

The birth of geometric group theory

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Geometric group theory is a relatively new field in mathematics which studies the relation between the algebraic structure of groups and the geometric structure of certain spaces on which they act. It emerged as a separate field in the eighties due to some mayor breakthroughs by Mikhael Gromov. I will focus on one of these results, namely the polynomial growth theorem, and explain how it is related to a type of dynamical systems called expanding maps.

Expanding maps are self-maps of a compact manifold which locally increase the distance between points. They were introduced by Stephen Smale in the sixties as one of the first examples combining the properties of chaos and structural stability, which are seemingly opposite. An open problem at that time was to describe the manifolds admitting such an expanding map. At first, almost nothing was known, except for a few topological properties and a weak geometric condition on the growth of the balls in the universal covering space. After 20 years without progress, M. Gromov single-handedly solved the problem by giving a deep relation between this growth and its fundamental group. I will end the talk by stating some open problems related to Anosov diffeomorphisms, a similar type of dynamical system which are much less understood.