

# INTEGRATION OVER THE $p$ -ADIC NUMBERS

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Completing the field of rational numbers  $\mathbb{Q}$  with respect to the usual absolute value, one gets the field of real numbers  $\mathbb{R}$ . There exists however, for each prime number  $p$ , also a  $p$ -adic absolute value on  $\mathbb{Q}$ . The completion with respect to this absolute value is called the field of  $p$ -adic numbers  $\mathbb{Q}_p$ . The  $p$ -adic numbers have a fascinating, and slightly peculiar structure. We will introduce the Haar measure on  $\mathbb{Q}_p$ , which allows us to talk about integration. If the function that we want to integrate, is of a certain form, then its integral can be calculated ‘in the same way’ for almost all prime numbers  $p$ . Using this theory we can study certain cases of the conjectures of Igusa and Denef-Sperber on exponential sums.