Colloquium on

Galois Geometry

To the memory of

Frédéric Vanhove

(1984 – 2013)

Abstracts

Quantum Walks and Distance-Regular Graphs

Chris Godsil

University of Waterloo

If A is the adjacency matrix of a graph X, then the unitary operators defined by

$$U(t) = \exp(-itA)$$

define what physicists call a *continuous quantum walk*. A number of questions of physical interest reduce to questions about the absolute values of entries of $U(t)$. We have found that we can make progress on these questions using standard tools from algebraic graph theory. In particular, association schemes provide useful tools for some of these questions, and distance-regular graphs provide many interesting examples. I will discuss recent progress in this area, some of which was joint work with Frdéric Vanhove and my students.

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More eigenvalue interlacing

Willem Haemers

Tilburg University

(Joint work with A. Abiad, M.A. Fiol and G. Perarnau)

We apply eigenvalue interlacing techniques for obtaining lower and upper bounds for the sums of Laplacian eigenvalues of graphs, and characterize equality. This leads to generalizations of, and variations on theorems by Grone, and Grone & Merris. As a consequence we obtain inequalities involving bounds for some well-known parameters of a graph, such as edge-connectivity, and the isoperimetric number.

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On the Maximum Size of M-Cliques of Generators on Hermitian Polar Spaces

Ferdinand Ihringer

Justus-Liebig-Universität Gießen

A *finite classical polar space* is an incidence geometry defined by a non-degenerate sesquilinear or quadratic form. The elements of maximal rank d of a polar space are called *generators*. Let $M \subseteq \{1, \ldots, d\}$. If we say that two generators are adjacent if they intersect in a space of codimension i for some $i \in M$, then this defines a graph with generators as its vertices. The cliques of this graph are called *M-cliques*. For some choices of M the determination of the maximum size of an M-clique and classifying all M-cliques obtaining this size are popular problems in finite geometry. In particular,

- for $M = \{1, \ldots, t-1, t\}$ an M-clique is an *Erdős-Ko-Rado set* on a polar space,
- for $M = \{d\}$ an M-clique is a *(partial) spread* on a polar space,
- for $M = \{t\}$ an M-clique is a *constant-distance subspace code* of generators on a polar space,

One of Frédéric Vanhove's main interests was to combine algebraic-combinatorial techniques with finite geometry to obtain new results on problems like these. The talk will take a survey through some interesting choices of M with a focus on the Hermitian polar spaces $H(2d-1, q^2)$, Frédéric Vanhove's results and some results obtained by the speaker.

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Girth and dual girth parameters in polynomial association schemes

Bill Martin

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Metric association schemes are better known as distance-regular graphs while their duals, cometric association schemes, are barely known at all.

In the first half of the talk, we discuss a conjecture of Suzuki and (independently) Koolen that the girth of a distance-regular graph other than a polygon cannot exceed 12. One may go further and ask for a "small" set of generators for the homotopy group of such a graph. (We will make this question precise.) Both questions have duals in the class of cometric association schemes. In the second part of the talk, we will introduce these, view their vertex sets as spherical codes, and consider the ideal of polynomials that vanish on this finite set. The structure of this ideal is largely governed by two "dual girth" parameters which we shall introduce. After exploring these parameters in some important cases, we shall employ the character theory of finite abelian groups to exhibit a duality between the homotopy in the first part of the talk and the algebraic geometry in the second part.

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Twisted Symplectic Polar Graphs

Akihiro Munemasa

Tohoku University, Sendai

The symplectic group $\mathrm{Sp}(2d, 2)$ acts on the set of nonzero vectors of the vector space $V(2d, 2)$ of dimension $2d$ over $\mathrm{GF}(2)$. Since this action has rank 3, this gives a strongly regular graph which we call a symplectic polar graph.

Let q be a power of 2. The orthogonal group $\mathrm{O}(3, q)$ acts on the projective plane $\mathrm{PG}(2, q)$ and leaves a quadric Q invariant. The group $\mathrm{O}(3, q)$ also fixes the nucleus, hence leaves the set X of points not on Q and distinct from the nucleus invariant. The set X carries a structure of a strongly regular graph, having the same parameters as the symplectic polar graph. It turns out that the latter graph is not isomorphic to the former. In this talk, we show that Gordon-Mills-Welch difference sets can be used to give a uniform construction of a family of strongly regular graphs containing these two graphs. We also show that when n is odd and $n \geq 5$, the obtained strongly regular graph forms a one of the relations of a 4-class association scheme, while it forms that of a 3-class association scheme when $n = 3$.

This work started as a joint work with Frédéric Vanhove, continued with collaboration with Bill Kantor and Andries Brouwer.

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Extremal Theorems in Polar Spaces

Valentina Pepe

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(Joint work with L. Storme and F. Vanhove)

I had the pleasure to work with Frédéric Vanhove on a classical problem of extremal combinatorics: the classification of the largest sets of pairwise intersecting generators of finite classical polar spaces.

Beyond the results we obtained, an interesting aspect of our research was that the problem had to be tackled with two different kinds of techniques: algebraic graph theory and finite geometry ones. The one without the other would not have worked.

I like to think that this encounter of two different but complementary mathematical approaches is perhaps a nice metaphor for our friendship.

References

[1] V. Pepe, L. Storme, and F. Vanhove, Theorems of Erdős-Ko-Rado type in polar spaces, *Journal of Combinatorial Theory Series A*, 118 (2011) pp. 1291-1312.

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