From Knudsen diffusion to stochastic thermodynamics

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Abstract:

The theory of stochastic dynamical systems provides a conceptually satisfying set of links connecting mechanical-geometric models of surface-gas interaction to (1) the classical distributions of Knudsen and Maxwell-Boltzmann describing the scattered velocities of a gas on a solid surface at equilibrium, (2) to the mathematical determination of Knudsen diffusivity in channels and, finally, (3) to the foundations of stochastic thermodynamics of simple ("nano-mechanical") heat engines.

The central character of the story is a classical surface scattering operator, P, derived from the interaction model. The random dynamical system we study is then a "multiple scattering" Markov chain (a "random billiard" system) with transition probabilities operator P.

Among other things we will see how the cosine law and, more generally, the surface Maxwellian arise as stationary distributions of the multiple scattering process, how Knudsen diffusivity relates to the geometrical-mechanical gas-surface interaction model via the spectrum of P, and how a variation on the theme of gas diffusion in channels can, somewhat surprisingly, also describe the workings of a minimalistic heat engine, which we explore via numerical simulation. The emphasis will be on mathematical ideas, although possible engineering applications could be a theme for discussion.