The quads have order (q, q). The generalized octagon has order $(q^2, 1)$.

> restart;

> with(LinearAlgebra) :

> $q := q;$

$$q := q \quad (1)$$

> $s := q; t2 := q; t := (1+1)\cdot t2;$

$$s := q$$
$$t2 := q$$
$$t := 2\,q \quad (2)$$

> $v := (s+1)\cdot(1 + s\cdot t + s^2\cdot t\cdot(t-t2) + s^3\cdot t\cdot(t-t2)^2 + s^4\cdot t2\cdot(t-t2)^3);$

$$v := (q+1)\,(q^8 + 2\,q^6 + 2\,q^4 + 2\,q^2 + 1) \quad (3)$$

> $M := Matrix\Big(\Big[[0, 1, 1, 0, 0, 0, 0, 0, 0, 0], [s, s-1, 0, 1, 0, 0, 0, 0, 0, 0], [s\cdot t, 0, s-1, t2, 1, 0, 0, 0, 0, 0], [0, s\cdot t, s\cdot t2, (t2+1)\cdot(s-1), 0, 1, 0, 0, 0, 0], [0, 0, s\cdot(t-t2), 0, s-1, t2, 1, 0, 0, 0], [0, 0, 0, s\cdot(t-t2), s\cdot t2, (t2+1)\cdot(s-1), 0, 1, 0, 0], \Big[0, 0, 0, 0, s\cdot(t-t2), 0, s-1, t2, \frac{t}{t2}, 0\Big], \Big[0, 0, 0, 0, 0, s\cdot(t-t2), s\cdot t2, (s-1)\cdot(t2+1), 0, \frac{t}{t2}\Big], \Big[0, 0, 0, 0, 0, 0, s\cdot(t-t2), 0, \frac{t}{t2}\cdot(s-1), t+1-\frac{t}{t2}\Big], \Big[0, 0, 0, 0, 0, 0, 0, s\cdot(t-t2), s\cdot\Big(t+1-\frac{t}{t2}\Big), (s-1)\cdot(t+1)\Big]\Big]\Big);$

$$M := [[0, 1, 1, 0, 0, 0, 0, 0, 0, 0], \quad (4)$$
$$[q, q-1, 0, 1, 0, 0, 0, 0, 0, 0],$$
$$[2\,q^2, 0, q-1, q, 1, 0, 0, 0, 0, 0],$$
$$[0, 2\,q^2, q^2, (q+1)\,(q-1), 0, 1, 0, 0, 0, 0],$$
$$[0, 0, q^2, 0, q-1, q, 1, 0, 0, 0],$$
$$[0, 0, 0, q^2, q^2, (q+1)\,(q-1), 0, 1, 0, 0],$$
$$[0, 0, 0, 0, q^2, 0, q-1, q, 2, 0],$$
$$[0, 0, 0, 0, 0, q^2, q^2, (q+1)\,(q-1), 0, 2],$$
$$[0, 0, 0, 0, 0, 0, q^2, 0, 2\,q-2, 2\,q-1],$$
$$[0, 0, 0, 0, 0, 0, 0, q^2, q\,(2\,q-1), (q-1)\,(2\,q+1)]]$$

> factor(CharacteristicPolynomial(M, x));

$$-(x+1)\,(2\,q^2+q-x)\,(q^2+q-x-1)\,(2\,q+1+x)\,(2\,q-1-x)\,(q-2-x)\,(2\,q^2 \quad (5)$$

$-x^2-2x-1)\,(q^4+2q^3-2q^2x-3q^2-2qx+x^2-2q+2x+1)$

 # The quads have order (q, q)

. The generalized octagon has order (q^2, q).

```
> s := q; t2 := q; t := (q + 1)·t2;
```

$$s := q$$
$$t2 := q$$
$$t := (q+1)\,q \quad \textbf{(6)}$$

```
> v := (s + 1)·(1 + s·t + s^2·t·(t − t2) + s^3·t·(t − t2)^2 + s^4·t2·(t − t2)^3);
```

$$v := (q+1)\left(1+q^2(q+1)+q^3(q+1)((q+1)q-q)+q^4(q+1)((q+1)q-q)^2+q^5((q+1)q-q)^3\right) \quad \textbf{(7)}$$

```
> M := Matrix([[0, 1, 1, 0, 0, 0, 0, 0, 0, 0], [s, s − 1, 0 , 1, 0, 0, 0, 0, 0, 0], [s·t, 0, s − 1, t2, 1,
    0, 0, 0, 0, 0], [0, s·t, s·t2, (t2 + 1)·(s − 1), 0, 1, 0, 0, 0, 0], [0, 0, s·(t − t2), 0, s − 1, t2,
    1, 0, 0, 0], [0, 0, 0, s·(t − t2), s·t2, (t2 + 1)·(s − 1), 0, 1, 0, 0], [0, 0, 0, 0, s·(t − t2), 0, s
    − 1, t2, t/t2, 0], [0, 0, 0, 0, 0, s·(t − t2), s·t2, (s − 1)·(t2 + 1), 0, t/t2], [0, 0, 0, 0, 0, 0, s
    ·(t − t2), 0, t/t2·(s − 1), t + 1 − t/t2], [0, 0, 0, 0, 0, 0, 0, s·(t − t2), s·(t + 1 − t/t2), (s
    − 1)·(t + 1)]]);
```

$$M := [[0, 1, 1, 0, 0, 0, 0, 0, 0, 0], \quad \textbf{(8)}$$
$$[q, q-1, 0, 1, 0, 0, 0, 0, 0, 0],$$
$$[q^2(q+1), 0, q-1, q, 1, 0, 0, 0, 0, 0],$$
$$[0, q^2(q+1), q^2, (q+1)(q-1), 0, 1, 0, 0, 0, 0],$$
$$[0, 0, q((q+1)q-q), 0, q-1, q, 1, 0, 0, 0],$$
$$[0, 0, 0, q((q+1)q-q), q^2, (q+1)(q-1), 0, 1, 0, 0],$$
$$[0, 0, 0, 0, q((q+1)q-q), 0, q-1, q, q+1, 0],$$
$$[0, 0, 0, 0, 0, q((q+1)q-q), q^2, (q+1)(q-1), 0, q+1],$$
$$[0, 0, 0, 0, 0, 0, q((q+1)q-q), 0, (q+1)(q-1), (q+1)q-q],$$
$$[0, 0, 0, 0, 0, 0, 0, q((q+1)q-q), q((q+1)q-q), (q-1)((q+1)q+1)]]$$

```
> factor(CharacteristicPolynomial(M, x));
```

$$(x+1)^2\,(q^2+q+x+1)\,(q^3+q^2+q-x)\,(2q^3-x^2-2x-1)\,(q^4-2q^2x-q^2 \quad \textbf{(9)}$$

$- 2qx + x^2 - 2q + 2x + 1)\,(q^2 + q - x - 1)^2$

The quads have order (q, q^2). The generalized octagon has order $(q^3, 1)$.

> $s := q;\ t2 := q^2;\ t := (1+1)\cdot t2;$

$$s := q$$
$$t2 := q^2$$
$$t := 2q^2 \qquad \textbf{(10)}$$

> $v := (s+1)\cdot(1 + s\cdot t + s^2\cdot t\cdot(t-t2) + s^3\cdot t\cdot(t-t2)^2 + s^4\cdot t2\cdot(t-t2)^3);$

$$v := (q+1)\,(q^{12} + 2q^9 + 2q^6 + 2q^3 + 1) \qquad \textbf{(11)}$$

> $M := Matrix\Big(\Big[[0, 1, 1, 0, 0, 0, 0, 0, 0, 0], [s, s-1, 0, 1, 0, 0, 0, 0, 0, 0], [s\cdot t, 0, s-1, t2, 1, 0, 0, 0, 0, 0], [0, s\cdot t, s\cdot t2, (t2+1)\cdot(s-1), 0, 1, 0, 0, 0, 0], [0, 0, s\cdot(t-t2), 0, s-1, t2, 1, 0, 0, 0], [0, 0, 0, s\cdot(t-t2), s\cdot t2, (t2+1)\cdot(s-1), 0, 1, 0, 0], \Big[0, 0, 0, 0, s\cdot(t-t2), 0, s-1, t2, \frac{t}{t2}, 0\Big], \Big[0, 0, 0, 0, 0, s\cdot(t-t2), s\cdot t2, (s-1)\cdot(t2+1), 0, \frac{t}{t2}\Big], \Big[0, 0, 0, 0, 0, 0, s\cdot(t-t2), 0, \frac{t}{t2}\cdot(s-1), t+1-\frac{t}{t2}\Big], \Big[0, 0, 0, 0, 0, 0, 0, s\cdot(t-t2), s\cdot\Big(t+1-\frac{t}{t2}\Big), (s-1)\cdot(t+1)\Big]\Big]\Big);$

$$
\begin{aligned}
M := [&[0, 1, 1, 0, 0, 0, 0, 0, 0, 0], \\
&[q, q-1, 0, 1, 0, 0, 0, 0, 0, 0], \\
&[2q^3, 0, q-1, q^2, 1, 0, 0, 0, 0, 0], \\
&[0, 2q^3, q^3, (q^2+1)\,(q-1), 0, 1, 0, 0, 0, 0], \\
&[0, 0, q^3, 0, q-1, q^2, 1, 0, 0, 0], \\
&[0, 0, 0, q^3, q^3, (q^2+1)\,(q-1), 0, 1, 0, 0], \\
&[0, 0, 0, 0, q^3, 0, q-1, q^2, 2, 0], \\
&[0, 0, 0, 0, 0, q^3, q^3, (q^2+1)\,(q-1), 0, 2], \\
&[0, 0, 0, 0, 0, 0, q^3, 0, 2q-2, 2q^2-1], \\
&[0, 0, 0, 0, 0, 0, 0, q^3, q\,(2q^2-1), (q-1)\,(2q^2+1)]]
\end{aligned} \qquad \textbf{(12)}
$$

> $factor(CharacteristicPolynomial(M, x));$

$$(2q-1-x)\,(2q^2+x+1)\,(2q^3+q-x)\,(q^3+q-x-1)\,(q-2-x)\,(q^2-q+x \qquad \textbf{(13)}$$

$$+1)\left(q^4-4q^3+2q^2x+3q^2-2qx+x^2-2q+2x+1\right)\left(q^6+2q^4-2q^3x-4q^3+q^2-2qx+x^2-2q+2x+1\right)$$

The quads have order (q, q^2). The generalized octagon has order (q^3, q^6).

> $s := q; t2 := q^2; t := (q^6+1)\cdot t2;$

$$s := q$$
$$t2 := q^2$$
$$t := \left(q^6+1\right)q^2 \quad \textbf{(14)}$$

> $v := (s+1)\cdot(1+s\cdot t+s^2\cdot t\cdot(t-t2)+s^3\cdot t\cdot(t-t2)^2+s^4\cdot t2\cdot(t-t2)^3);$

$$v := (q+1)\left(1+q^3\left(q^6+1\right)+q^4\left(q^6+1\right)\left(\left(q^6+1\right)q^2-q^2\right)+q^5\left(q^6+1\right)\left(\left(q^6+1\right)q^2-q^2\right)^2+q^6\left(\left(q^6+1\right)q^2-q^2\right)^3\right) \quad \textbf{(15)}$$

> $M := Matrix\left(\left[[0,1,1,0,0,0,0,0,0,0],[s,s-1,0,1,0,0,0,0,0,0],[s\cdot t,0,s-1,t2,1,0,0,0,0,0],[0,s\cdot t,s\cdot t2,(t2+1)\cdot(s-1),0,1,0,0,0,0],[0,0,s\cdot(t-t2),0,s-1,t2,1,0,0,0],[0,0,0,s\cdot(t-t2),s\cdot t2,(t2+1)\cdot(s-1),0,1,0,0],\left[0,0,0,0,s\cdot(t-t2),0,s-1,t2,\frac{t}{t2},0\right],\left[0,0,0,0,0,s\cdot(t-t2),s\cdot t2,(s-1)\cdot(t2+1),0,\frac{t}{t2}\right],\left[0,0,0,0,0,0,s\cdot(t-t2),0,\frac{t}{t2}\cdot(s-1),t+1-\frac{t}{t2}\right],\left[0,0,0,0,0,0,0,s\cdot(t-t2),s\cdot\left(t+1-\frac{t}{t2}\right),(s-1)\cdot(t+1)\right]\right]\right);$

$$M := [[0,1,1,0,0,0,0,0,0,0], \quad \textbf{(16)}$$
$$[q,q-1,0,1,0,0,0,0,0,0],$$
$$[q^3\left(q^6+1\right),0,q-1,q^2,1,0,0,0,0,0],$$
$$[0,q^3\left(q^6+1\right),q^3,\left(q^2+1\right)(q-1),0,1,0,0,0,0],$$
$$[0,0,q\left(\left(q^6+1\right)q^2-q^2\right),0,q-1,q^2,1,0,0,0],$$
$$[0,0,0,q\left(\left(q^6+1\right)q^2-q^2\right),q^3,\left(q^2+1\right)(q-1),0,1,0,0],$$
$$[0,0,0,0,q\left(\left(q^6+1\right)q^2-q^2\right),0,q-1,q^2,q^6+1,0],$$
$$[0,0,0,0,0,q\left(\left(q^6+1\right)q^2-q^2\right),q^3,\left(q^2+1\right)(q-1),0,q^6+1],$$
$$[0,0,0,0,0,0,q\left(\left(q^6+1\right)q^2-q^2\right),0,\left(q^6+1\right)(q-1),\left(q^6+1\right)q^2-q^6],$$
$$[0,0,0,0,0,0,0,q\left(\left(q^6+1\right)q^2-q^2\right),q\left(\left(q^6+1\right)q^2-q^6\right),(q-1)\left(\left(q^6+1\right)q^2\right.$$

$+ 1)]]$

> factor(CharacteristicPolynomial(M, x));

$$-(q^9+q^3+q-x)\,(q^2-q+x+1)\,(q^8+q^2+x+1)\,(q^6-q+x+1)\,(q^3+q-x-1)\,(q^7+q-x-1)\,(2q^9-q^6-2q^4+2q^3x+2q^3-q^2+2qx-x^2+2q-2x-1)\,(2q^9-q^4+2q^3-2q^2x-3q^2+2qx-x^2+2q-2x-1) \quad (17)$$

The quads have order (q^2, q^4) . The generalized octagon has order (q^6, q^3).

> s := q^2; t2 := q^4; t := (q^3+1)·t2;

$$s := q^2$$

$$t2 := q^4$$

$$t := (q^3+1)\,q^4 \quad (18)$$

> v := (s+1)·(1 + s·t + s^2·t·(t − t2) + s^3·t·(t − t2)^2 + s^4·t2·(t − t2)^3);

$$v := (q^2+1)\,\left(1+q^6\,(q^3+1)+q^8\,(q^3+1)\,((q^3+1)\,q^4-q^4)+q^{10}\,(q^3+1)\,((q^3+1)\,q^4-q^4)^2+q^{12}\,((q^3+1)\,q^4-q^4)^3\right) \quad (19)$$

> M := Matrix([[0, 1, 1, 0, 0, 0, 0, 0, 0, 0], [s, s − 1, 0 , 1, 0, 0, 0, 0, 0, 0], [s·t, 0, s − 1, t2, 1, 0, 0, 0, 0, 0], [0, s·t, s·t2, (t2 + 1)·(s − 1), 0, 1, 0, 0, 0, 0], [0, 0, s·(t − t2), 0, s − 1, t2, 1, 0, 0, 0], [0, 0, 0, s·(t − t2), s·t2, (t2 + 1)·(s − 1), 0, 1, 0, 0], [0, 0, 0, 0, s·(t − t2), 0, s − 1, t2, $\frac{t}{t2}$, 0], [0, 0, 0, 0, 0, s·(t − t2), s·t2, (s − 1)·(t2 + 1), 0, $\frac{t}{t2}$], [0, 0, 0, 0, 0, 0, s·(t − t2), 0, $\frac{t}{t2}$·(s − 1), t + 1 − $\frac{t}{t2}$], [0, 0, 0, 0, 0, 0, 0, s·(t − t2), s·$\left(t + 1 - \frac{t}{t2}\right)$, (s − 1)·(t + 1)]]);

$$M := [[0, 1, 1, 0, 0, 0, 0, 0, 0, 0], \quad (20)$$
$$[q^2, q^2-1, 0, 1, 0, 0, 0, 0, 0, 0],$$
$$[q^6\,(q^3+1), 0, q^2-1, q^4, 1, 0, 0, 0, 0, 0],$$
$$[0, q^6\,(q^3+1), q^6, (q^4+1)\,(q^2-1), 0, 1, 0, 0, 0, 0],$$
$$[0, 0, q^2\,((q^3+1)\,q^4-q^4), 0, q^2-1, q^4, 1, 0, 0, 0],$$
$$[0, 0, 0, q^2\,((q^3+1)\,q^4-q^4), q^6, (q^4+1)\,(q^2-1), 0, 1, 0, 0],$$
$$[0, 0, 0, 0, q^2\,((q^3+1)\,q^4-q^4), 0, q^2-1, q^4, q^3+1, 0],$$

$[0, 0, 0, 0, 0, q^2 ((q^3+1) q^4 - q^4), q^6, (q^4+1)(q^2-1), 0, q^3+1],$
$[0, 0, 0, 0, 0, 0, q^2 ((q^3+1) q^4 - q^4), 0, (q^3+1)(q^2-1), (q^3+1) q^4 - q^3],$
$[0, 0, 0, 0, 0, 0, 0, q^2 ((q^3+1) q^4 - q^4), q^2 ((q^3+1) q^4 - q^3), (q^2-1) ((q^3+1) q^4 + 1)]]$

> *factor*(*CharacteristicPolynomial*(*M*, *x*));

$$(q^6+q^2-x-1)(q^5+q^2-x-1)(q^7+q^4+x+1)(q^9+q^6+q^2-x)(q^4-q^2+x+1)(q^3-q^2+x+1)(q^{12}-2q^9+2q^8-2q^6x-2q^6+q^4-2q^2x-2q^2+x^2+2x+1)(2q^9-q^8+2q^6-2q^4x-3q^4+2q^2x+2q^2-x^2-2x-1) \quad \textbf{(21)}$$

>