

Half-normal lattice paths

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The enumeration of lattice paths is a classical topic in combinatorics which is up to now a very active field of research. They have many applications in chemistry, physics, mathematics, and computer science. For example lattice paths are used as the solution of integer programming problems, in cryptanalysis, in crystallography, and as models in queueing theory.

We start with a short introduction of these objects and for our purposes classical solution strategies (e.g., generating functions and analytic combinatorics). Then we will turn to more advanced counting by simultaneously observing several parameters using bivariate generating functions. These results give us qualitative answers to questions on the probability that a certain path ends at a given altitude, or that it has a certain number of returns to zero.

In the last part we present the concept of schemes that can be used as blackboxes to identify the limiting distribution for a combinatorial class. This is achieved by determining analytical and algebraic properties of the associated bivariate generating function. In particular, we state sufficient conditions implying a half-normal limiting distribution. We end with some applications to lattice path enumeration.