);
```

$$e4 := 1260 \quad (21)$$

```
> e5 := expand(evalm([0, 1, 0, 0, 0].M^3)[1]·e1);
```

$$e5 := 51660 \quad (22)$$

```
> V := transpose(vandermonde([l1, l2, l3, l4, l5]));
```

$$V := \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ -6 & -1 & 2 & 5 & 12 \\ 36 & 1 & 4 & 25 & 144 \\ -216 & -1 & 8 & 125 & 1728 \\ 1296 & 1 & 16 & 625 & 20736 \end{bmatrix} \quad (23)$$

```
> W := evalm( inverse(V).transpose(Matrix([e1, e2, e3, e4, e5])));
```

(24)

$$W := \begin{bmatrix} 14 \\ 56 \\ 14 \\ 20 \\ 1 \end{bmatrix} \quad (24)$$

>

>

>

> $M := M2;$

$$M := \begin{bmatrix} 0 & 0 & 1 & 0 & 0 \\ 0 & 2 & 2 & 0 & 2 \\ 12 & 2 & 5 & 0 & 1 \\ 0 & 0 & 0 & 6 & 4 \\ 0 & 8 & 4 & 6 & 5 \end{bmatrix} \quad (25)$$

> $eigvals := eigvals2;$

$$eigvals := 12, 3, 7, -2, -2 \quad (26)$$

> $l1 := -2;$

$$l1 := -2 \quad (27)$$

> $l2 := 3;$

$$l2 := 3 \quad (28)$$

> $l3 := 7;$

$$l3 := 7 \quad (29)$$

> $l4 := 12;$

$$l4 := 12 \quad (30)$$

> $e1 := 105;$

$$e1 := 105 \quad (31)$$

> $e2 := 0;$

$$e2 := 0 \quad (32)$$

> $e3 := expand(evalm([0, 0, 1, 0, 0].M)[1] \cdot e1);$

$$e3 := 1260 \quad (33)$$

> $e4 := expand\left(evalm\left([0, 0, 1, 0, 0].M^2\right)[1] \cdot e1\right);$

$$e4 := 6300 \quad (34)$$

```
> V := transpose(vandermonde([l1, l2, l3, l4]));
```

$$V := \begin{bmatrix} 1 & 1 & 1 & 1 \\ -2 & 3 & 7 & 12 \\ 4 & 9 & 49 & 144 \\ -8 & 27 & 343 & 1728 \end{bmatrix} \tag{35}$$

```
> W := evalm( inverse(V).transpose(Matrix([e1, e2, e3, e4])));
```

$$W := \begin{bmatrix} 76 \\ 14 \\ 14 \\ 1 \end{bmatrix} \tag{36}$$

```
>
>
>
> M := M3;
```

$$M := \begin{bmatrix} 0 & 0 & 0 & 1 & 0 \\ 0 & 8 & 0 & 3 & 4 \\ 0 & 0 & 0 & 6 & 4 \\ 32 & 8 & 16 & 10 & 8 \\ 0 & 16 & 16 & 12 & 16 \end{bmatrix} \tag{37}$$

```
> eigvals := eigvals3;
```

$$eigvals := -8, 4, 8, -2, 32 \tag{38}$$

```
> l1 := -8;
```

$$l1 := -8 \tag{39}$$

```
> l2 := -2;
```

$$l2 := -2 \tag{40}$$

```
> l3 := 4;
```

$$l3 := 4 \tag{41}$$

```
> l4 := 8;
```

$$l4 := 8 \tag{42}$$

```
> l5 := 32;
```

$$l5 := 32 \tag{43}$$

```
> e1 := 105;
```

$$e1 := 105 \tag{44}$$

```
> e2 := 0;
```

$$e2 := 0 \tag{45}$$

```
> e3 := expand(evalm([0, 0, 0, 1, 0].M)[1] · e1);
```

$$e3 := 3360 \tag{46}$$

```
> e4 := expand(evalm([0, 0, 0, 1, 0].M^2)[1]·e1);
```

$$e4 := 33600 \tag{47}$$

```
> e5 := expand(evalm([0, 0, 0, 1, 0].M^3)[1]·e1);
```

$$e5 := 1169280 \tag{48}$$

```
> V := transpose(vandermonde([l1, l2, l3, l4, l5]));
```

$$V := \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ -8 & -2 & 4 & 8 & 32 \\ 64 & 4 & 16 & 64 & 1024 \\ -512 & -8 & 64 & 512 & 32768 \\ 4096 & 16 & 256 & 4096 & 1048576 \end{bmatrix} \tag{49}$$

```
> W := evalm( inverse(V).transpose(Matrix([e1, e2, e3, e4, e5])));
```

$$W := \begin{bmatrix} 14 \\ 56 \\ 20 \\ 14 \\ 1 \end{bmatrix} \tag{50}$$

```
>
>
>
> M := M4;
```

$$M := \begin{bmatrix} 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 8 & 6 & 6 \\ 0 & 8 & 4 & 6 & 5 \\ 0 & 16 & 16 & 12 & 16 \\ 48 & 24 & 20 & 24 & 20 \end{bmatrix} \tag{51}$$

```
> eigvals := eigvals4;
```

$$\tag{52}$$

$$eigvals := 4, -8, -6, 48, -2 \quad (52)$$

```
> l1 := -8;
```

$$l1 := -8 \quad (53)$$

```
> l2 := -6;
```

$$l2 := -6 \quad (54)$$

```
> l3 := -2;
```

$$l3 := -2 \quad (55)$$

```
> l4 := 4;
```

$$l4 := 4 \quad (56)$$

```
> l5 := 48;
```

$$l5 := 48 \quad (57)$$

```
> e1 := 105;
```

$$e1 := 105 \quad (58)$$

```
> e2 := 0;
```

$$e2 := 0 \quad (59)$$

```
> e3 := expand(evalm([0, 0, 0, 0, 1].M)[1] · e1);
```

$$e3 := 5040 \quad (60)$$

```
> e4 := expand(evalm([0, 0, 0, 0, 1].M^2)[1]·e1);
```

$$e4 := 100800 \quad (61)$$

```
> e5 := expand(evalm([0, 0, 0, 0, 1].M^3)[1]·e1);
```

$$e5 := 5423040 \quad (62)$$

```
> V := transpose(vandermonde([l1, l2, l3, l4, l5]));
```

$$V := \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ -8 & -6 & -2 & 4 & 48 \\ 64 & 36 & 4 & 16 & 2304 \\ -512 & -216 & -8 & 64 & 110592 \\ 4096 & 1296 & 16 & 256 & 5308416 \end{bmatrix} \quad (63)$$

```
> W := evalm( inverse(V).transpose(Matrix([e1, e2, e3, e4, e5])));
```

(64)

$$W := \begin{bmatrix} 20 \\ 14 \\ 14 \\ 56 \\ 1 \end{bmatrix} \quad (64)$$

```
> with(CurveFitting);
```

$$[ArrayInterpolation, BSpline, BSplineCurve, Interactive, LeastSquares, Lowess, PolynomialInterpolation, RationalInterpolation, Spline, ThieleInterpolation] \quad (65)$$

```
> P2a := PolynomialInterpolation([[-6, 3], [-1, -2], [2, 7], [5, -2], [12, 12]], x);
```

$$P2a := \frac{2}{143} x^4 - \frac{3}{22} x^3 - \frac{92}{143} x^2 + \frac{1139}{286} x + \frac{354}{143} \quad (66)$$

```
> P2b := PolynomialInterpolation([[-6, 8], [-1, -2], [2, -8], [5, 4], [12, 32]], x);
```

$$P2b := -\frac{4}{429} x^4 + \frac{1}{11} x^3 + \frac{109}{143} x^2 - \frac{1282}{429} x - \frac{808}{143} \quad (67)$$

```
> P2 := PolynomialInterpolation([[-6, 11], [-1, -4], [2, -1], [5, 2], [12, 44]], x);
```

$$P2 := \frac{2}{429} x^4 - \frac{1}{22} x^3 + \frac{17}{143} x^2 + \frac{853}{858} x - \frac{454}{143} \quad (68)$$

```
> P3 := PolynomialInterpolation([[-6, -6], [-1, 4], [2, -2], [5, -8], [12, 48]], x);
```

$$P3 := \frac{1}{572} x^4 + \frac{1}{22} x^3 - \frac{189}{572} x^2 - \frac{519}{286} x + \frac{366}{143} \quad (69)$$

```
>
>
```