Galois Geometries and Coding Theory

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The geometric nature of certain optimality problems in coding theory has been long known. The connection between the geometry of special point sets in suitably chosen projective and affine spaces and linear codes over certain algebraic structures (finite fields, semifields, special rings) has been exploited repeatedly during the years.

In the last decade, a substantial progress has been made in the fields of finite geometry and coding theory. Yet many challenging problems remain unsolved. The goal of this talk is to state some of the basic results in both areas, to survey the recent progress and to formulate some interesting (in our view) open problems.

The talk covers the following topics:

1. Basic facts from coding theory
   - 1.1. Linear codes over finite fields and rings
   - 1.2. Equivalent codes, the automorphism group of a linear code
   - 1.3. The spectrum of a linear code. MacWilliams identities
   - 1.4. General bounds for linear codes
2. Finite geometries
   - 2.1. The projective geometries $\mathrm{PG}(V, K)$ and $\mathrm{PHG}(M_R)$
   - 2.2. Collineations in $\mathrm{PG}(V, K)$ and $\mathrm{PHG}(M_R)$
   - 2.3. Linear codes as sets of points in $\mathrm{PG}(k-1, q)$
3. Special sets of points in $\mathrm{PG}(N, q)$
   - 3.1. κ-arcs in $\mathrm{PG}(N, q)$
   - 3.2 (κ, ν)-arcs in $\mathrm{PG}(2, q)$
   - 3.3. (κ, ν)-caps in $\mathrm{PG}(N, q)$
   - 3.4. Multiple blocking sets and minihypers
4. Some special families of linear codes
   - 4.1. MDS-codes