# Nonlinear perfect codes and their impact on designs and geometry

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Let $\mathbb{F}_q^n$ be a vector space of dimension $n$ over $GF(q)$. A subset $C$ of $\mathbb{F}_q^n$ is said to be a $q$-ary perfect code if for some integer $r \geq 0$ every $x \in \mathbb{F}_q^n$ is within distance $r$ from exactly one codeword of $C$. It is known that the only parameters for nontrivial perfect codes are those of the two Golay codes and the $q$-ary 1-perfect codes, where $q$ is a prime or prime power. It is also known that the linear 1-perfect codes are unique up to equivalence. They are the well-known Hamming codes and exist for all $m \geq 2$. Nonlinear 1-perfect $q$-ary codes also exist for $q = 2, m \geq 4$; $q \geq 3, m \geq 3$; and for $q$ a prime power, $q \neq 4$, $m \geq 2$.

The current main results about perfect codes will be given, as well as their connexions with design theory and projective geometry.