

# 1 De belangrijkste regels van de substitutiecalculi

<table border="1"> <tr><td>1</td><td>List L1</td><td></td></tr> <tr><td>2</td><td>List L2</td><td></td></tr> <tr><td>3</td><td>Term <math>t</math></td><td></td></tr> <tr><td>SubL1 4</td><td>Var <math>x</math></td><td></td></tr> <tr><td>5</td><td><math>\vdash L2 = \text{subst}(t, x, L1)</math></td><td>prem</td></tr> <tr><td>6</td><td><math>\vdash \neg(x \in \text{vars}(t))</math></td><td>prem</td></tr> <tr><td>7</td><td><math>\vdash \neg(x \in \text{freevar}(L2))</math></td><td>SubL1 (5, 6)</td></tr> </table>	1	List L1		2	List L2		3	Term $t$		SubL1 4	Var $x$		5	$\vdash L2 = \text{subst}(t, x, L1)$	prem	6	$\vdash \neg(x \in \text{vars}(t))$	prem	7	$\vdash \neg(x \in \text{freevar}(L2))$	SubL1 (5, 6)	<table border="1"> <tr><td>1</td><td>List L1</td><td></td></tr> <tr><td>2</td><td>List L2</td><td></td></tr> <tr><td>3</td><td>Term <math>t</math></td><td></td></tr> <tr><td>4</td><td>Var <math>x</math></td><td></td></tr> <tr><td>SubL2 5</td><td>Var <math>y</math></td><td></td></tr> <tr><td>6</td><td><math>\vdash L2 = \text{subst}(t, x, L1)</math></td><td>prem</td></tr> <tr><td>7</td><td><math>\neg(x = y)</math></td><td>prem</td></tr> <tr><td>8</td><td><math>\vdash \neg(y \in \text{freevar}(L2))</math></td><td>prem</td></tr> <tr><td>9</td><td><math>\vdash \neg(y \in \text{freevar}(L1))</math></td><td>SubL2 (6, 8)</td></tr> </table>	1	List L1		2	List L2		3	Term $t$		4	Var $x$		SubL2 5	Var $y$		6	$\vdash L2 = \text{subst}(t, x, L1)$	prem	7	$\neg(x = y)$	prem	8	$\vdash \neg(y \in \text{freevar}(L2))$	prem	9	$\vdash \neg(y \in \text{freevar}(L1))$	SubL2 (6, 8)						
1	List L1																																																						
2	List L2																																																						
3	Term $t$																																																						
SubL1 4	Var $x$																																																						
5	$\vdash L2 = \text{subst}(t, x, L1)$	prem																																																					
6	$\vdash \neg(x \in \text{vars}(t))$	prem																																																					
7	$\vdash \neg(x \in \text{freevar}(L2))$	SubL1 (5, 6)																																																					
1	List L1																																																						
2	List L2																																																						
3	Term $t$																																																						
4	Var $x$																																																						
SubL2 5	Var $y$																																																						
6	$\vdash L2 = \text{subst}(t, x, L1)$	prem																																																					
7	$\neg(x = y)$	prem																																																					
8	$\vdash \neg(y \in \text{freevar}(L2))$	prem																																																					
9	$\vdash \neg(y \in \text{freevar}(L1))$	SubL2 (6, 8)																																																					
<table border="1"> <tr><td>1</td><td>List L1</td><td></td></tr> <tr><td>2</td><td>List L2</td><td></td></tr> <tr><td>3</td><td>Term <math>t</math></td><td></td></tr> <tr><td>4</td><td>Var <math>x</math></td><td></td></tr> <tr><td>SubL4 5</td><td>Var <math>y</math></td><td></td></tr> <tr><td>6</td><td><math>\vdash L2 = \text{subst}(t, x, L1)</math></td><td>prem</td></tr> <tr><td>7</td><td><math>\vdash \neg(y \in \text{freevar}(L1))</math></td><td>prem</td></tr> <tr><td>8</td><td><math>\vdash \neg(y \in \text{vars}(t))</math></td><td>prem</td></tr> <tr><td>9</td><td><math>\vdash \neg(y \in \text{freevar}(L2))</math></td><td>SubL4 (6, 7, 8)</td></tr> </table>	1	List L1		2	List L2		3	Term $t$		4	Var $x$		SubL4 5	Var $y$		6	$\vdash L2 = \text{subst}(t, x, L1)$	prem	7	$\vdash \neg(y \in \text{freevar}(L1))$	prem	8	$\vdash \neg(y \in \text{vars}(t))$	prem	9	$\vdash \neg(y \in \text{freevar}(L2))$	SubL4 (6, 7, 8)	<table border="1"> <tr><td>1</td><td>List <math>A</math></td><td></td></tr> <tr><td>2</td><td>List <math>B</math></td><td></td></tr> <tr><td>3</td><td>List <math>a</math></td><td></td></tr> <tr><td>4</td><td>List <math>b</math></td><td></td></tr> <tr><td>SubList 5</td><td>Term <math>t</math></td><td></td></tr> <tr><td>6</td><td>Var <math>x</math></td><td></td></tr> <tr><td>7</td><td><math>\vdash A = \text{subst}(t, x, a)</math></td><td>prem</td></tr> <tr><td>8</td><td><math>\vdash B = \text{subst}(t, x, b)</math></td><td>prem</td></tr> <tr><td>9</td><td><math>\vdash (A, B) = \text{subst}(t, x, (a, b))</math></td><td>SubList (7, 8)</td></tr> </table>	1	List $A$		2	List $B$		3	List $a$		4	List $b$		SubList 5	Term $t$		6	Var $x$		7	$\vdash A = \text{subst}(t, x, a)$	prem	8	$\vdash B = \text{subst}(t, x, b)$	prem	9	$\vdash (A, B) = \text{subst}(t, x, (a, b))$	SubList (7, 8)
1	List L1																																																						
2	List L2																																																						
3	Term $t$																																																						
4	Var $x$																																																						
SubL4 5	Var $y$																																																						
6	$\vdash L2 = \text{subst}(t, x, L1)$	prem																																																					
7	$\vdash \neg(y \in \text{freevar}(L1))$	prem																																																					
8	$\vdash \neg(y \in \text{vars}(t))$	prem																																																					
9	$\vdash \neg(y \in \text{freevar}(L2))$	SubL4 (6, 7, 8)																																																					
1	List $A$																																																						
2	List $B$																																																						
3	List $a$																																																						
4	List $b$																																																						
SubList 5	Term $t$																																																						
6	Var $x$																																																						
7	$\vdash A = \text{subst}(t, x, a)$	prem																																																					
8	$\vdash B = \text{subst}(t, x, b)$	prem																																																					
9	$\vdash (A, B) = \text{subst}(t, x, (a, b))$	SubList (7, 8)																																																					
<table border="1"> <tr><td>1</td><td>List <math>L</math></td><td></td></tr> <tr><td>2</td><td>Term <math>t</math></td><td></td></tr> <tr><td>SubList2 3</td><td>Var <math>x</math></td><td></td></tr> <tr><td>4</td><td><math>\vdash \neg(x \in \text{freevar}(L))</math></td><td>prem</td></tr> <tr><td>5</td><td><math>\vdash L = \text{subst}(t, x, L)</math></td><td>SubList2 (4)</td></tr> </table>	1	List $L$		2	Term $t$		SubList2 3	Var $x$		4	$\vdash \neg(x \in \text{freevar}(L))$	prem	5	$\vdash L = \text{subst}(t, x, L)$	SubList2 (4)	<table border="1"> <tr><td>1</td><td>Term <math>a</math></td><td></td></tr> <tr><td>2</td><td>Term <math>b</math></td><td></td></tr> <tr><td>3</td><td>Term <math>A</math></td><td></td></tr> <tr><td>4</td><td>Term <math>B</math></td><td></td></tr> <tr><td>SubEq 5</td><td>Var <math>x</math></td><td></td></tr> <tr><td>6</td><td>Term <math>t</math></td><td></td></tr> <tr><td>7</td><td><math>\vdash A = \text{subst}(t, x, a)</math></td><td>prem</td></tr> <tr><td>8</td><td><math>\vdash B = \text{subst}(t, x, b)</math></td><td>prem</td></tr> <tr><td>9</td><td><math>\vdash (A = B) = \text{subst}(t, x, a = b)</math></td><td>SubEq (7, 8)</td></tr> </table>	1	Term $a$		2	Term $b$		3	Term $A$		4	Term $B$		SubEq 5	Var $x$		6	Term $t$		7	$\vdash A = \text{subst}(t, x, a)$	prem	8	$\vdash B = \text{subst}(t, x, b)$	prem	9	$\vdash (A = B) = \text{subst}(t, x, a = b)$	SubEq (7, 8)												
1	List $L$																																																						
2	Term $t$																																																						
SubList2 3	Var $x$																																																						
4	$\vdash \neg(x \in \text{freevar}(L))$	prem																																																					
5	$\vdash L = \text{subst}(t, x, L)$	SubList2 (4)																																																					
1	Term $a$																																																						
2	Term $b$																																																						
3	Term $A$																																																						
4	Term $B$																																																						
SubEq 5	Var $x$																																																						
6	Term $t$																																																						
7	$\vdash A = \text{subst}(t, x, a)$	prem																																																					
8	$\vdash B = \text{subst}(t, x, b)$	prem																																																					
9	$\vdash (A = B) = \text{subst}(t, x, a = b)$	SubEq (7, 8)																																																					
<table border="1"> <tr><td>1</td><td>Pred <math>p</math></td><td></td></tr> <tr><td>2</td><td>Term <math>t</math></td><td></td></tr> <tr><td>3</td><td>Var <math>x</math></td><td></td></tr> <tr><td>SubPred 4</td><td>List args</td><td></td></tr> <tr><td>5</td><td>List Args</td><td></td></tr> <tr><td>6</td><td><math>\vdash \text{Args} = \text{subst}(t, x, \text{args})</math></td><td>prem</td></tr> <tr><td>7</td><td><math>\vdash p(\text{Args}) = \text{subst}(t, x, p(\text{args}))</math></td><td>SubPred (6)</td></tr> </table>	1	Pred $p$		2	Term $t$		3	Var $x$		SubPred 4	List args		5	List Args		6	$\vdash \text{Args} = \text{subst}(t, x, \text{args})$	prem	7	$\vdash p(\text{Args}) = \text{subst}(t, x, p(\text{args}))$	SubPred (6)	<table border="1"> <tr><td>1</td><td>Term <math>a</math></td><td></td></tr> <tr><td>2</td><td>Var <math>x</math></td><td></td></tr> <tr><td>SubTerm 3</td><td>Term <math>t</math></td><td></td></tr> <tr><td>4</td><td><math>\vdash \neg(x \in \text{freevar}(a))</math></td><td>prem</td></tr> <tr><td>5</td><td><math>\vdash a = \text{subst}(t, x, a)</math></td><td>SubTerm (4)</td></tr> </table>	1	Term $a$		2	Var $x$		SubTerm 3	Term $t$		4	$\vdash \neg(x \in \text{freevar}(a))$	prem	5	$\vdash a = \text{subst}(t, x, a)$	SubTerm (4)																		
1	Pred $p$																																																						
2	Term $t$																																																						
3	Var $x$																																																						
SubPred 4	List args																																																						
5	List Args																																																						
6	$\vdash \text{Args} = \text{subst}(t, x, \text{args})$	prem																																																					
7	$\vdash p(\text{Args}) = \text{subst}(t, x, p(\text{args}))$	SubPred (6)																																																					
1	Term $a$																																																						
2	Var $x$																																																						
SubTerm 3	Term $t$																																																						
4	$\vdash \neg(x \in \text{freevar}(a))$	prem																																																					
5	$\vdash a = \text{subst}(t, x, a)$	SubTerm (4)																																																					
<table border="1"> <tr><td>1</td><td>Term <math>a</math></td><td></td></tr> <tr><td>SubVar 2</td><td>Var <math>x</math></td><td></td></tr> <tr><td>3</td><td><math>\vdash a = \text{subst}(a, x, x)</math></td><td>SubVar</td></tr> </table>	1	Term $a$		SubVar 2	Var $x$		3	$\vdash a = \text{subst}(a, x, x)$	SubVar	<table border="1"> <tr><td>1</td><td>Const <math>c</math></td><td></td></tr> <tr><td>2</td><td>Term <math>t</math></td><td></td></tr> <tr><td>3</td><td>Var <math>x</math></td><td></td></tr> <tr><td>SubConst 4</td><td><math>\vdash c = \text{subst}(t, x, c)</math></td><td>SubConst</td></tr> </table>	1	Const $c$		2	Term $t$		3	Var $x$		SubConst 4	$\vdash c = \text{subst}(t, x, c)$	SubConst																																	
1	Term $a$																																																						
SubVar 2	Var $x$																																																						
3	$\vdash a = \text{subst}(a, x, x)$	SubVar																																																					
1	Const $c$																																																						
2	Term $t$																																																						
3	Var $x$																																																						
SubConst 4	$\vdash c = \text{subst}(t, x, c)$	SubConst																																																					
<table border="1"> <tr><td>1</td><td>Term <math>t</math></td><td></td></tr> <tr><td>2</td><td>Var <math>x</math></td><td></td></tr> <tr><td>SubG1 3</td><td>Formula <math>f</math></td><td></td></tr> <tr><td>4</td><td><math>() \vdash (\forall x : f) = \text{subst}(t, x, \forall x : f)</math></td><td>SubG1</td></tr> </table>	1	Term $t$		2	Var $x$		SubG1 3	Formula $f$		4	$() \vdash (\forall x : f) = \text{subst}(t, x, \forall x : f)$	SubG1	<table border="1"> <tr><td>1</td><td>Term <math>t</math></td><td></td></tr> <tr><td>2</td><td>Var <math>x</math></td><td></td></tr> <tr><td>3</td><td>Var <math>y</math></td><td></td></tr> <tr><td>SubG2 4</td><td>Formula <math>f</math></td><td></td></tr> <tr><td>5</td><td><math>\vdash f = \text{subst}(t, x, f)</math></td><td>prem</td></tr> <tr><td>6</td><td><math>\vdash (\forall y : f) = \text{subst}(t, x, \forall y : f)</math></td><td>SubG2 (5)</td></tr> </table>	1	Term $t$		2	Var $x$		3	Var $y$		SubG2 4	Formula $f$		5	$\vdash f = \text{subst}(t, x, f)$	prem	6	$\vdash (\forall y : f) = \text{subst}(t, x, \forall y : f)$	SubG2 (5)																								
1	Term $t$																																																						
2	Var $x$																																																						
SubG1 3	Formula $f$																																																						
4	$() \vdash (\forall x : f) = \text{subst}(t, x, \forall x : f)$	SubG1																																																					
1	Term $t$																																																						
2	Var $x$																																																						
3	Var $y$																																																						
SubG2 4	Formula $f$																																																						
5	$\vdash f = \text{subst}(t, x, f)$	prem																																																					
6	$\vdash (\forall y : f) = \text{subst}(t, x, \forall y : f)$	SubG2 (5)																																																					
<table border="1"> <tr><td>1</td><td>Formula <math>f</math></td><td></td></tr> <tr><td>2</td><td>Var <math>x</math></td><td></td></tr> <tr><td>FreeG 3</td><td><math>\vdash \neg(x \in \text{freevar}(\forall x : f))</math></td><td>FreeG</td></tr> </table>	1	Formula $f$		2	Var $x$		FreeG 3	$\vdash \neg(x \in \text{freevar}(\forall x : f))$	FreeG	<table border="1"> <tr><td>1</td><td>Formula <math>f</math></td><td></td></tr> <tr><td>2</td><td>Var <math>x</math></td><td></td></tr> <tr><td>3</td><td>Var <math>y</math></td><td></td></tr> <tr><td>FreeG2 4</td><td><math>\vdash \neg(x \in \text{freevar}(f))</math></td><td>prem</td></tr> <tr><td>5</td><td><math>\neg(x = y)</math></td><td>prem</td></tr> <tr><td>6</td><td><math>\vdash \neg(x \in \text{freevar}(\forall y : f))</math></td><td>FreeG2 (4, 5)</td></tr> </table>	1	Formula $f$		2	Var $x$		3	Var $y$		FreeG2 4	$\vdash \neg(x \in \text{freevar}(f))$	prem	5	$\neg(x = y)$	prem	6	$\vdash \neg(x \in \text{freevar}(\forall y : f))$	FreeG2 (4, 5)																											
1	Formula $f$																																																						
2	Var $x$																																																						
FreeG 3	$\vdash \neg(x \in \text{freevar}(\forall x : f))$	FreeG																																																					
1	Formula $f$																																																						
2	Var $x$																																																						
3	Var $y$																																																						
FreeG2 4	$\vdash \neg(x \in \text{freevar}(f))$	prem																																																					
5	$\neg(x = y)$	prem																																																					
6	$\vdash \neg(x \in \text{freevar}(\forall y : f))$	FreeG2 (4, 5)																																																					

FreeNil	1	Var $x$	
	2	$\vdash \neg(x \in \text{freevar}())$	FreeNil

	1	List $L$	
	2	List $M$	
FreeList	3	Var $x$	
	4	$\vdash \neg(x \in \text{freevar}(L))$	prem
	5	$\vdash \neg(x \in \text{freevar}(M))$	prem
	6	$\vdash \neg(x \in \text{freevar}(L, M))$	FreeList (4, 5)

VarVar	1	Var $x$	
	2	Var $y$	
	3	$\neg(x = y)$	prem
	4	$\vdash \neg(x \in \text{vars}(y))$	VarVar

	1	List L1	
	2	List L2	
	3	Term $t$	
FV1	4	Var $x$	
	5	$\vdash L2 = \text{subst}(t, x, L1)$	prem
	6	$\vdash \neg(x \in \text{vars}(t))$	prem
	7	$\vdash \neg(x \in \text{freevar}(L2))$	FV1 (5, 6)

## 2 Voorbeeldbewijzen

1		Var $x$	
2		Term $a$	
3		List L1	
4		List L2	
5		Pred $p$	
6		$\vdash \neg(x \in \text{freevar}(a))$	prem
7		$\vdash L2 = \text{subst}(a, x, L1)$	prem
8	L1	$\vdash (x = a) \Rightarrow p(x, a)$	prem
9	L1, $x = a$	$\vdash p(x, a)$	IE1 (8)
10		$\vdash a = \text{subst}(a, x, x)$	SubVar
11		$\vdash a = \text{subst}(a, x, a)$	SubTerm (6)
12		$\vdash (a, a) = \text{subst}(a, x, (x, a))$	SubList (10, 11)
13		$\vdash p(a, a) = \text{subst}(a, x, p(x, a))$	SubPred (12)
14		$\vdash (a = a) = \text{subst}(a, x, x = a)$	SubEq (10, 11)
15		$\vdash (L2, a = a) = \text{subst}(a, x, (L1, x = a))$	SubList (7, 14)
16	L2, $a = a$	$\vdash p(a, a)$	Stx $a x$ (9, 13, 15)
17		$\vdash a = a$	$E$
18	L2	$\vdash p(a, a)$	SN (16, 17)
19	L2	$\vdash \exists x : p(x, a)$	ExPxt (18, 13)

1		Const $a$	
2		Var $x$	
3		List $L$	
4		List L1	
5		Pred $p$	
6		$\vdash L1 = \text{subst}(a, x, L)$	prem
7	L	$\vdash x = a \Rightarrow p(x, a)$	prem
8	L, $x = a$	$\vdash p(x, a)$	IE1 (7)
9		$\vdash a = \text{subst}(a, x, x)$	SubVar
10		$\vdash a = \text{subst}(a, x, a)$	SubConst
11	()	$\vdash (a, a) = \text{subst}(a, x, (x, a))$	SubList (9, 10)
12	()	$\vdash p(a, a) = \text{subst}(a, x, p(x, a))$	SubPred (11)
13		$\vdash (a = a) = \text{subst}(a, x, (x = a))$	SubEq (9, 10)
14		$\vdash (L1, a = a) = \text{subst}(a, x, (L, x = a))$	SubList (6, 13)
15	L1, $a = a$	$\vdash p(a, a)$	Stx $a x$ (8, 14, 12)
16	()	$\vdash a = a$	$E$
17	L1	$\vdash p(a, a)$	SN (15, 16)
18	L1	$\vdash \exists x : p(x, a)$	ExPxt (17, 12)