GENERALISED VERONESE VARIETIES

9TH SHANGHAI CONFERENCE ON COMBINATORICS

MAY 2017

GHENT UNIVERSITY

ANNELEEN DE SCHEPPER

OVERVIEW

(generalised)
Veronese varieties

a little bit of
algebra

point-line geometries
with 3 axioms

OVERVIEW

(generalised)
Veronese varieties

description

examples

a little bit of
algebra

point-line geometries
with 3 axioms

EXAMPLE 1

Let $\mathbb{K}$ be a field. The Veronese variety $\mathscr{V}(\mathbb{K})$ is defined as follows

EXAMPLE 1

Let $\mathbb{K}$ be a field. The Veronese variety $\mathscr{V}(\mathbb{K})$ is defined as follows

$$\rho: PG(2,K) \to PG(5,K): (x,y,z) \mapsto (x^2, y^2, z^2; yz, zx, xy)$$

EXAMPLE 1

Let $\mathbb{K}$ be a field. The Veronese variety $\mathscr{V}(\mathbb{K})$ is defined as follows

$$\rho: PG(2,K) \to PG(5,K): (x,y,z) \mapsto (x^2, y^2, z^2; yz, zx, xy)$$

point → **point**

EXAMPLE 1

Let $\mathbb{K}$ be a field. The Veronese variety $\mathscr{V}(\mathbb{K})$ is defined as follows

$$\rho: PG(2,K) \to PG(5,K): (x,y,z) \mapsto (x^2, y^2, z^2; yz, zx, xy)$$

point → **point**

line → ?

EXAMPLE 1

Let $\mathbb{K}$ be a field. The Veronese variety $\mathscr{V}(\mathbb{K})$ is defined as follows

$$\rho: PG(2,K) \to PG(5,K): (x,y,z) \mapsto (x^2, y^2, z^2; yz, zx, xy)$$

point → **point**

line → ?

$(0,y,z) \mapsto (0, y^2, z^2; yz, 0, 0)$ satisfies $X_1X_2=X_3^2$, $X_0=X_4=X_5=0$

EXAMPLE 1

Let $\mathbb{K}$ be a field. The Veronese variety $\mathscr{V}(\mathbb{K})$ is defined as follows

$$\rho: PG(2,K) \to PG(5,K): (x,y,z) \mapsto (x^2, y^2, z^2; yz, zx, xy)$$

point → **point**

line → **conic in a plane**

$(0,y,z) \mapsto (0, y^2, z^2; yz, 0, 0)$ satisfies $X_1X_2=X_3^2$, $X_0=X_4=X_5=0$

EXAMPLE 1

Let $\mathbb{K}$ be a field. The Veronese variety $\mathscr{V}(\mathbb{K})$ is defined as follows

$$\boldsymbol{\rho}: PG(2,K) \to PG(5,K): (x,y,z) \mapsto (x^2, y^2, z^2; yz, zx, xy)$$

point → **point**

line → **conic in a plane**

$(0,y,z) \mapsto (0, y^2, z^2; yz, 0, 0)$ satisfies $X_1X_2=X_3^2$, $X_0=X_4=X_5=0$

Note: this plane contains no other points of im($\boldsymbol{\rho}$).

EXAMPLE 1

Let $\mathbb{K}$ be a field. The Veronese variety $\mathscr{V}(\mathbb{K})$ is defined as follows

$$\boldsymbol{\rho}: PG(2,K) \to PG(5,K): (x,y,z) \mapsto (x^2, y^2, z^2; yz, zx, xy)$$

point $\to$ **point**

line $\rightsquigarrow$ **plane intersecting im(ρ) in a conic ("conic-place")**

$(0,y,z) \mapsto (0, y^2, z^2; yz, 0, 0)$ satisfies $X_1X_2=X_3^2$, $X_0=X_4=X_5=0$

Note: this plane contains no other points of im(**ρ**).

EXAMPLE 1

Let $\mathbb{K}$ be a field. The Veronese variety $\mathscr{V}(\mathbb{K})$ is defined as follows

$$\rho\colon PG(2,K) \to PG(5,K)\colon (x,y,z) \mapsto (x^2, y^2, z^2; yz, zx, xy)$$

point $\to$ **point**

line $\rightsquigarrow$ **plane intersecting im(ρ) in a conic ("conic-place")**

$(0,y,z) \mapsto (0, y^2, z^2; yz, 0, 0)$ satisfies $X_1X_2=X_3^2$, $X_0=X_4=X_5=0$

This variety of **points** and **conic-planes** has the following **properties**.

EXAMPLE 1

Let $\mathbb{K}$ be a field. The Veronese variety $\mathscr{V}(\mathbb{K})$ is defined as follows

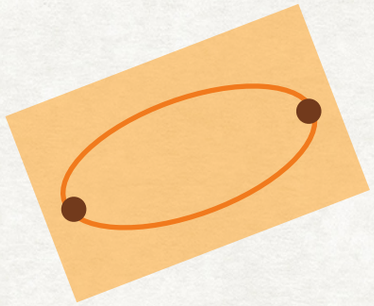
$$\rho\colon PG(2,K) \to PG(5,K)\colon (x,y,z) \mapsto (x^2, y^2, z^2; yz, zx, xy)$$

point $\to$ **point**

line $\rightsquigarrow$ **plane intersecting im(ρ) in a conic ("conic-place")**

$(0,y,z) \mapsto (0, y^2, z^2; yz, 0, 0)$ satisfies $X_1X_2=X_3^2$, $X_0=X_4=X_5=0$

This variety of **points** and **conic-planes** has the following **properties**.



each two **points** of im(ρ) belong to a **conic-plane**

EXAMPLE 1

Let $\mathbb{K}$ be a field. The Veronese variety $\mathscr{V}(\mathbb{K})$ is defined as follows

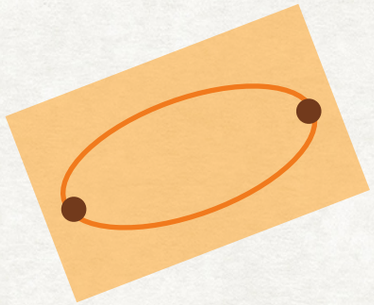
$$\rho\colon PG(2,K) \to PG(5,K)\colon (x,y,z) \mapsto (x^2, y^2, z^2; yz, zx, xy)$$

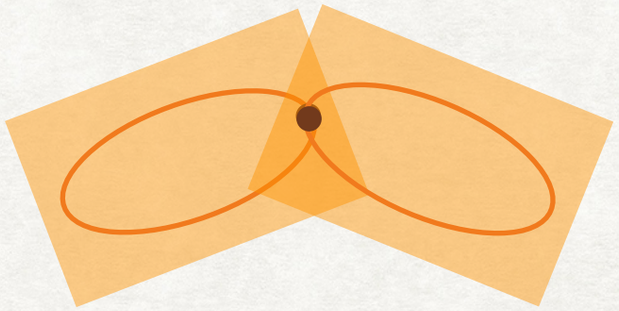
point $\to$ **point**

line $\rightsquigarrow$ **plane intersecting im(ρ) in a conic ("conic-place")**

$(0,y,z) \mapsto (0, y^2, z^2; yz, 0, 0)$ satisfies $X_1X_2=X_3^2$, $X_0=X_4=X_5=0$

This variety of **points** and **conic-planes** has the following **properties**.





each two **points** of im(ρ) belong to a **conic-plane**

the **conic-planes** intersect in **points** of im(ρ)

EXAMPLE 1

Let $\mathbb{K}$ be a field. The Veronese variety $\mathscr{V}(\mathbb{K})$ is defined as follows

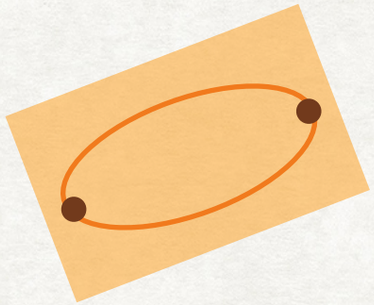
$$\rho\colon PG(2,K) \to PG(5,K)\colon (x,y,z) \mapsto (x^2, y^2, z^2; yz, zx, xy)$$

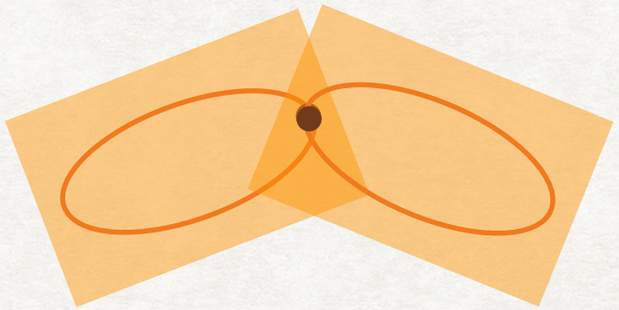
point $\to$ **point**

line $\rightsquigarrow$ **plane intersecting im(ρ) in a conic ("conic-place")**

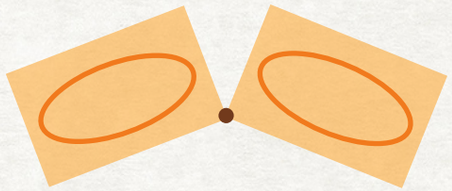
$(0,y,z) \mapsto (0, y^2, z^2; yz, 0, 0)$ satisfies $X_1X_2=X_3^2$, $X_0=X_4=X_5=0$

This variety of **points** and **conic-planes** has the following **properties**.

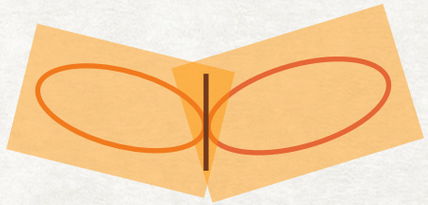




not



nor



each two **points** of im(ρ) belong to a **conic-plane**

the **conic-planes** intersect in **points** of im(ρ)

EXAMPLE 1

Let $\mathbb{K}$ be a field. The Veronese variety $\mathscr{V}(\mathbb{K})$ is defined as follows

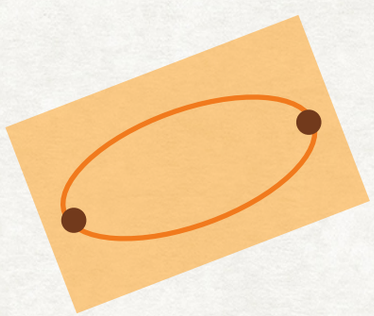
$$\rho\colon PG(2,K) \to PG(5,K)\colon (x,y,z) \mapsto (x^2, y^2, z^2; yz, zx, xy)$$

point $\to$ **point**

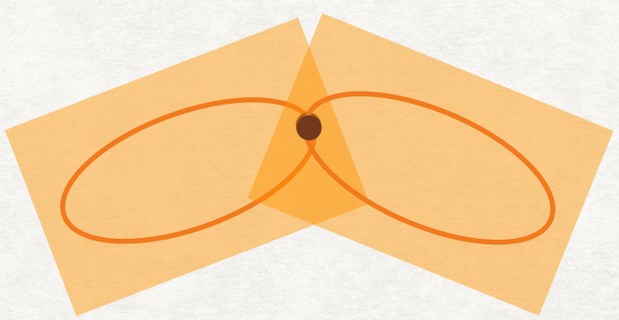
line $\rightsquigarrow$ **plane intersecting im(ρ) in a conic ("conic-place")**

$(0,y,z) \mapsto (0, y^2, z^2; yz, 0, 0)$ satisfies $X_1X_2=X_3^2$, $X_0=X_4=X_5=0$

This variety of **points** and **conic-planes** has the following **properties**.



each two **points** of im(ρ) belong to a **conic-plane**



the **conic-planes** intersect in **points** of im(ρ)

the tangent space of a **point** is a plane

the span of all tangent lines through a point is the tangent space of that point

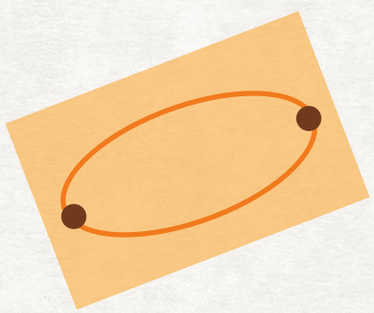
EXAMPLE 1

Axiomatic description of the Veronese variety $\mathscr{V}(K)$

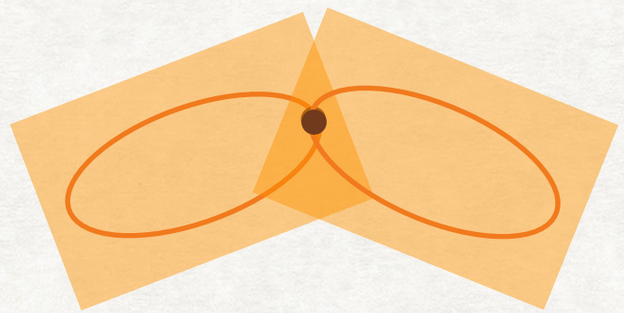
X = spanning **point** set of PG(N,K)

Ξ = family of **planes** ξ in PG(N,K) s.th. $\xi \cap X$ is an **conic** in ξ

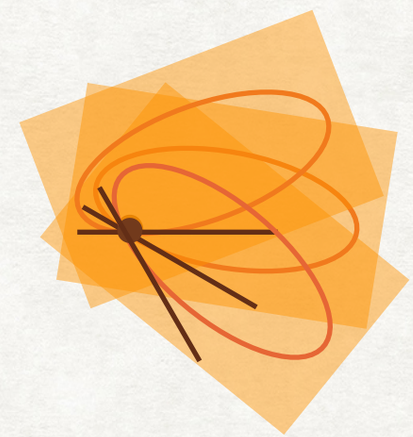
such that the following **properties** are satisfied



each two **points** of X belong to a **plane** of Ξ



two **planes** of Ξ intersect in **points** of X



the tangent space of a **point** of X is a plane

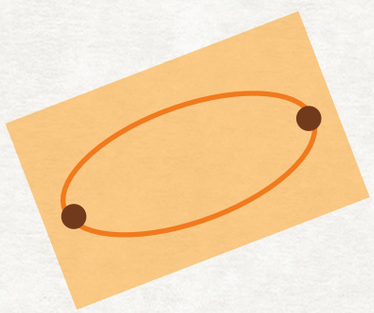
EXAMPLE 1

Axiomatic description of the Veronese variety $\mathscr{V}(K)$

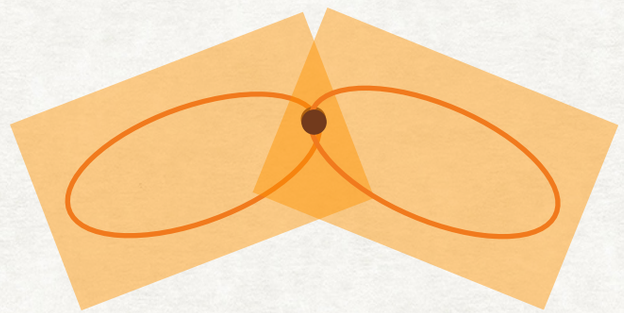
X = spanning **point** set of PG(N,K)

Ξ = family of **planes** ξ in PG(N,K) s.th. ξ ∩ X is an **conic** in ξ

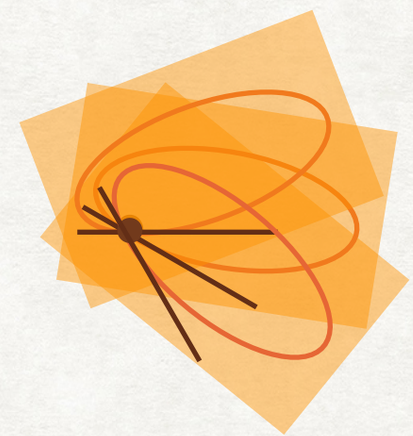
such that the following **properties** are satisfied



each two **points** of **X** belong to a **plane** of **Ξ**



two **planes** of **Ξ** intersect in **points** of **X**



the tangent space of a **point** of **X** is a plane

Mazzocca, Melone (1984)

If K=F(q) with q odd, and N=5:

(**X**, **Ξ**) is projectively unique and hence equivalent with $\mathscr{V}(K)$.

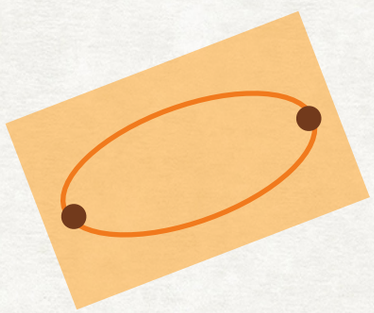
EXAMPLE 1

Axiomatic description of the Veronese variety $\mathscr{V}(K)$

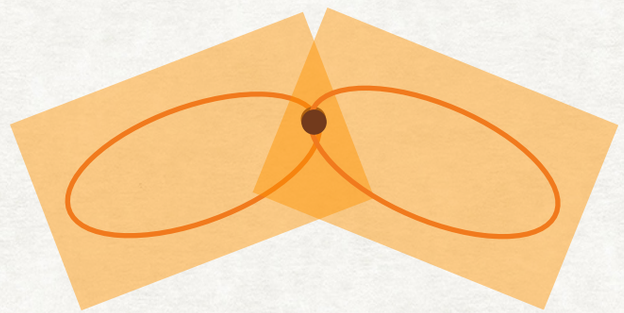
X = spanning **point** set of PG(N,K)

Ξ = family of **planes** ξ in PG(N,K) s.th. ξ ∩ X is an **conic** in ξ

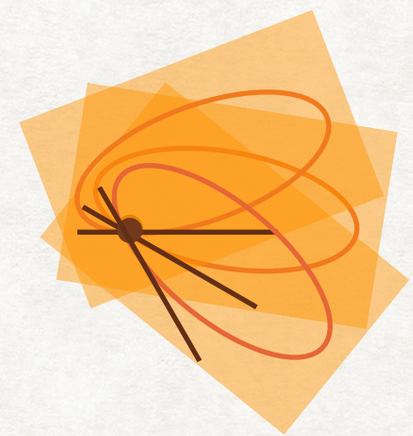
such that the following **properties** are satisfied



each two **points** of X belong to a **plane** of Ξ



two **planes** of Ξ intersect in **points** of X



the tangent space of a **point** of X is a plane

Hirschfeld, Thas (1991)

If K=F(q):

(X, Ξ) is projectively unique and hence equivalent with $\mathscr{V}(K)$.

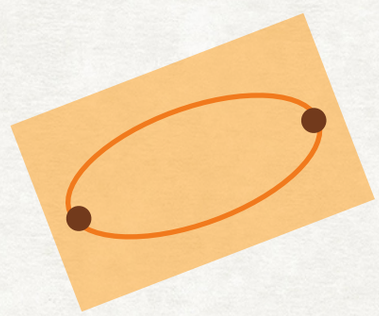
EXAMPLE 1

Axiomatic description of the Veronese variety $\mathscr{V}(K)$

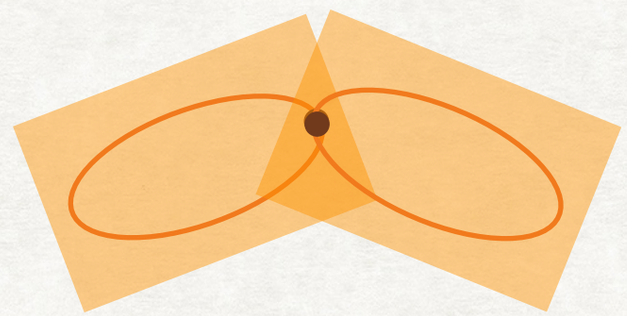
X = spanning point set of PG(N,K)

Ξ = family of planes ξ in PG(N,K) s.th. ξ ∩ X is an conic in ξ

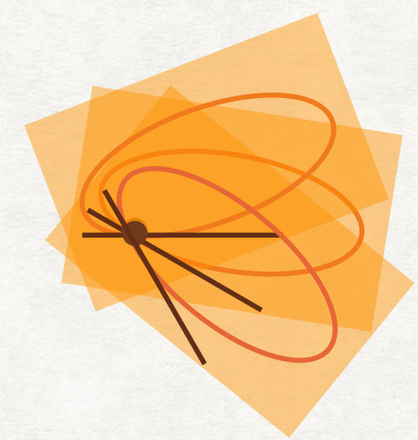
such that the following **properties** are satisfied



each two points of X belong to a plane of Ξ



two planes of Ξ intersect in points of X



the tangent space of a point of X is a plane

Thas, Van Maldeghem (2004)

If K=F(q) and we use ovals instead of conics:

(X, Ξ) is projectively unique and hence equivalent with $\mathscr{V}(K)$.

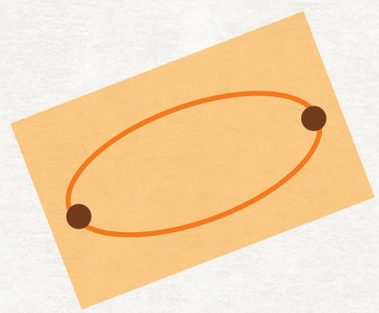
EXAMPLE 1

Axiomatic description of the Veronese variety $\mathscr{V}(K)$

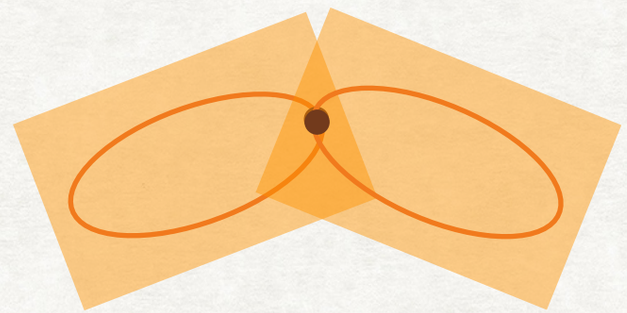
X = spanning **point** set of PG(N,K)

Ξ = family of **planes** ξ in PG(N,K) s.th. $\xi \cap X$ is an **conic** in ξ

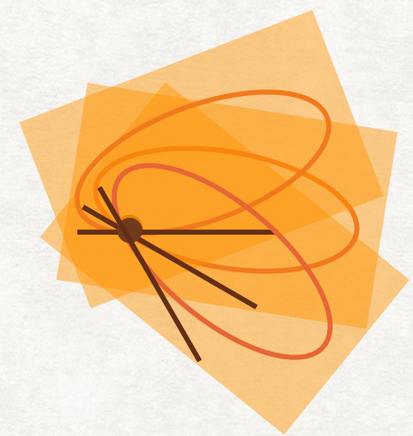
such that the following **properties** are satisfied



each two **points** of X belong to a **plane** of Ξ



two **planes** of Ξ intersect in **points** of X



the tangent space of a **point** of X is a plane

Schillewaert, Van Maldeghem (2013)

For any field K (and still using ovals):

(X, Ξ) is projectively unique and hence equivalent with $\mathscr{V}(K)$.

EXAMPLE 1

Axiomatic description of the Veronese variety $\mathscr{V}(K)$

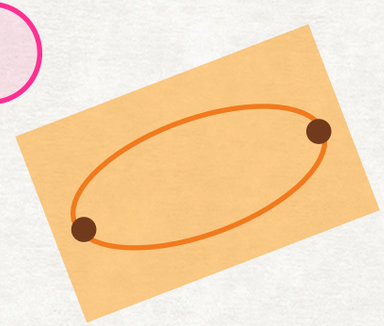
(X, Ξ) is an MM-set

X = spanning **point** set of PG(N,K)

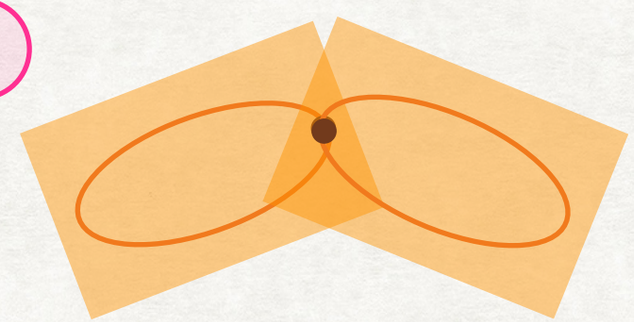
Ξ = family of **planes** ξ in PG(N,K) s.th. $\xi \cap X$ is an **conic** in ξ

such that the following **properties** are satisfied

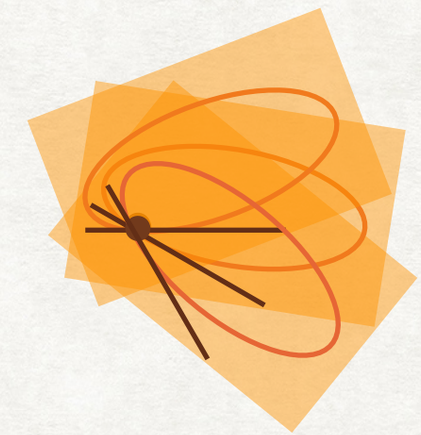
MM1



MM2



MM3



each two **points** of X belong to a **plane** of Ξ

two **planes** of Ξ intersect in **points** of X

the tangent space of a **point** of X is a plane

Schillewaert, Van Maldeghem (2013)

For any field K (and still using ovals):

(X, Ξ) is projectively unique and hence equivalent with $\mathscr{V}(K)$.

OTHER VERONESE VARIETIES

OTHER VERONESE VARIETIES

To each quadratic alternative algebra we associate a Veronese variety.

OTHER VERONESE VARIETIES

Let $\mathbb{K}$ be a field, char(K) ≠ 2 (for simplicity).

To each quadratic alternative algebra we associate a Veronese variety.

OTHER VERONESE VARIETIES

Let $\mathbb{K}$ be a field, char(K) ≠ 2 (for simplicity).

To each quadratic **alternative** algebra we associate a Veronese variety.

x(xy) = (xx)y and (xy)y=x(yy) for all x,y in A

OTHER VERONESE VARIETIES

Let $\mathbb{K}$ be a field, char(K) ≠ 2 (for simplicity).

To each quadratic alternative algebra we associate a Veronese variety.

OTHER VERONESE VARIETIES

Let $\mathbb{K}$ be a field, char(K) ≠ 2 (for simplicity).

To each **quadratic** alternative algebra we associate a Veronese variety.

each elt. of A is the **root of quadratic equation over K**

OTHER VERONESE VARIETIES

Let K be a field, char(K) ≠ 2 (for simplicity).

To each **quadratic** alternative algebra we associate a Veronese variety.

each elt. of A is the **root of quadratic equation over K**

$\forall x \in A \backslash K$ there are (unique) $t_A(x)$, $n_A(x) \in K$ such that

$$x^2 - t_A(x)\, x + n_A(x) = 0$$

OTHER VERONESE VARIETIES

Let K be a field, char(K) ≠ 2 (for simplicity).

To each **quadratic** alternative algebra we associate a Veronese variety.

each elt. of A is the **root of quadratic equation over K**

$\forall x \in A \backslash K$ there are (unique) $t_A(x)$, $n_A(x) \in K$ such that

$$x^2 - t_A(x)\, x + n_A(x) = 0$$

Denote the other root by $\underline{x}$ and put $\underline{x} = x$ for $x \in K$.

OTHER VERONESE VARIETIES

Let K be a field, char(K) ≠ 2 (for simplicity).

To each **quadratic** alternative algebra we associate a Veronese variety.

each elt. of A is the **root of quadratic equation over K**

$\forall x \in A \backslash K$ there are (unique) $t_A(x)$, $n_A(x) \in K$ such that

$$x^2 - t_A(x)\, x + n_A(x) = 0$$

Denote the other root by $\underline{x}$ and put $\underline{x} = x$ for $x \in K$.

Then $x \mapsto \underline{x}$ is an **involution**, inducing a **norm function** on A:

$$n_A : A \to K : x \mapsto x\underline{x}$$

OTHER VERONESE VARIETIES

Let K be a field, char(K) ≠ 2 (for simplicity).

To each **quadratic** alternative algebra we associate a Veronese variety.

each elt. of A is the **root of quadratic equation over K**

$\forall x \in A \backslash K$ there are (unique) $t_A(x)$, $n_A(x) \in K$ such that

$$x^2 - t_A(x)\, x + n_A(x) = 0$$

Denote the other root by $\underline{x}$ and put $\underline{x} = x$ for $x \in K$.

Then $x \mapsto \underline{x}$ is an **involution**, inducing a **norm function** on A:

$$n_A : A \to K : x \mapsto x\underline{x}$$

Property: x is <u>invertible</u> iff $n_A(x) \neq 0$;

if so, $x^{-1} = n_A(x)^{-1}\, \underline{x}$

OTHER VERONESE VARIETIES

Let K be a field, char(K) ≠ 2 (for simplicity).

To each **quadratic** alternative algebra we associate a Veronese variety.

each elt. of A is the **root of quadratic equation over K**

$\forall x \in A \backslash K$ there are (unique) $t_A(x)$, $n_A(x) \in K$ such that

$$x^2 - t_A(x)\, x + n_A(x) = 0$$

Denote the other root by $\underline{x}$ and put $\underline{x} = x$ for $x \in K$.

Then $x \mapsto \underline{x}$ is an **involution**, inducing a **norm function** on A:

$$n_A : A \to K : x \mapsto x\underline{x}$$

Property: x is <u>invertible</u> iff $n_A(x) \neq 0$;

if so, $x^{-1} = n_A(x)^{-1}\, \underline{x}$

If $n_A(x) = 0$ implies $x = 0$

A is a division algebra

OTHER VERONESE VARIETIES

Let K be a field, char(K) ≠ 2 (for simplicity).

To each **quadratic** alternative algebra we associate a Veronese variety.

each elt. of A is the **root of quadratic equation over K**

$\forall x \in A \backslash K$ there are (unique) $t_A(x)$, $n_A(x) \in K$ such that

$$x^2 - t_A(x)\, x + n_A(x) = 0$$

Denote the other root by $\underline{x}$ and put $\underline{x} = x$ for $x \in K$.

Then $x \mapsto \underline{x}$ is an **involution**, inducing a **norm function** on A:

$$n_A : A \to K : x \mapsto x\underline{x}$$

Property: x is <u>invertible</u> iff $n_A(x) \neq 0$;

if so, $x^{-1} = n_A(x)^{-1}\, \underline{x}$

If $n_A(x) = 0$ implies $x = 0$
A is a division algebra

If not
A is a split algebra

OTHER VERONESE VARIETIES

Let K be a field, char(K) ≠ 2 (for simplicity).

To each **quadratic** alternative algebra we associate a Veronese variety.

each elt. of A is the **root of quadratic equation over K**

$\forall x \in A \backslash K$ there are (unique) $t_A(x)$, $n_A(x) \in K$ such that

$$x^2 - t_A(x)\, x + n_A(x) = 0$$

Denote the other root by $\underline{x}$ and put $\underline{x} = x$ for $x \in K$.

Then $x \mapsto \underline{x}$ is an **involution**, inducing a **norm function** on A:

$$n_A : A \to K : x \mapsto x\underline{x}$$

Property: x is <u>invertible</u> iff $n_A(x) \neq 0$;

if so, $x^{-1} = n_A(x)^{-1}\, \underline{x}$

If $n_A(x) = 0$ implies $x = 0$
A is a division algebra

If not
A is a split algebra

Example split
<u>2x2 matrices over K</u>
$n(M) = \det(M)$
$\underline{M} = \mathrm{Adj}(M)$

OTHER VERONESE VARIETIES

Let $\mathbb{K}$ be a field, char(K) ≠ 2 (for simplicity).

To each quadratic alternative algebra we associate a Veronese variety.

OTHER VERONESE VARIETIES

Let $\mathbb{K}$ be a field, char(K) ≠ 2 (for simplicity).

To each quadratic alternative algebra we associate a Veronese variety.

dim(A/K) ∈ {1,2,4,8} and A is one of the following:

OTHER VERONESE VARIETIES

Let K be a field, char(K) ≠ 2 (for simplicity).

To each quadratic alternative algebra we associate a Veronese variety.

dim(A/K) ∈ {1,2,4,8} and A is one of the following:

dim	division	split	peculiarities
1	commutative **field** K		$x = \underline{x}$, $n(x)=x^2$
2	L **quadr. extension** $K[\sqrt{a}]$ with $a \notin K^2$	L′ **quadr. extension** $K[\sqrt{a}]$ with $a \in K^2$	commutative ring
4	H **quaternion** division algebra over K	H′ split **quaternions** over K (2x2 matrices)	associative ring
8	O **octonion division algebra** over K	O′ split **octonions** over K	strictly alternative ring

OTHER VERONESE VARIETIES

Let $\mathbb{K}$ be a field, char(K) ≠ 2 (for simplicity).

To each quadratic alternative algebra we associate a Veronese variety.

dim(A/K) ∈ {1,2,4,8} and A is one of the following:

dim	division	split	peculiarities
1	commutative **field** K		$x = \underline{x}$, $n(x)=x^2$
2	L **quadr. extension** $K[\sqrt{a}]$ with $a \notin K^2$	L′ **quadr. extension** $K[\sqrt{a}]$ with $a \in K^2$	commutative ring
4	H **quaternion** division algebra over K	H′ split **quaternions** over K (2x2 matrices)	associative ring
8	O **octonion division algebra** over K	O′ split **octonions** over K	strictly alternative ring

Remark: If K is a finite field, there is no division quaternion algebra nor an octonion algebra over it.

ASSOCIATED VERONESE VARIETIES

Example 1. The Veronese variety $\mathscr{V}(K)$ - K a field

ρ: PG(2,K) → PG(5,K): $(x,y,z) \mapsto (x^2, y^2, z^2; yz, zx, xy)$

line ➜ conic

$(0,y,z) \mapsto (0, y^2, z^2; yz, 0, 0)$ satisfies $X_1X_2=X_3^2$

ASSOCIATED VERONESE VARIETIES

Example 1. The Veronese variety $\mathscr{V}(K)$ - K a field

ρ: PG(2,K) → PG(5,K): $(x,y,z) \mapsto (x^2, y^2, z^2; yz, zx, xy)$

line ➜ conic

$(0,y,z) \mapsto (0, y^2, z^2; yz, 0, 0)$ satisfies $X_1X_2=X_3^2$

Example 2. The Veronese variety $\mathscr{V}(R)$ - $R = K[\sqrt{a}]$

ASSOCIATED VERONESE VARIETIES

Example 1. The Veronese variety $\mathscr{V}(K)$ - K a field

ρ: PG(2,K) → PG(5,K): $(x,y,z) \mapsto (x^2, y^2, z^2; yz, zx, xy)$

line ➜ conic

$(0,y,z) \mapsto (0, y^2, z^2; yz, 0, 0)$ satisfies $X_1X_2=X_3^2$

Example 2. The Veronese variety $\mathscr{V}(R)$ - $R = K[\sqrt{a}]$

➜ rewrite ρ, using that $x\underline{x} = x^2 = n(x)$ for $x \in K$

ASSOCIATED VERONESE VARIETIES

Example 1. The Veronese variety $\mathscr{V}(K)$ - K a field

ρ: PG(2,K) → PG(5,K): $(x,y,z) \mapsto (x\underline{x}, y\underline{y}, z\underline{z}; y\underline{z}, z\underline{x}, x\underline{y})$

line ➜ conic

$(0,y,z) \mapsto (0, y\underline{y}, z\underline{z}; y\underline{z}, 0, 0)$ satisfies $X_1X_2 = X_3\underline{X_3} = n(X_3) = X_3^2$

Example 2. The Veronese variety $\mathscr{V}(R)$ - $R = K[\sqrt{a}]$

➜ rewrite ρ, using that $x\underline{x} = x^2 = n(x)$ for $x \in K$

ASSOCIATED VERONESE VARIETIES

Example 1. The Veronese variety $\mathscr{V}(K)$ - K a field

ρ: PG(2,K) → PG(5,K): $(x,y,z) \mapsto (x\underline{x}, y\underline{y}, z\underline{z}; y\underline{z}, z\underline{x}, x\underline{y})$

line ➜ conic

$(0,y,z) \mapsto (0, y\underline{y}, z\underline{z}; y\underline{z}, 0, 0)$ satisfies $X_1X_2 = X_3\underline{X_3} = n(X_3) = X_3^2$

Example 2. The Veronese variety $\mathscr{V}(R)$ - $R = K[\sqrt{a}]$

$X_0 \quad X_1 \quad X_2 \quad (X_3, X_4) \quad (X_5, X_6) \quad (X_7, X_8)$

ρ: PG(2,R) → PG(8,K): $(x,y,z) \mapsto (x\underline{x}, y\underline{y}, z\underline{z}; \; y\underline{z} \;, \; z\underline{x} \;, \; x\underline{y} \;)$

ASSOCIATED VERONESE VARIETIES

Example 1. The Veronese variety $\mathscr{V}(K)$ - K a field

ρ: PG(2,K) → PG(5,K): $(x,y,z) \mapsto (x\underline{x}, y\underline{y}, z\underline{z}; y\underline{z}, z\underline{x}, x\underline{y})$

line ➜ conic

$(0,y,z) \mapsto (0, y\underline{y}, z\underline{z}; y\underline{z}, 0, 0)$ satisfies $X_1X_2 = X_3\underline{X_3} = n(X_3) = X_3^2$

Example 2. The Veronese variety $\mathscr{V}(R)$ - $R = K[\sqrt{a}]$

$X_0 \quad X_1 \quad X_2 \quad (X_3, X_4) \quad (X_5, X_6) \quad (X_7, X_8)$

ρ: PG(2,R) → PG(8,K): $(x,y,z) \mapsto (x\underline{x}, y\underline{y}, z\underline{z}; \; y\underline{z} \;, \; z\underline{x} \;, \; x\underline{y} \;)$

Warning: if R = L′ (split), then there is no projective plane over it.

ASSOCIATED VERONESE VARIETIES

Example 1. The Veronese variety $\mathscr{V}(K)$ - K a field

ρ: PG(2,K) → PG(5,K): $(x,y,z) \mapsto (x\underline{x}, y\underline{y}, z\underline{z}; y\underline{z}, z\underline{x}, x\underline{y})$

line ➜ conic

$(0,y,z) \mapsto (0, y\underline{y}, z\underline{z}; y\underline{z}, 0, 0)$ satisfies $X_1X_2 = X_3\underline{X_3} = n(X_3) = X_3^2$

Example 2. The Veronese variety $\mathscr{V}(R)$ - $R = K[\sqrt{a}]$

$X_0 \quad X_1 \quad X_2 \quad (X_3, X_4) \quad (X_5, X_6) \quad (X_7, X_8)$

ρ: PG(2,R) → PG(8,K): $(x,y,z) \mapsto (x\underline{x}, y\underline{y}, z\underline{z}; \; y\underline{z} \;, \; z\underline{x} \;, \; x\underline{y} \;)$

Warning: if R = L′ (split), then there is no projective plane over it.

➜ take a ring geometry G(2,R) as follows

<u>points</u> : $\{(x,y,z)R^* \mid x, y, z \in R \;\&\; (x,y,z)r = 0 \text{ for } r \in R \text{ implies } r = 0\}$

<u>lines</u> : $\{R^*[a,b,c] \mid a, b, c \in R \;\&\; r[a,b,c] = 0 \text{ for } r \in R \text{ implies } r = 0\}$

<u>incidence</u>: $ax + by + cz = 0$

If R = L (division algebra), then G(2,L)=PG(2,L)

ASSOCIATED VERONESE VARIETIES

Example 1. The Veronese variety $\mathscr{V}(K)$ - K a field

ρ: PG(2,K) → PG(5,K): $(x,y,z) \mapsto (x\underline{x}, y\underline{y}, z\underline{z}; y\underline{z}, z\underline{x}, x\underline{y})$

line ➜ conic

$(0,y,z) \mapsto (0, y\underline{y}, z\underline{z}; y\underline{z}, 0, 0)$ satisfies $X_1X_2 = X_3\underline{X_3} = n(X_3) = X_3^2$

Example 2. The Veronese variety $\mathscr{V}(R)$ - $R = K[\sqrt{a}]$

$X_0 \quad X_1 \quad X_2 \quad (X_3, X_4) \quad (X_5, X_6) \quad (X_7, X_8)$

ρ: G(2,R) → PG(8,K): $(x,y,z) \mapsto (x\underline{x}, y\underline{y}, z\underline{z}; \; y\underline{z} \;, \; z\underline{x} \;, \; x\underline{y} \;)$

Warning: if R = L′ (split), then there is no projective plane over it.

➜ take a ring geometry G(2,R) as follows

points : $\{(x,y,z)R^* \mid x, y, z \in R \;\&\; (x,y,z)r = 0 \text{ for } r \in R \text{ implies } r = 0\}$

lines : $\{R^*[a,b,c] \mid a, b, c \in R \;\&\; r[a,b,c] = 0 \text{ for } r \in R \text{ implies } r = 0\}$

incidence: $ax + by + cz = 0$

If R = L (division algebra), then G(2,L)=PG(2,L)

ASSOCIATED VERONESE VARIETIES

Example 1. The Veronese variety $\mathscr{V}(K)$ - K a field

ρ: PG(2,K) → PG(5,K): $(x,y,z) \mapsto (x\underline{x}, y\underline{y}, z\underline{z}; y\underline{z}, z\underline{x}, x\underline{y})$

line ➜ conic

$(0,y,z) \mapsto (0, y\underline{y}, z\underline{z}; y\underline{z}, 0, 0)$ satisfies $X_1X_2 = X_3\underline{X_3} = n(X_3) = X_3^2$

Example 2. The Veronese variety $\mathscr{V}(R)$ - $R = K[\sqrt{a}]$

$X_0 \quad X_1 \quad X_2 \quad (X_3, X_4) \quad (X_5, X_6) \quad (X_7, X_8)$

ρ: G(2,R) → PG(8,K): $(x,y,z) \mapsto (x\underline{x}, y\underline{y}, z\underline{z}; \; y\underline{z} \;, \; z\underline{x} \;, \; x\underline{y} \;)$

ASSOCIATED VERONESE VARIETIES

Example 1. The Veronese variety $\mathscr{V}(K)$ - K a field

ρ: PG(2,K) → PG(5,K): $(x,y,z) \mapsto (x\underline{x}, y\underline{y}, z\underline{z}; y\underline{z}, z\underline{x}, x\underline{y})$

line ➜ conic

$(0,y,z) \mapsto (0, y\underline{y}, z\underline{z}; y\underline{z}, 0, 0)$ satisfies $X_1X_2 = X_3\underline{X_3} = n(X_3) = X_3^2$

Example 2. The Veronese variety $\mathscr{V}(R)$ - $R = K[\sqrt{a}]$

$X_0 \quad X_1 \quad X_2 \quad (X_3, X_4) \quad (X_5, X_6) \quad (X_7, X_8)$

ρ: G(2,R) → PG(8,K): $(x,y,z) \mapsto (x\underline{x}, y\underline{y}, z\underline{z}; \; y\underline{z} \;, \; z\underline{x} \;, \; x\underline{y} \;)$

$(0,y,z) \mapsto (0, y\underline{y}, z\underline{z}; y\underline{z}, 0, 0)$ satisfies $X_1X_2 = n(X_3, X_4) = X_3^2 - aX_4^2$

ASSOCIATED VERONESE VARIETIES

Example 1. The Veronese variety $\mathscr{V}(K)$ - K a field

ρ: PG(2,K) → PG(5,K): $(x,y,z) \mapsto (x\underline{x}, y\underline{y}, z\underline{z}; y\underline{z}, z\underline{x}, x\underline{y})$

line ➜ conic

$(0,y,z) \mapsto (0, y\underline{y}, z\underline{z}; y\underline{z}, 0, 0)$ satisfies $X_1X_2 = X_3\underline{X_3} = n(X_3) = X_3^2$

Example 2. The Veronese variety $\mathscr{V}(R)$ - $R = K[\sqrt{a}]$

$X_0 \quad X_1 \quad X_2 \quad (X_3, X_4) \quad (X_5, X_6) \quad (X_7, X_8)$

ρ: G(2,R) → PG(8,K): $(x,y,z) \mapsto (x\underline{x}, y\underline{y}, z\underline{z}; \; y\underline{z} \;, \; z\underline{x} \;, \; x\underline{y} \;)$

$(0,y,z) \mapsto (0, y\underline{y}, z\underline{z}; y\underline{z}, 0, 0)$ satisfies $X_1X_2 = n(X_3, X_4) = X_3^2 - aX_4^2$

- If $a \notin K^2$ (i.e., R division), we obtain an elliptic quadric in PG(3,K)
- If $a \in K^2$ (i.e., R split), we obtain a hyperbolic quadric in PG(3,K)

ASSOCIATED VERONESE VARIETIES

Example 1. The Veronese variety $\mathscr{V}(K)$ - K a field

ρ: PG(2,K) → PG(5,K): $(x,y,z) \mapsto (x\underline{x}, y\underline{y}, z\underline{z}; y\underline{z}, z\underline{x}, x\underline{y})$

line ➜ conic

$(0,y,z) \mapsto (0, y\underline{y}, z\underline{z}; y\underline{z}, 0, 0)$ satisfies $X_1X_2 = X_3\underline{X_3} = n(X_3) = X_3^2$

Example 2. The Veronese variety $\mathscr{V}(R)$ - $R = K[\sqrt{a}]$

$X_0 \quad X_1 \quad X_2 \quad (X_3, X_4) \quad (X_5, X_6) \quad (X_7, X_8)$

ρ: G(2,R) → PG(8,K): $(x,y,z) \mapsto (x\underline{x}, y\underline{y}, z\underline{z}; \; y\underline{z} \;, \; z\underline{x} \;, \; x\underline{y} \;)$

$(0,y,z) \mapsto (0, y\underline{y}, z\underline{z}; y\underline{z}, 0, 0)$ satisfies $X_1X_2 = n(X_3, X_4) = X_3^2 - aX_4^2$

Axiomatic description of the Veronese variety $\mathscr{V}(R)$

X = spanning point set of PG(N,K)

Ξ = family of 3-spaces ξ in PG(N,K) s.th. $\xi \cap X$ is an ell./hyp. quadric in ξ

each two points of X belong to a [3] of Ξ	two [3]s of Ξ intersect in points of X	the tangent space of a point is contained in a [4]

ASSOCIATED VERONESE VARIETIES

What do we obtain for the other quadratic alternative algebras over K?

dim	division	split	peculiarities
1	commutative **field** K		$x = \underline{x}$, $n(x)=x^2$
2	L' **quadr. extension** $K[\sqrt{a}]$ with $a \notin K^2$	L' **quadr. extension** $K[\sqrt{a}]$ with $a \in K^2$	commutative ring
4	H **quaternion** division algebra over K	H' split **quaternions** over K (2x2 matrices)	associative ring
8	O **octonion division algebra** over K	O' split **octonions** over K	strictly alternative ring

ASSOCIATED VERONESE VARIETIES

What do we obtain for the other quadratic alternative algebras over K?

The quadrics are the following.

dim	division	split	dim(im(ρ))
1	**oval** in PG(2,K)		5
2	**elliptic quadric** in PG(3,K)	**hyperbolic quadric** in PG(3,K)	8
4	**elliptic quadric** in PG(5,K)	**hyperbolic quadric** in PG(5,K)	14
8	**elliptic quadric** in PG(9,K)	**hyperbolic quadric** in PG(9,K)	26

ASSOCIATED VERONESE VARIETIES

What do we obtain for the other quadratic alternative algebras over K?

The quadrics are the following.

dim	division	split	dim(im(ρ))
1	**oval** in PG(2,K)		5
2	**elliptic quadric** in PG(3,K)	**hyperbolic quadric** in PG(3,K)	8
4	**elliptic quadric** in PG(5,K)	**hyperbolic quadric** in PG(5,K)	14
8	**elliptic quadric** in PG(9,K)	**hyperbolic quadric** in PG(9,K)	26
d-1	quadric of minimal Witt index in PG(d,K)	quadric of maximal Witt index in PG(d,K)	3d-1

ASSOCIATED VERONESE VARIETIES

What do we obtain for the other quadratic alternative algebras over K?

The Veronese varieties are isomorphic to:

dim	division	split	dim. of var.
1	PG(2,K)		5
2	PG(2,L)	Segre variety $S_{2,2}(K)$	8
4	PG(2,H)	Line Grassmannian of PG(5,K)	14
8	PG(2,O)	$E_6(K)$-variety	26

ASSOCIATED VERONESE VARIETIES

What do we obtain for the other quadratic alternative algebras over K?

The Veronese varieties are isomorphic to:

dim	division	split	dim. of var.
1	PG(2,K)		5
2	PG(2,L)	Segre variety $S_{2,2}(K)$	8
4	PG(2,H)	Line Grassmannian of PG(5,K)	14
8	PG(2,O)	$E_6(K)$-variety	26
d-1	quadric of minimal Witt index in PG(d,K)	quadric of maximal Witt index in PG(d,K)	3d-1

These Veronese varieties are MM-sets with quadrics in PG(d-1,K) of *min/max* Witt index.

ASSOCIATED VERONESE VARIETIES

What do we obtain for the other quadratic alternative algebras over K?

The Veronese varieties are isomorphic to:

dim	division	split	dim. of var.
1	PG(2,K)		5
2	PG(2,L)	Segre variety $S_{2,2}(K)$	8
4	PG(2,H)	Line Grassmannian of PG(5,K)	14
8	PG(2,O)	$E_6(K)$-variety	26
d-1	quadric of minimal Witt index in PG(d,K)	quadric of maximal Witt index in PG(d,K)	3d-1

These Veronese varieties are MM-sets with quadrics in PG(d-1,K) of *min/max* Witt index.
Moreover, these are ***the only ones*** (up to a projectivity)

ASSOCIATED VERONESE VARIETIES

What do we obtain for the other quadratic alternative algebras over K?

The Veronese varieties are isomorphic to:

dim	division	split	dim. of var.
1	PG(2,K)		5
2	PG(2,L)	Segre variety $S_{2,2}(K)$	8
4	PG(2,H)	Line Grassmannian of PG(5,K)	14
8	PG(2,O)	$E_6(K)$-variety	26
d-1	quadric of minimal Witt index in PG(d,K)	quadric of maximal Witt index in PG(d,K)	3d-1

These Veronese varieties are MM-sets with quadrics in PG(d-1,K) of *min/max* Witt index.
Moreover, these are ***the only ones*** (up to a projectivity)
(unless if N<3d-1, then there also are 3 subvarieties when A is split).

ASSOCIATED VERONESE VARIETIES

What do we obtain for the other quadratic alternative algebras over K?

The Veronese varieties are isomorphic to:

dim	division	split	dim. of var.
1	PG(2,K)		5
2	PG(2,L)	Segre variety $S_{2,2}(K)$	8
4	PG(2,H)	Line Grassmannian of PG(5,K)	14
8	PG(2,O)	$E_6(K)$-variety	26
d-1	quadric of minimal Witt index in PG(d,K)	quadric of maximal Witt index in PG(d,K)	3d-1

These Veronese varieties are MM-sets with quadrics in PG(d-1,K) of *min/max* Witt index.
Moreover, these are ***the only ones*** (up to a projectivity)
(unless if N<3d-1, then there also are 3 subvarieties when A is split).
(Krauss), Schillewaert, Van Maldeghem (2015)

GENERALISATIONS

OTHER SUCH SETS?

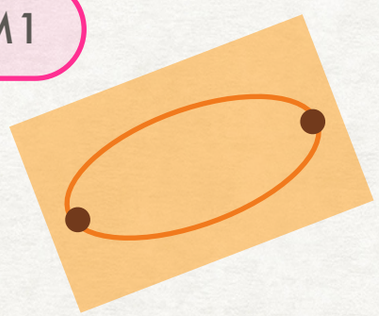
Axiomatic description.

X = spanning point set of PG(N,K)

Ξ = family of d-spaces ξ in PG(N,K)

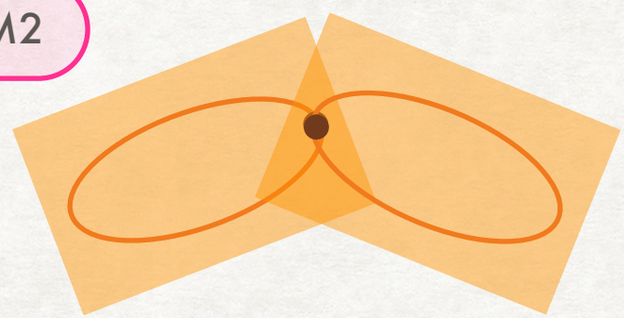
s.th. ξ ∩ X is a ?????

MM1



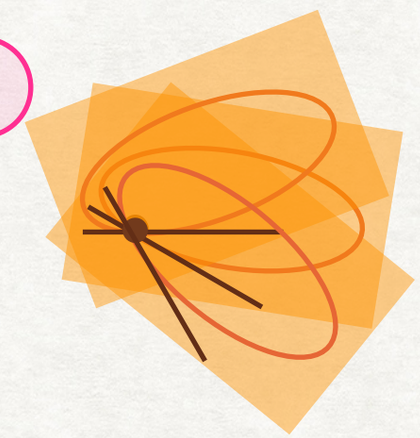
each two points of X belong to a [d] of Ξ

MM2



two [d]s of Ξ intersect in points of X

MM3



the tangent space of a point of X is contained in a [2(d-1)]

OTHER SUCH SETS?

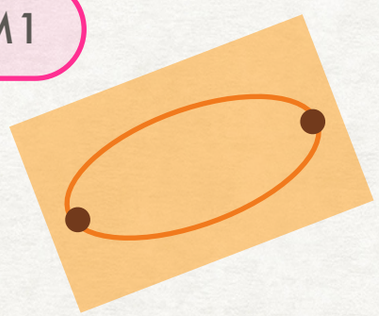
Axiomatic description.

X = spanning point set of PG(N,K)

Ξ = family of d-spaces ξ in PG(N,K)

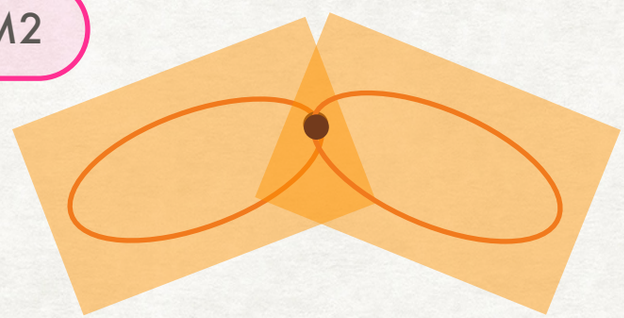
s.th. ξ ∩ X is a ?????

MM1



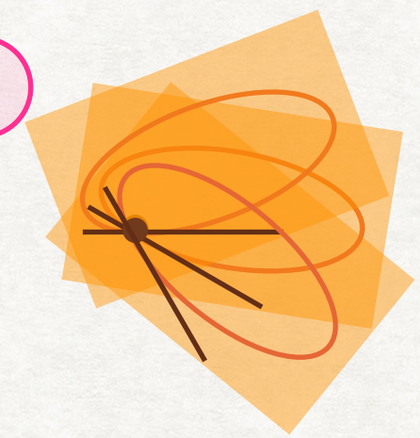
each two points of X belong to a [d] of Ξ

MM2



two [d]s of Ξ intersect in points of X

MM3



the tangent space of a point of X is contained in a [2(d-1)]

Quadrics of a Witt index which is not minimal nor maximal?

Singular quadrics: cones over quadrics of min/max Witt index

OTHER SUCH SETS?

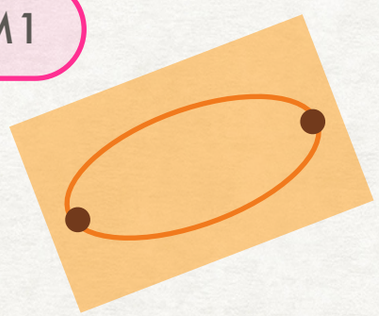
Axiomatic description.

X = spanning point set of PG(N,K)

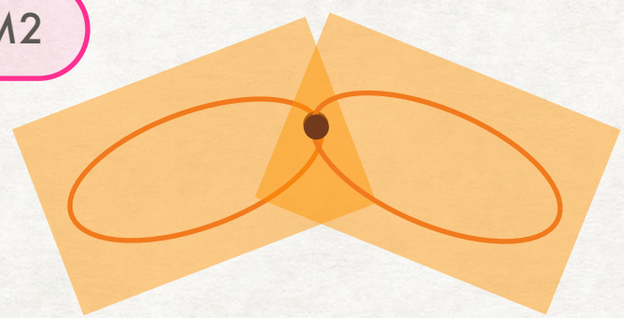
Ξ = family of d-spaces ξ in PG(N,K)

s.th. ξ ∩ X is a ?????

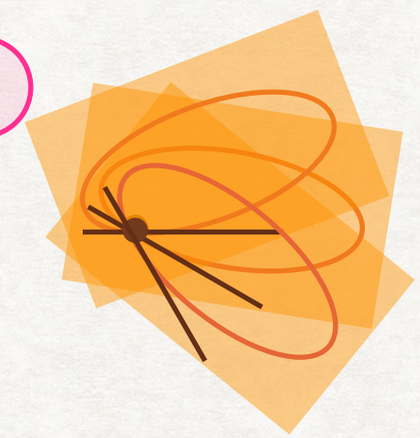
MM1



MM2



MM3



each two points of X belong to a [d] of Ξ

two [d]s of Ξ intersect in points of X

the tangent space of a point of X is contained in a [2(d-1)]

Quadrics of a Witt index which is not minimal nor maximal?

Probably not!

(conj.)

Singular quadrics: cones over quadrics of min/max Witt index

OTHER SUCH SETS?

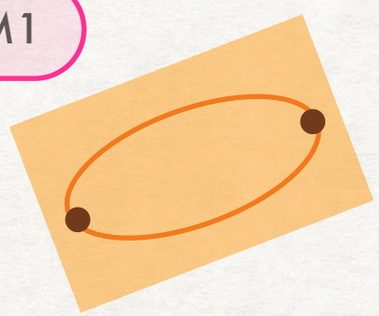
Axiomatic description.

X = spanning point set of PG(N,K)

Ξ = family of d-spaces ξ in PG(N,K)

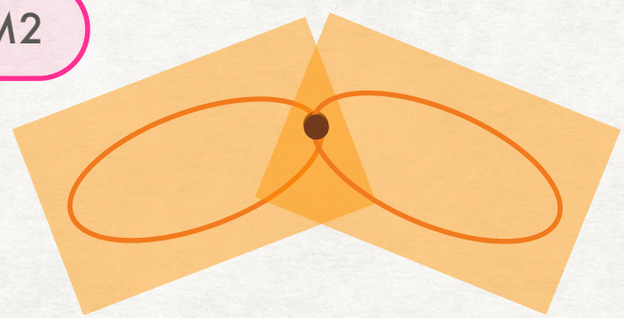
s.th. ξ ∩ X is a ?????

MM1



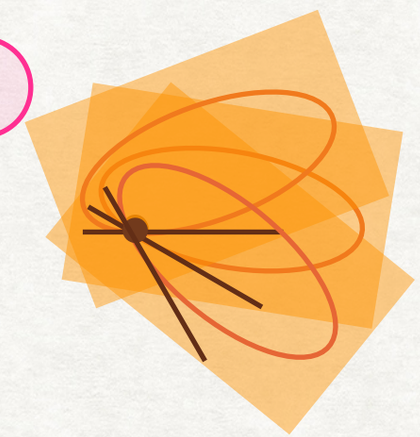
each two points of X belong to a [d] of Ξ

MM2



two [d]s of Ξ intersect in points of X

MM3



the tangent space of a point of X is contained in a [2(d-1)]

Quadrics of a Witt index which is not minimal nor maximal?

Probably not!

(conj.)

Singular quadrics: cones over quadrics of min/max Witt index

Yes!

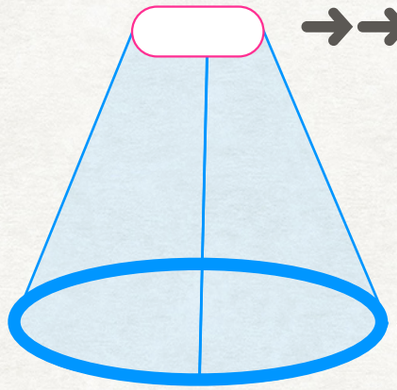
SINGULAR QUADRICS AS SYMPS

Axiomatic description.

X = spanning **point** set of PG(N,K)

Ξ = family of **d'-spaces** ξ in PG(N,K)

$d'=d+v+1$ s.th. $\xi \cap X$ is a (d,v)-tube



vertex of dim v

ovoid in PG(d,K)

SINGULAR QUADRICS AS SYMPS

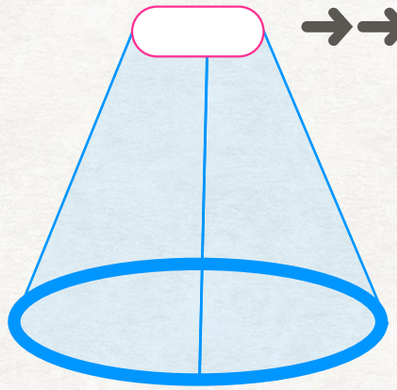
Axiomatic description.

X = spanning **point** set of PG(N,K)

Ξ = family of **d'-spaces** ξ in PG(N,K)

d'=d+v+1 s.th. ξ ∩ X is a (d,v)-tube

Y = set of **vertices**



vertex of dim v (excluded!)

ovoid in PG(d,K)

SINGULAR QUADRICS AS SYMPS

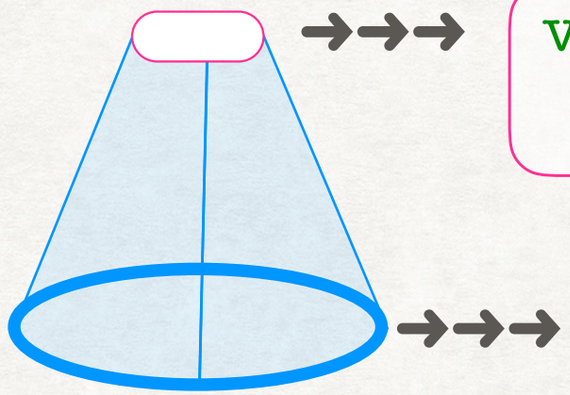
Axiomatic description.

X = spanning **point** set of PG(N,K)

Ξ = family of **d'-spaces** ξ in PG(N,K)

d'=d+v+1 s.th. ξ ∩ X is a (d,v)-tube

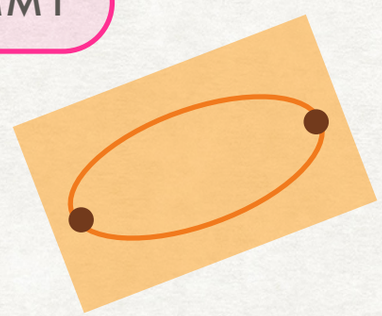
Y = set of **vertices**



vertex of dim v (excluded!)

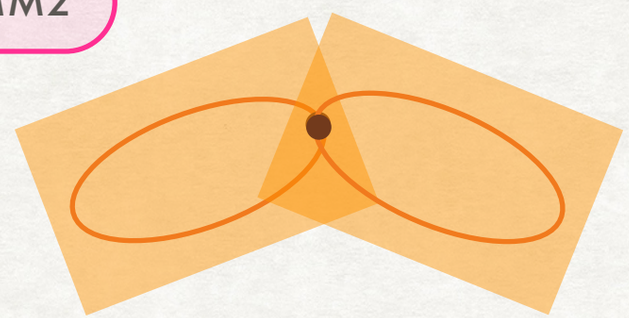
ovoid in PG(d,K)

MM1



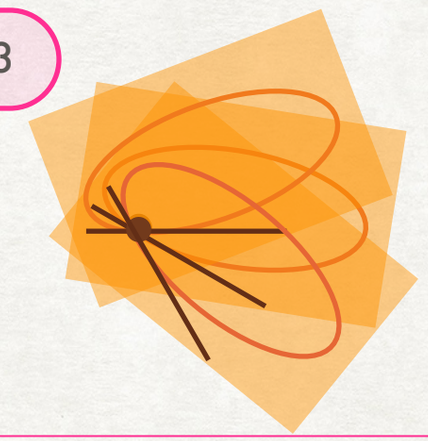
each two **points** of **X** belong to a [d'] of **Ξ**

MM2



two [d']s of **Ξ** intersect in **points** of **X**∪**Y**

MM3



the tangent space of a **point** of **X** is contained in a [2(d'-1)]

SINGULAR QUADRICS AS SYMPS

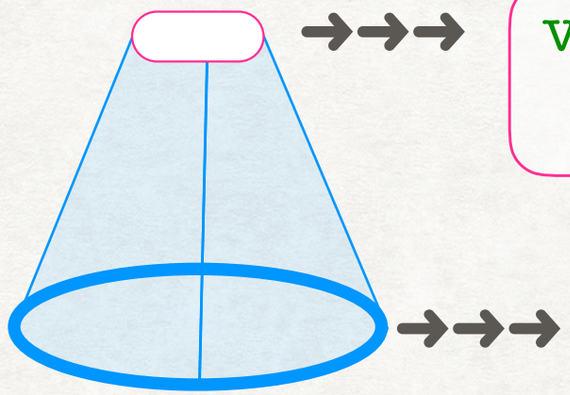
Axiomatic description.

X = spanning **point** set of PG(N,K)

Ξ = family of **d'-spaces** ξ in PG(N,K)

d'=d+v+1 s.th. ξ ∩ X is a (d,v)-tube

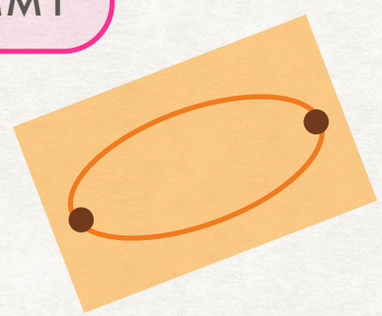
Y = set of **vertices**



vertex of dim v (excluded!)

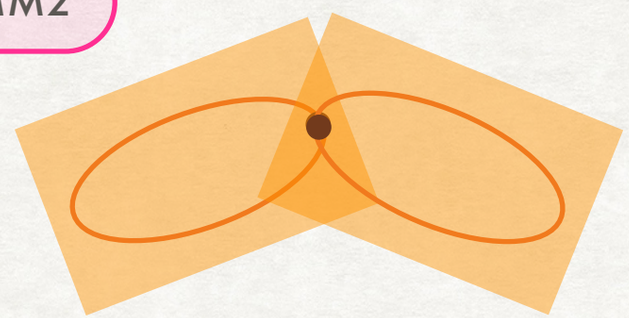
ovoid in PG(d,K)

MM1



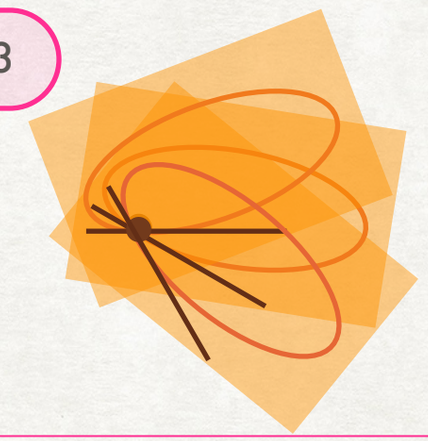
each two **points** of **X** belong to a [d'] of **Ξ**

MM2



two [d']s of **Ξ** intersect in **points** of **X**∪**Y**

MM3



the tangent space of a **point** of **X** is contained in a [2(d'-1)]

Warning: You might want to avoid that two quads intersect in points of Y only

SINGULAR QUADRICS AS SYMPS

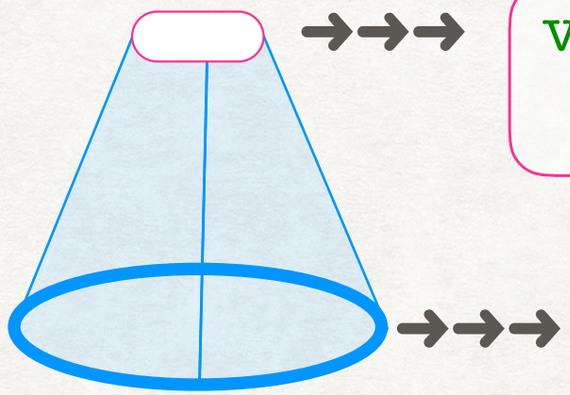
Axiomatic description.

X = spanning **point** set of PG(N,K)

Ξ = family of **d'-spaces** ξ in PG(N,K)

d'=d+v+1 s.th. ξ ∩ X is a (d,v)-tube

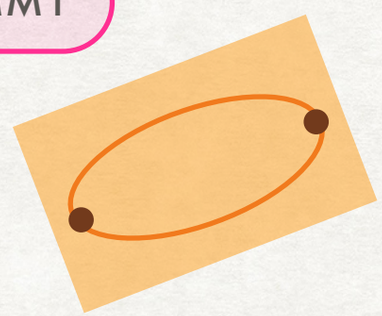
Y = set of **vertices**



vertex of dim v (excluded!)

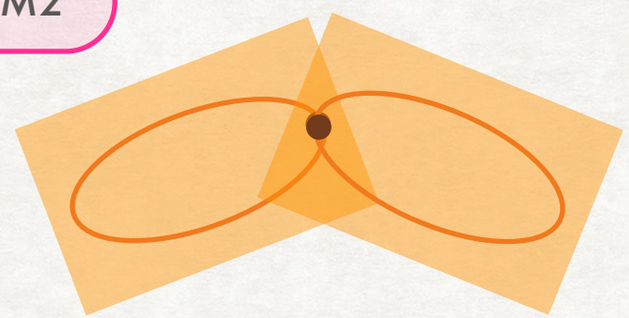
ovoid in PG(d,K)

MM1



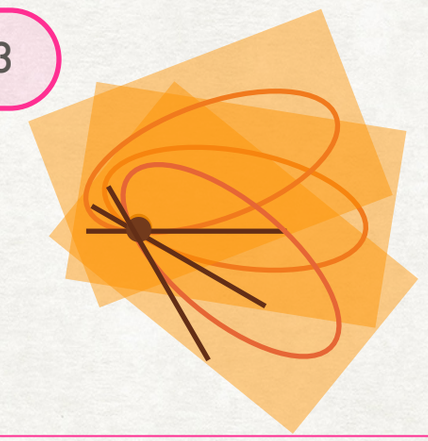
each two **points** of **X** belong to a [d'] of **Ξ**

MM2'



two [d']s of **Ξ** intersect in **points** of **X**∪**Y**

MM3



the tangent space of a **point** of **X** is contained in a [2(d'-1)]

Warning: You might want to avoid that two quads intersect in points of Y only

SINGULAR QUADRICS AS SYMPS

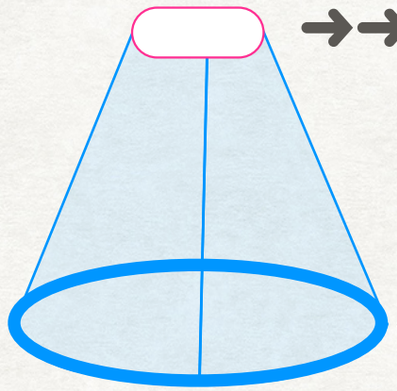
Axiomatic description.

X = spanning **point** set of PG(N,K)

Ξ = family of **d'-spaces** ξ in PG(N,K)

d'=d+v+1 s.th. ξ ∩ X is a (d,v)-tube

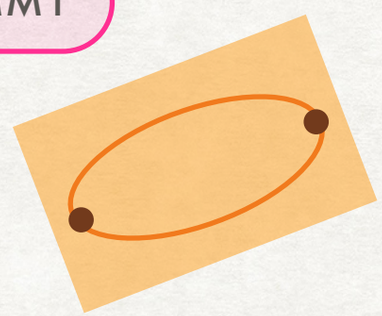
Y = set of **vertices**



vertex of dim v (excluded!)

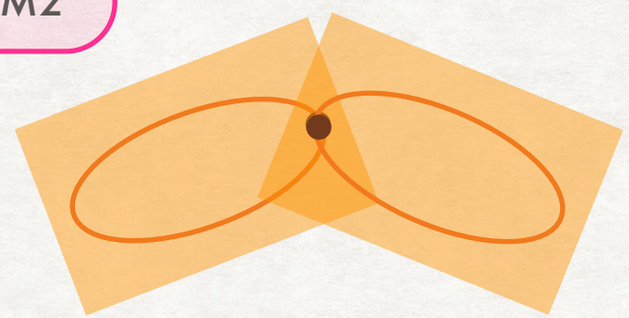
ovoid in PG(d,K)

MM1



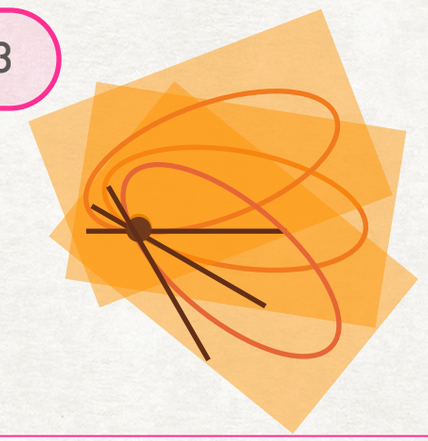
each two **points** of **X** belong to a [d'] of **Ξ**

MM2'



two [d']s of **Ξ** intersect in **points** of **X**∪**Y** but never in Y only

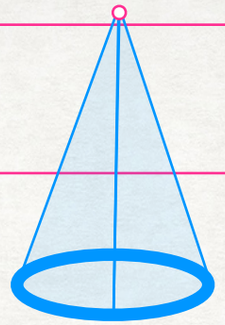
MM3



the tangent space of a **point** of **X** is contained in a [2(d'-1)]

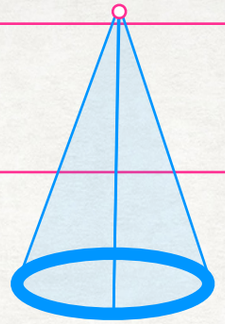
The pair (**X**, **Ξ**) together with MM1, MM2' and MM3 is called a singular MM-set.

CASE 1: ONE POINT AS A VERTEX



For any field K, let (X, Ξ) be a singular MM-set with (d,0)-tubes.

CASE 1: ONE POINT AS A VERTEX

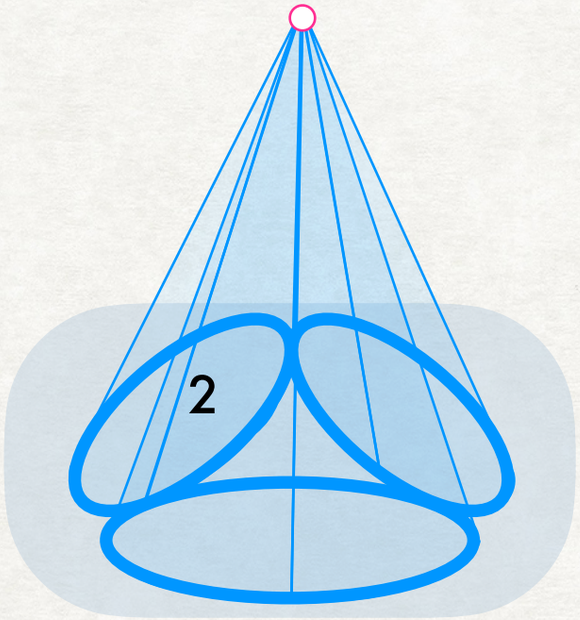


For any field K, let (**X**, **Ξ**) be a singular MM-set with (d,**0**)-tubes.

d=2

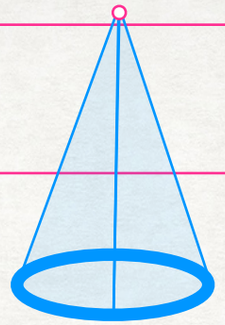
Schillewaert, Van Maldeghem (2015)

(1) (**X**, **Ξ**) is <u>projectively equivalent</u> to a point-cone over $\mathscr{V}$(K)



$\mathscr{V}$(K)

CASE 1: ONE POINT AS A VERTEX



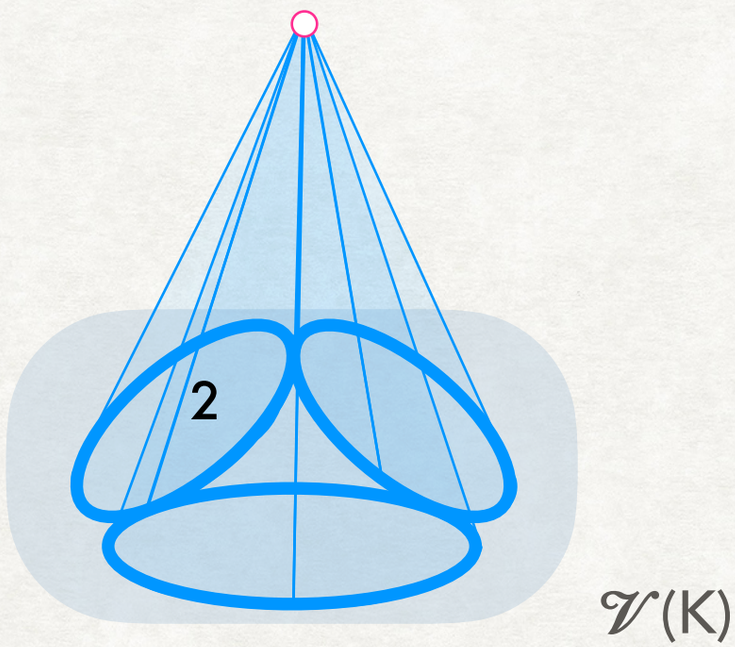
For any field K, let (**X**, **Ξ**) be a singular MM-set with (d,**0**)-tubes.

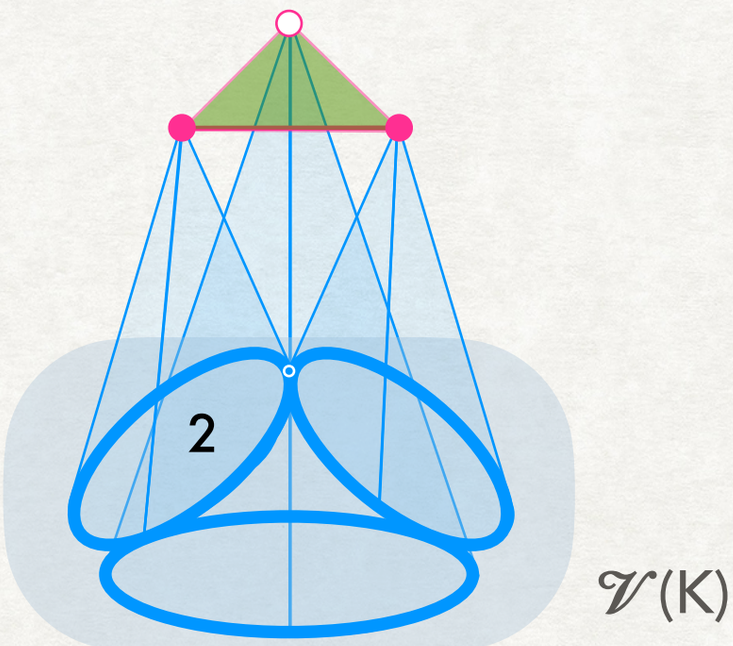
d=2

Schillewaert, Van Maldeghem (2015)

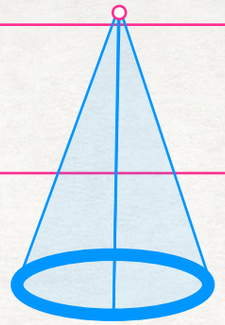
(1) (**X**, **Ξ**) is <u>projectively equivalent</u> to a point-cone over $\mathscr{V}$(K)

(2) (**X**, **Ξ**) is <u>projectively equivalent</u> to a Hjelmslevian projective plane.





CASE 1: ONE POINT AS A VERTEX



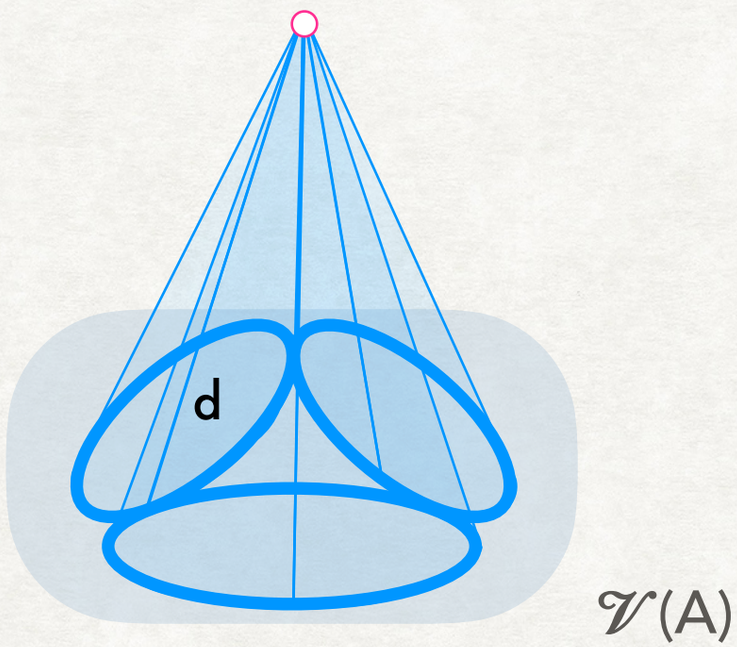
For any field K, let (**X**, **Ξ**) be a singular MM-set with (d,**0**)-tubes.

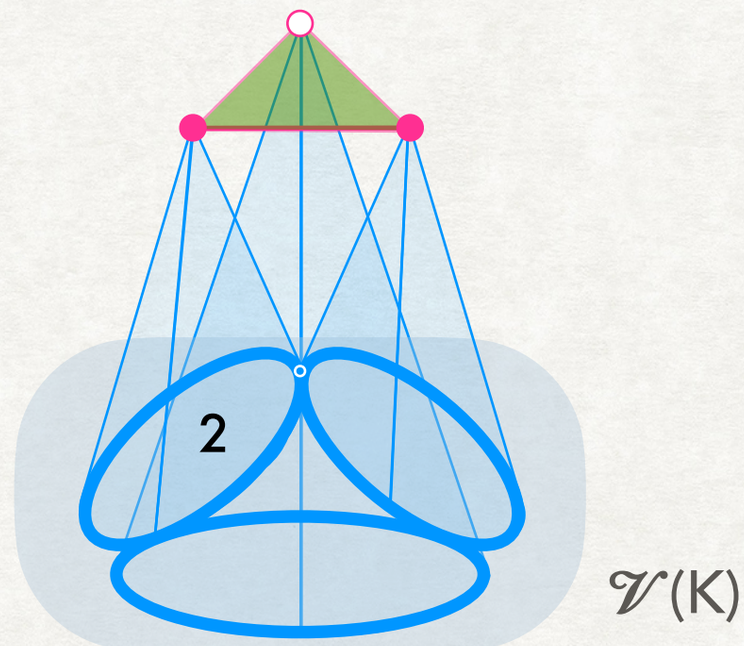
d=2

Schillewaert, Van Maldeghem (2015)

(1) (**X**, **Ξ**) is <u>projectively equivalent</u> to a point-cone over $\mathscr{V}$(K)

(2) (**X**, **Ξ**) is <u>projectively equivalent</u> to a Hjelmslevian projective plane.





d>2

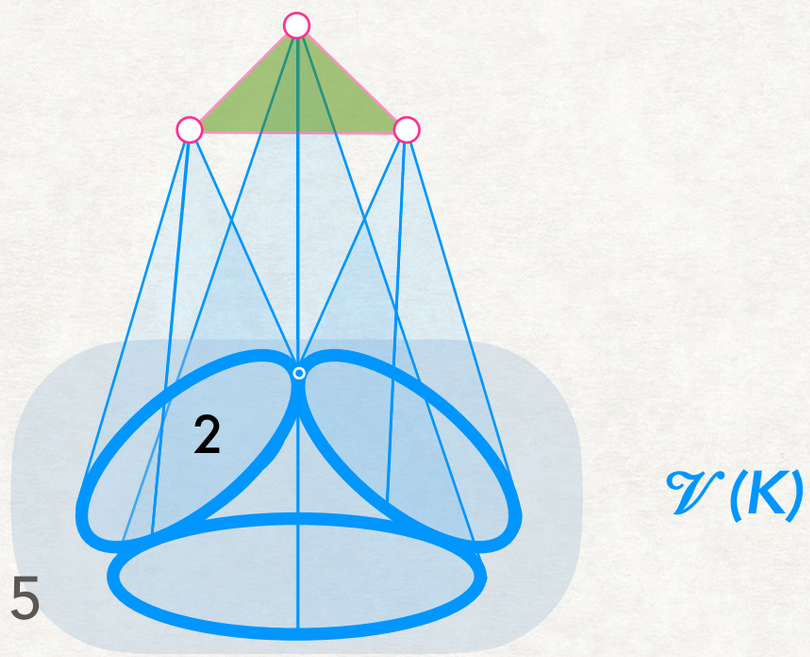
ADS, Van Maldeghem (2017)

(**X**, **Ξ**) is <u>projectively equivalent</u> to a cone over $\mathscr{V}$(A), A=K, L, H or O.

There are no non-trivial cases.

HJELMSLEVIAN PROJECTIVE PLANES

($\mathbf{X}$, Ξ) is <u>projectively equivalent</u> to a Hjelmslevian projective plane.

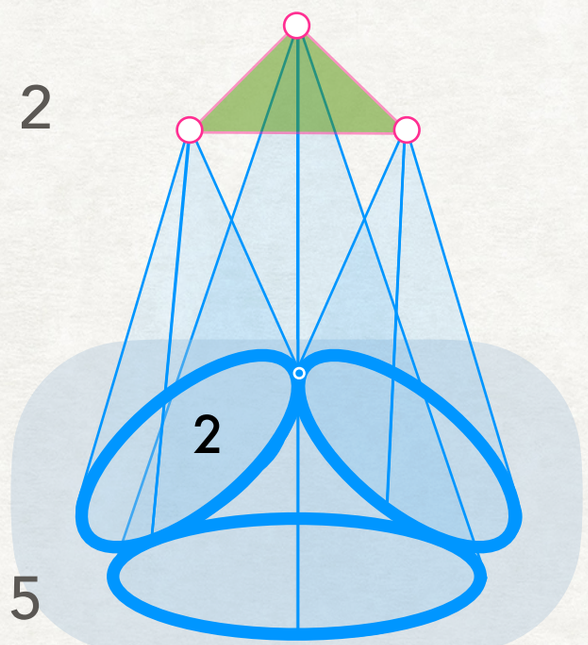


$\mathscr{V}(K)$

HJELMSLEVIAN PROJECTIVE PLANES

($\mathbf{X}$, Ξ) is <u>projectively equivalent</u> to a Hjelmslevian projective plane.

8



PG(2,K)

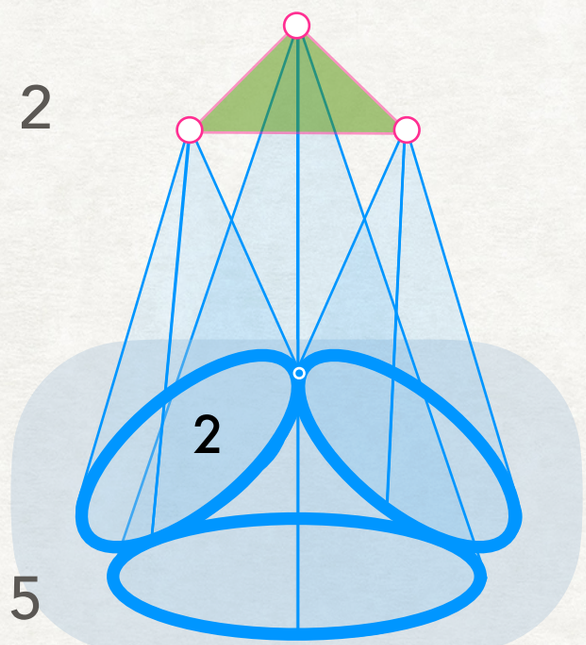
$\mathscr{V}$(K)

The **vertices** form a projective plane over K.

HJELMSLEVIAN PROJECTIVE PLANES

($\mathbf{X}$, Ξ) is <u>projectively equivalent</u> to a Hjelmslevian projective plane.

8



PG(2,K)

$\mathscr{V}(K)$

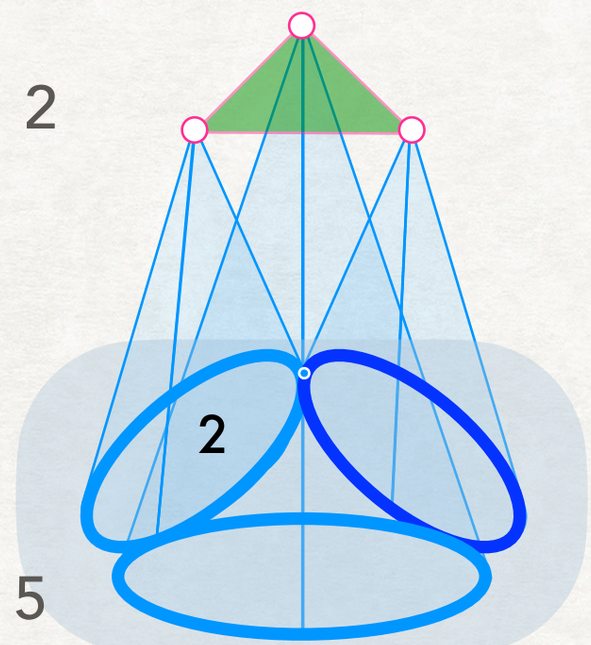
The **vertices** form a projective plane over K.

In a complementary subspace, the **points of X** form the Veronese variety $\mathscr{V}(K)$.

HJELMSLEVIAN PROJECTIVE PLANES

($\mathbf{X}$, Ξ) is <u>projectively equivalent</u> to a Hjelmslevian projective plane.

8



PG(2,K)

$\mathscr{V}(K)$

conic

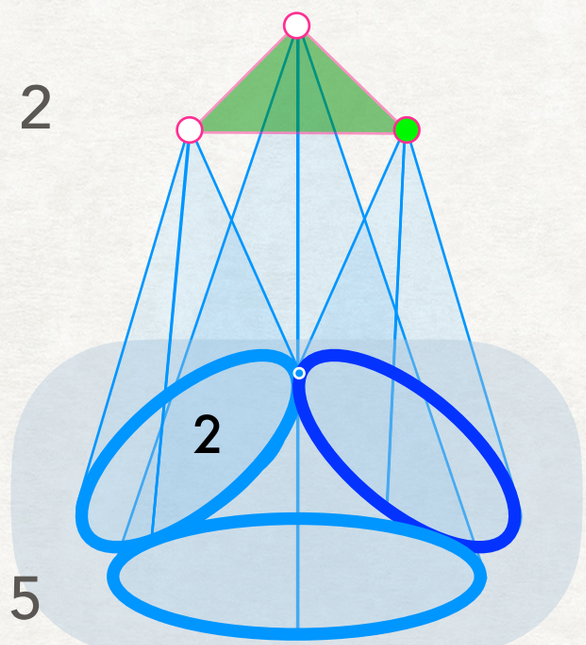
The **vertices** form a projective plane over K.

In a complementary subspace, the **points of X** form the Veronese variety $\mathscr{V}(K)$.

HJELMSLEVIAN PROJECTIVE PLANES

($\mathbf{X}$, Ξ) is <u>projectively equivalent</u> to a Hjelmslevian projective plane.

8



PG(2,K)

its vertex

$\mathscr{V}(K)$

conic

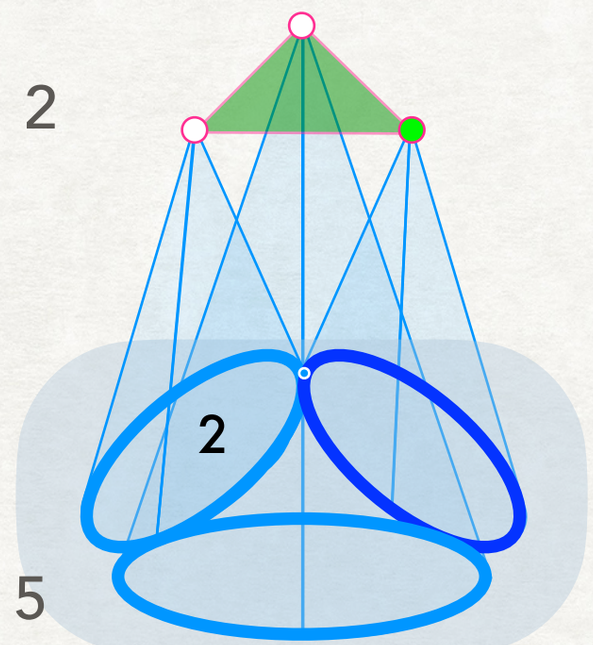
The **vertices** form a projective plane over K.

In a complementary subspace, the **points of X** form the Veronese variety $\mathscr{V}(K)$.

HJELMSLEVIAN PROJECTIVE PLANES

($\mathbf{X}$, Ξ) is <u>projectively equivalent</u> to a Hjelmslevian projective plane.

8



PG(2,K)

its vertex

χ

$\mathscr{V}(K)$

$\approx PG(2,K)$

conic

The **vertices** form a projective plane over K.

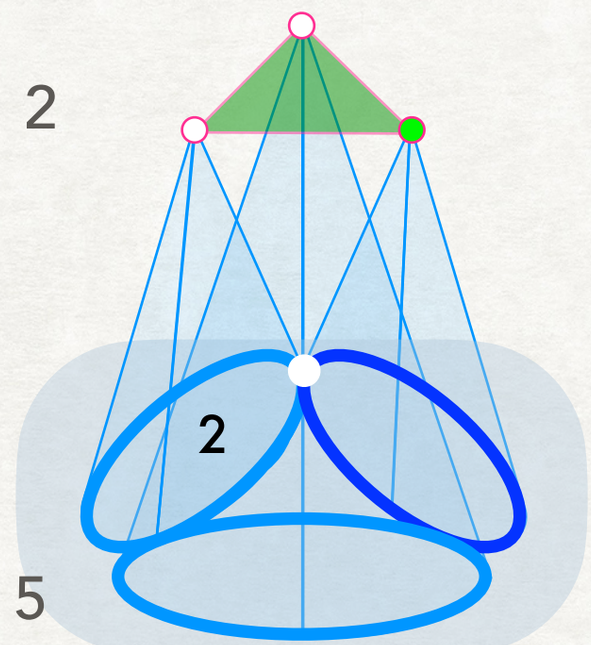
In a complementary subspace, the **points of X** form the Veronese variety $\mathscr{V}(K)$.

The mapping χ is a **linear duality** between $\mathscr{V}(K)$ and **PG(2,K)**.

HJELMSLEVIAN PROJECTIVE PLANES

(X, Ξ) is <u>projectively equivalent</u> to a Hjelmslevian projective plane.

8



PG(2,K)

its vertex

χ

$\mathscr{V}(K)$

≈ PG(2,K)

conic

point

The **vertices** form a projective plane over K.

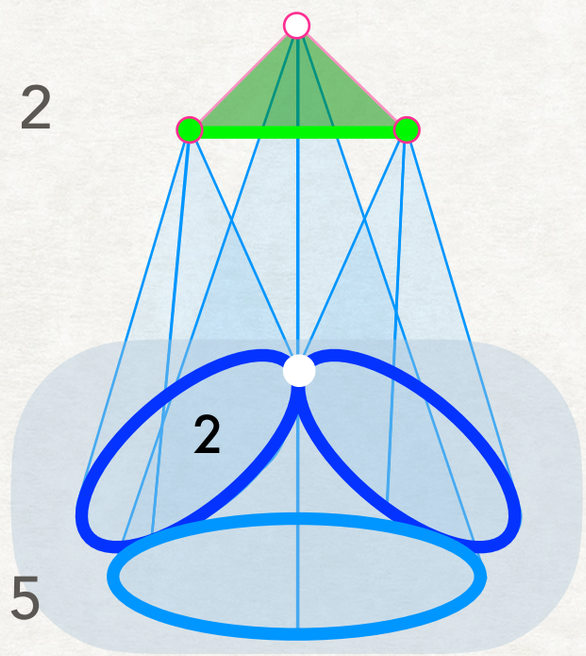
In a complementary subspace, the **points of X** form the Veronese variety $\mathscr{V}(K)$.

The mapping χ is a **linear duality** between $\mathscr{V}(K)$ and **PG(2,K)**.

HJELMSLEVIAN PROJECTIVE PLANES

($\mathbf{X}$, Ξ) is <u>projectively equivalent</u> to a Hjelmslevian projective plane.

8



PG(2,K)

$\mathscr{V}(K)$
$\approx$ *PG(2,K)*

its vertex

χ

conic

vertices of the conics through it

χ

point

The **vertices** form a projective plane over K.

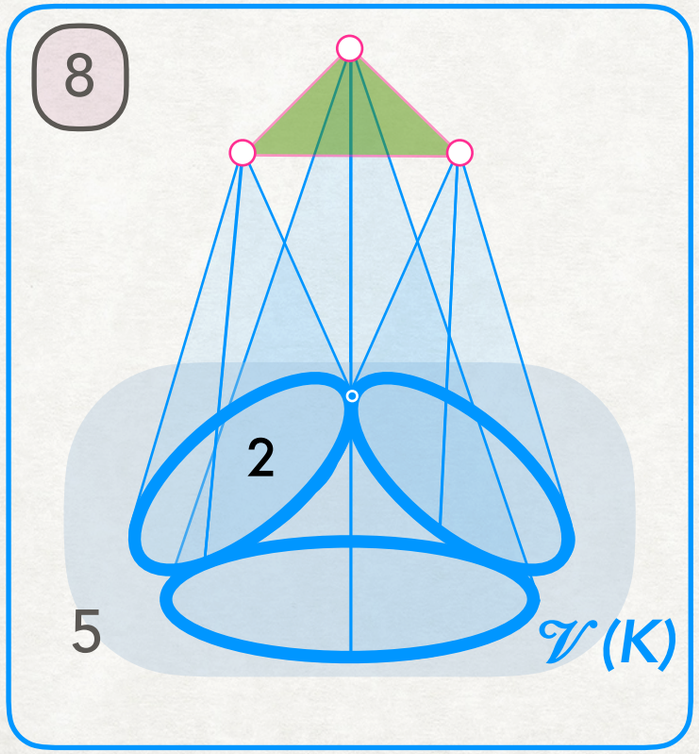
In a complementary subspace, the **points of X** form the Veronese variety $\mathscr{V}(K)$.

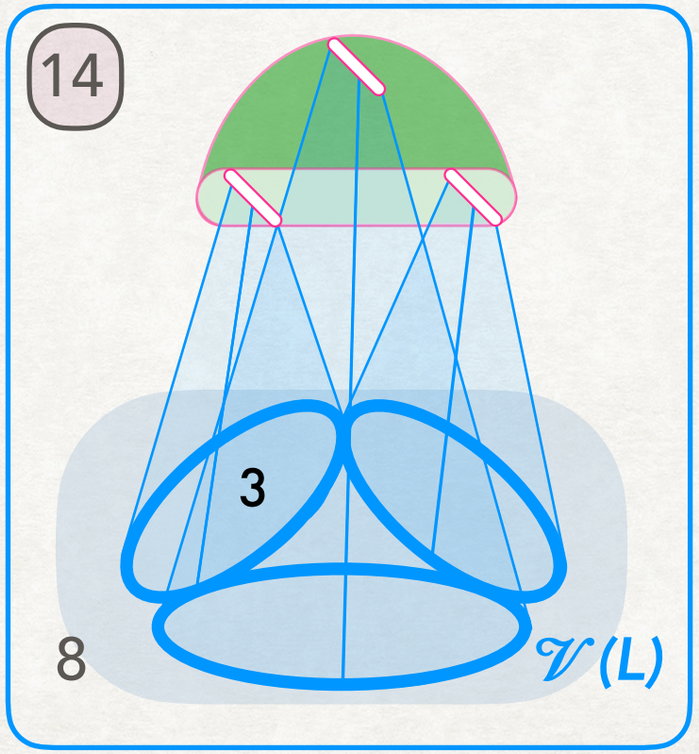
The mapping χ is a **linear duality** between $\mathscr{V}(K)$ and **PG(2,K)**.

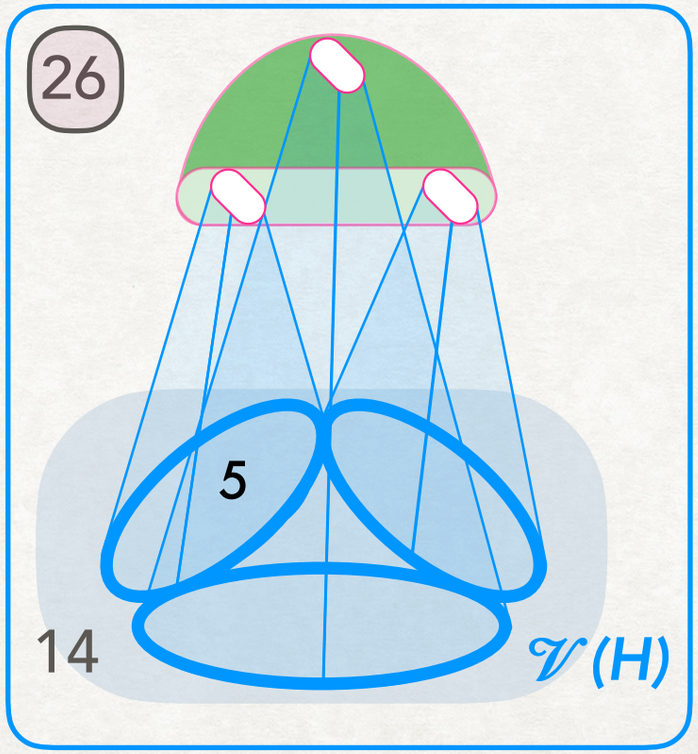
The affine planes obtained by joining x and $\chi(x)$, (x of $\mathscr{V}(K)$) *give us* X.

SIMILAR CONSTRUCTIONS

	division	split	singular	dim(im(ρ))
d-1	quadrics of <u>minimal</u> Witt index in PG(d,K)	quadrics of <u>maximal</u> Witt index in PG(d,K)	<u>cones</u> over quadrics of minimal Witt index	5, 8, 14, 26

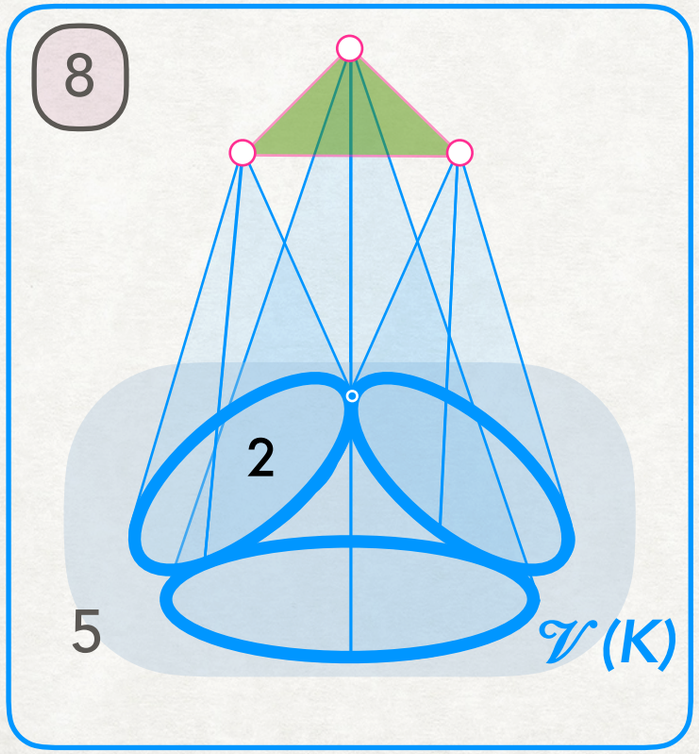


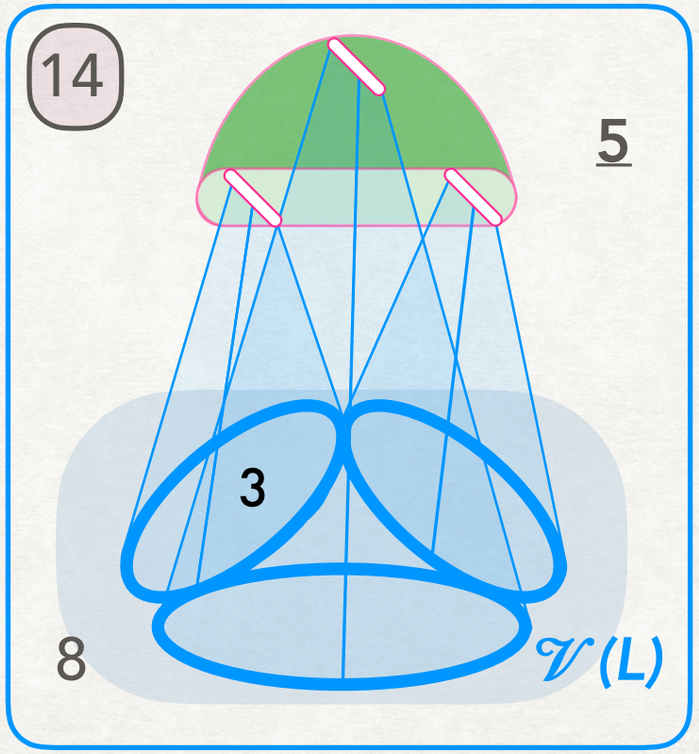


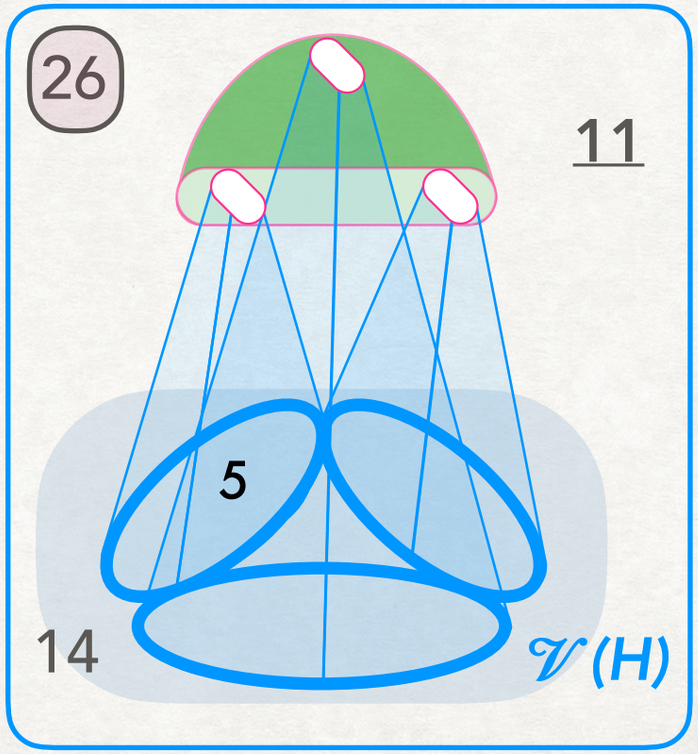


SIMILAR CONSTRUCTIONS

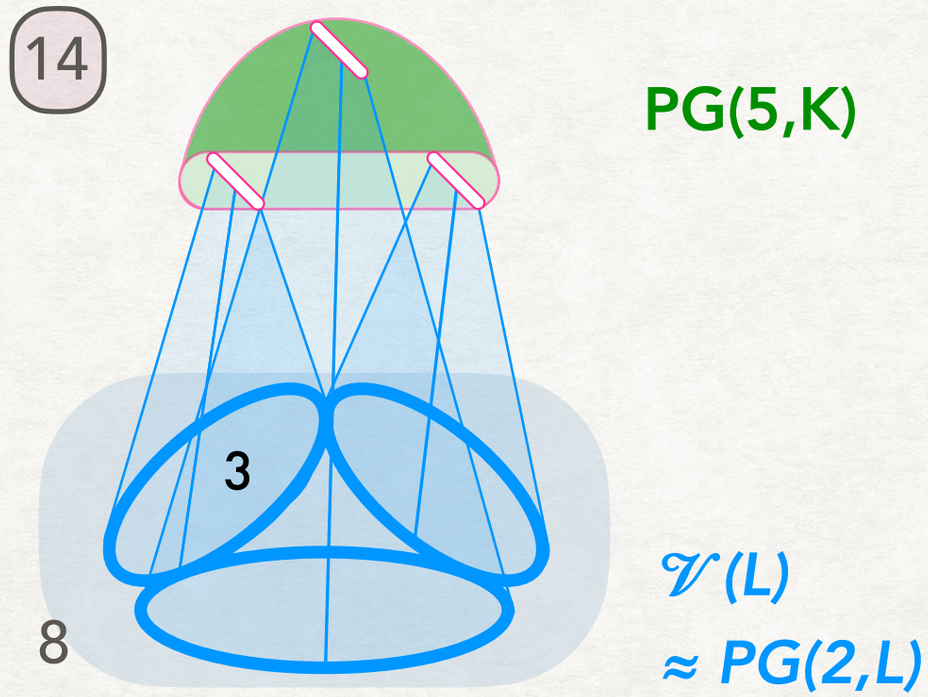
	division	split	singular	dim(im(ρ))
d-1	quadrics of <u>minimal</u> Witt index in PG(d,K)	quadrics of <u>maximal</u> Witt index in PG(d,K)	<u>cones</u> over quadrics of minimal Witt index	5, 8, 14, 26







SIMILAR CONSTRUCTIONS



its vertex

χ

quadric

vertices of the conics through it

χ

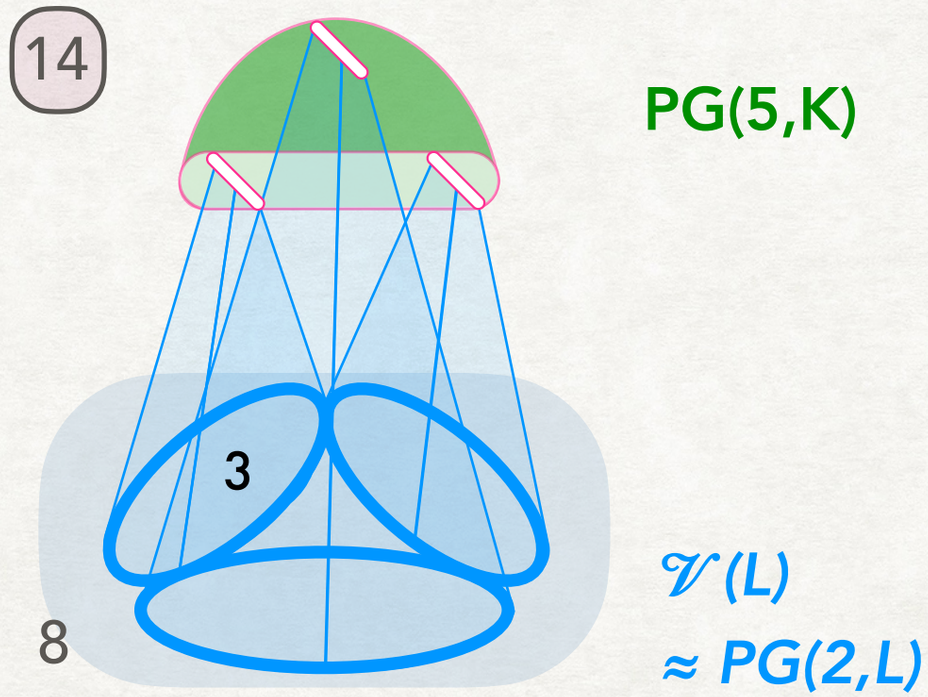
point

The **vertices** form a projective 5-space over K.

In a complementary subspace, the **points of X** form the Veronese variety $\mathscr{V}(L)$.

The mapping χ is a **linear duality** between $\mathscr{V}(L)$ and **the top structure**.

SIMILAR CONSTRUCTIONS



its vertex

χ

quadric

vertices of the conics through it

χ

point

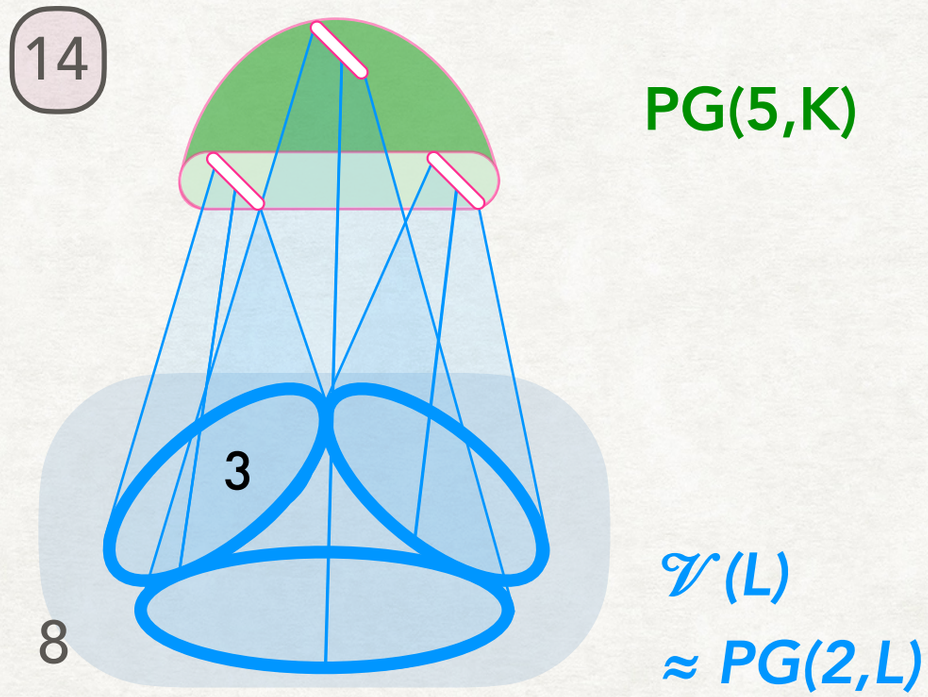
The **vertices** form a projective 5-space over K.

In a complementary subspace, the **points of X** form the Veronese variety $\mathscr{V}(L)$.

The mapping χ is a **linear duality** between $\mathscr{V}(L)$ and **the top structure**.

$\mathscr{V}(L) \approx$ PG(2,L) ➜ represent PG(2,L) as a regular line spread in PG(5,K)

SIMILAR CONSTRUCTIONS



its vertex

χ

quadric

vertices of the conics through it

χ

point

The **vertices** form a projective 5-space over K.

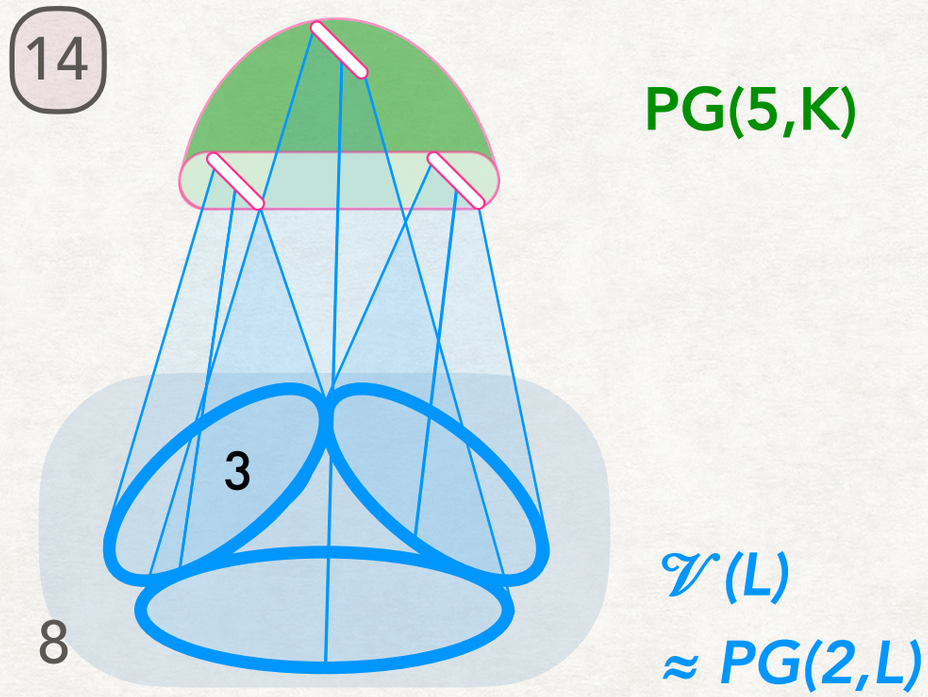
In a complementary subspace, the **points of X** form the Veronese variety $\mathscr{V}(L)$.

The mapping χ is a **linear duality** between $\mathscr{V}(L)$ and **the top structure**.

$\mathscr{V}(L) \approx$ PG(2,L) ➜ represent PG(2,L) as a regular line spread in PG(5,K)

points in PG(2,K) = 1-spaces in V(3,K) ➜ 2-spaces in V(6,K) = lines in PG(5,K)

SIMILAR CONSTRUCTIONS



its vertex (a line)	vertices of the conics through it (regular spread in 3-space)
↑ χ	↑ χ
quadric	point

The **vertices** form a projective 5-space over K.

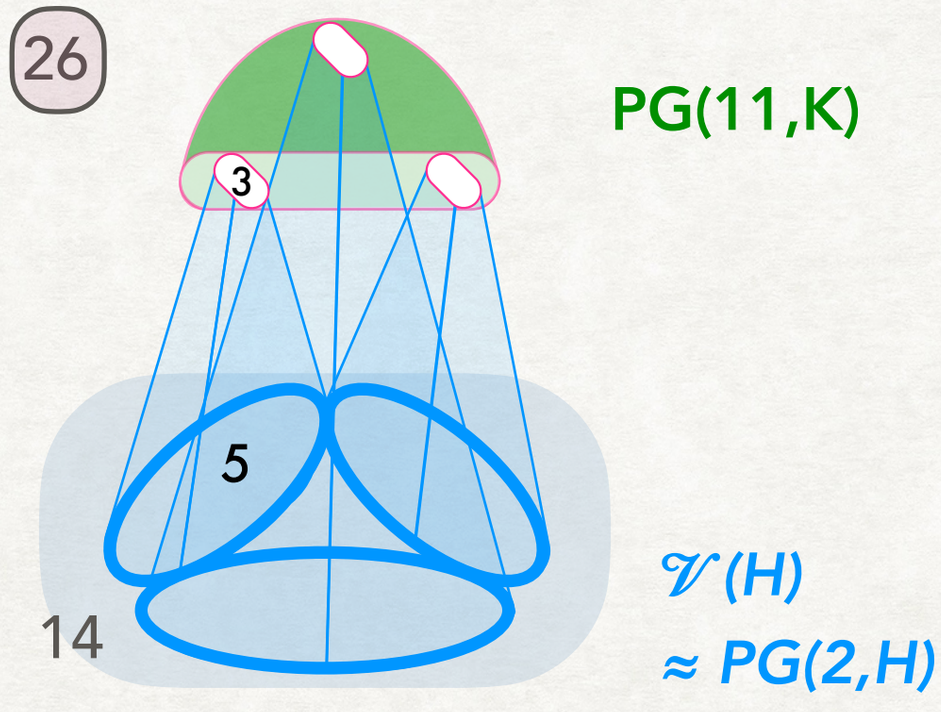
In a complementary subspace, the **points of X** form the Veronese variety $\mathscr{V}(L)$.

The mapping χ is a **linear duality** between $\mathscr{V}(L)$ and **the top structure**.

$\mathscr{V}(L) \approx$ PG(2,L) ➜ represent PG(2,L) as a regular line spread in PG(5,K)

points in PG(2,K) = 1-spaces in V(3,K) ➜ 2-spaces in V(6,K) = lines in PG(5,K)

SIMILAR CONSTRUCTIONS



its vertex (a [3])	vertices of the conics through it (regular [3] spread in 7-space)
↑ χ	↑ χ
quadric	point

The **vertices** form a projective 11-space over K.

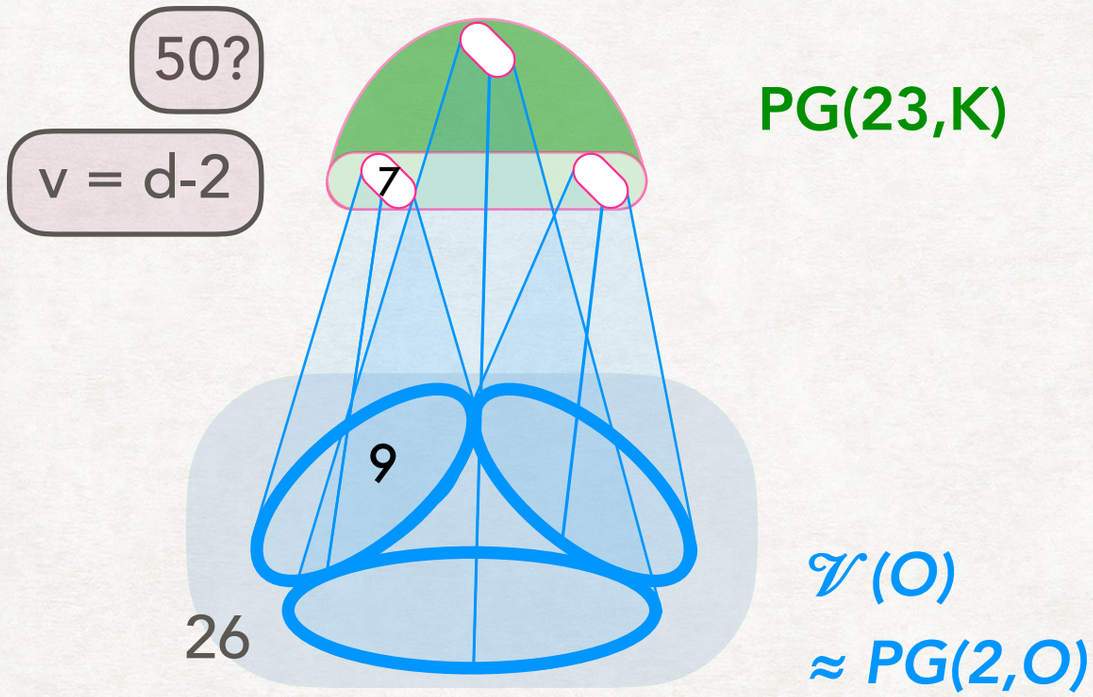
In a complementary subspace, the **points of X** form the Veronese variety *𝒱(H)*.

The mapping χ is a **linear duality** between *𝒱(H)* and **the top structure**.

𝒱(H) ≈ PG(2,H) ➜ represent PG(2,H) as a regular [3] spread in PG(11,K)

points in PG(2,K) = 1-spaces in V(3,K) ➜ 4-spaces in V(12,K) = 3-spaces in PG(11,K)

SIMILAR CONSTRUCTIONS



its vertex (a [7])

vertices of the conics through it (regular [7] spread in 15-space)

χ ↑ quadric

χ ↑ point

The **vertices** form a projective 23-space over K.

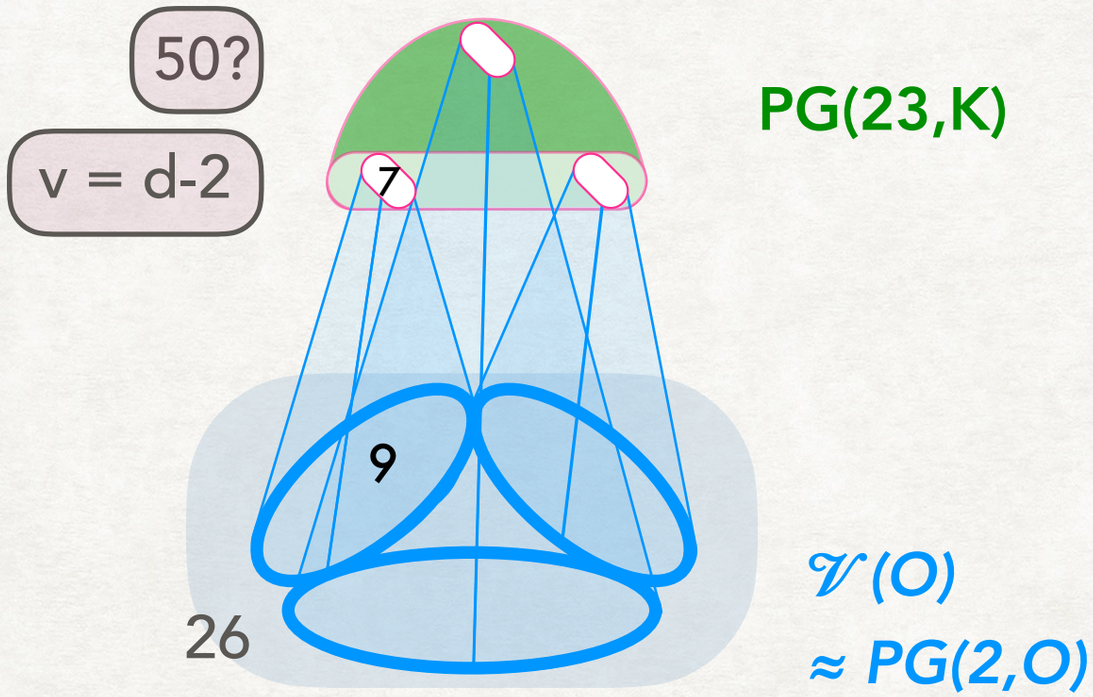
In a complementary subspace, the **points of X** form a the Veronese variety $\mathscr{V}(O)$.

The mapping χ is a **linear duality** between $\mathscr{V}(O)$ and **the top structure**.

$\mathscr{V}(O) \approx$ PG(2,O) ➜ represent PG(2,O) as a regular [7] spread in PG(23,K)

points in PG(2,K) = 1-spaces in V(3,K) ➜ 8-spaces in V(24,K) = 7-spaces in PG(23,K)

SIMILAR CONSTRUCTIONS



its vertex (a [7])	vertices of the conics through it (regular [7] spread in 15-space)
↑ χ	↑ χ
quadric	point

The **vertices** form a projective 23-space over K.

In a complementary subspace, the **points of X** form a the Veronese variety $\mathscr{V}(O)$.

The mapping χ is a **linear duality** between $\mathscr{V}(O)$ and **the top structure**.

$\mathscr{V}(O) \approx$ PG(2,O) ➜ represent PG(2,O) as a regular [7] spread in PG(23,K)

Warning: The top plane is Desarguesian, whereas PG(2,O) is not. <u>This does not work!</u>

CASE 2: A HIGHER-DIMENSIONAL VERTEX

For any field K, let (X, Ξ) be a singular MM*-set with (d,v)-tubes, v>0.

CASE 2: A HIGHER-DIMENSIONAL VERTEX

For any field K, let ($\mathbf{X}$, Ξ) be a singular MM*-set with (d,**v**)-tubes, v>0.

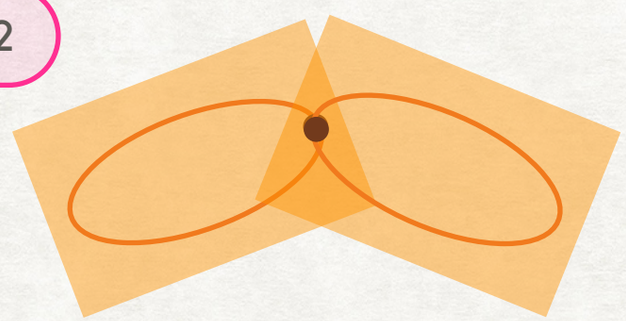
MM2′ is replaced by MM2*:

CASE 2: A HIGHER-DIMENSIONAL VERTEX

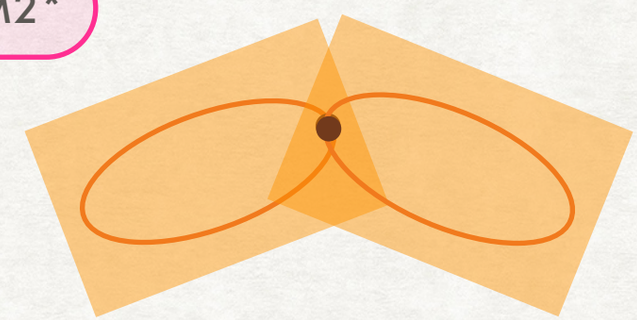
For any field K, let (**X**, **Ξ**) be a singular MM*-set with (d,**v**)-tubes, v>0.

MM2′ is replaced by MM2*:

MM2



MM2*



→ two [d′]s of **Ξ** intersect in **points** of **X**

→ two [d′]s of **Ξ** intersect in **points** of **X**∪**Y** and always contain a point of X

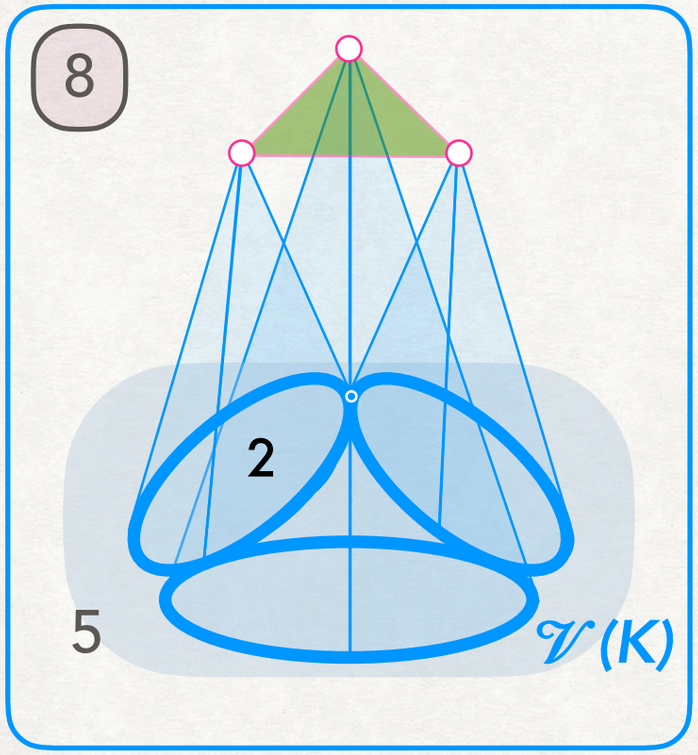
CASE 2: A HIGHER-DIMENSIONAL VERTEX

For any field K, let (X, Ξ) be a singular MM*-set with (d,v)-tubes, v>0.

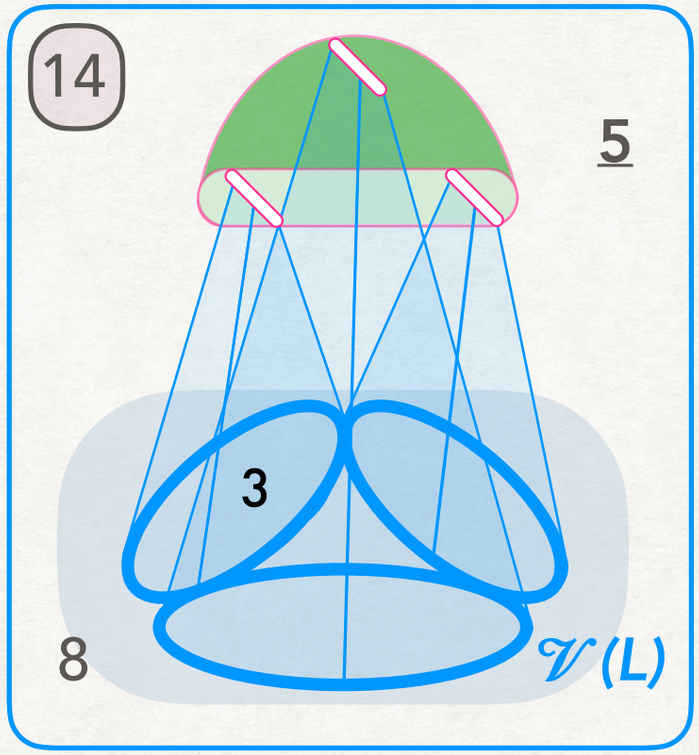
v>0
d>1

ADS, Van Maldeghem (2017)

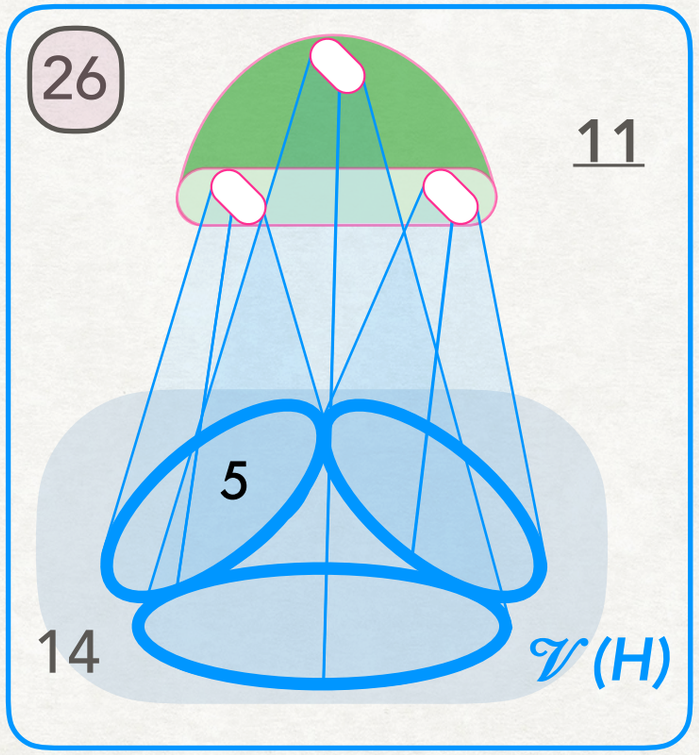
(1) (X, Ξ) is <u>projectively equivalent</u> to one of the following



$\mathscr{HV}$(K)



$\mathscr{HV}$(L)



$\mathscr{HV}$(H)

CASE 2: A HIGHER-DIMENSIONAL VERTEX

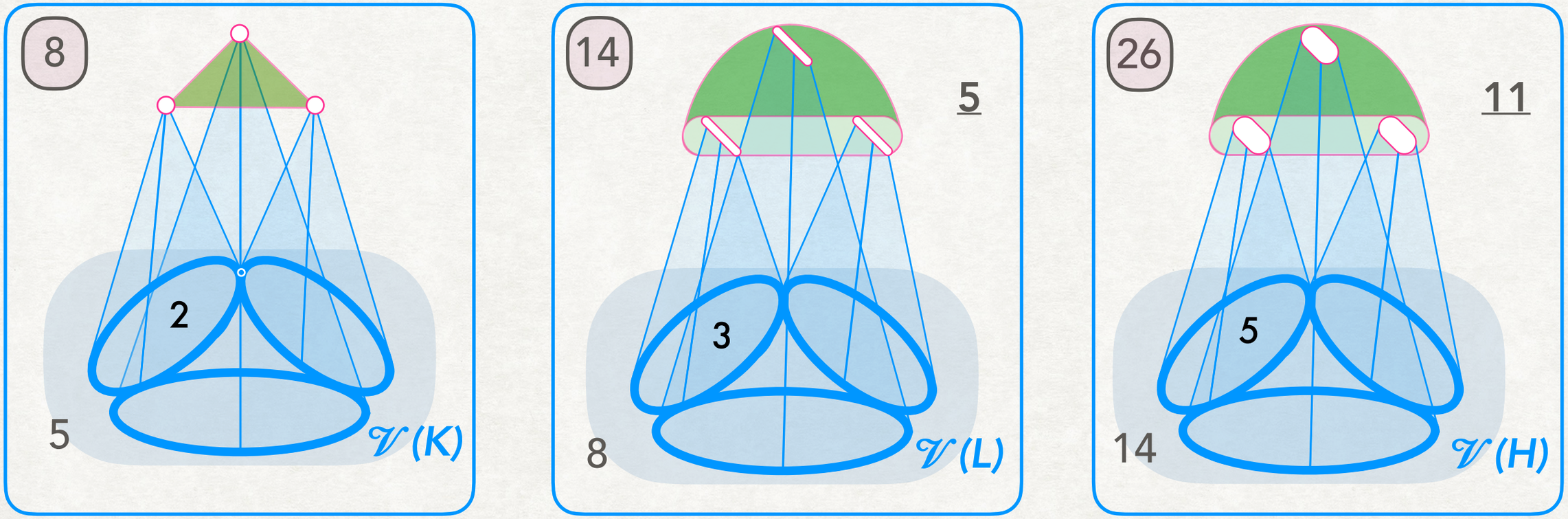
For any field K, let (X, Ξ) be a singular MM*-set with (d,v)-tubes, v>0.

v>0
d>1

ADS, Van Maldeghem (2017)

(1) (X, Ξ) is <u>projectively equivalent</u> to one of the following

(2) (X, Ξ) is <u>trivial</u> (a cone over $\mathscr{V}$(A) or over $\mathscr{HV}$(K), $\mathscr{HV}$(L), $\mathscr{HV}$(H))



$\mathscr{HV}$(K) $\mathscr{HV}$(L) $\mathscr{HV}$(H)

Thanks for your attention!