1 De belangrijkste regels van de substitutiecalculus

SubL1	1		List L1	
	2		List L2	
	3		Term t	
	4		Var x	
	5	$\vdash$	$\mathrm{L2} = \mathrm{subst}\,(t, x, \mathrm{L1})$	prem
	6	$\vdash$	$\neg\,(x \in \mathrm{vars}\,(t))$	prem
	7	$\vdash$	$\neg\,(x \in \mathrm{freevar}\,(\mathrm{L2}))$	SubL1 (5, 6)

SubL2	1		List L1	
	2		List L2	
	3		Term t	
	4		Var x	
	5		Var y	
	6	$\vdash$	$\mathrm{L2} = \mathrm{subst}\,(t, x, \mathrm{L1})$	prem
	7		$\neg\,(x = y)$	prem
	8	$\vdash$	$\neg\,(y \in \mathrm{freevar}\,(\mathrm{L2}))$	prem
	9	$\vdash$	$\neg\,(y \in \mathrm{freevar}\,(\mathrm{L1}))$	SubL2 (6, 8)

SubL4	1		List L1	
	2		List L2	
	3		Term t	
	4		Var x	
	5		Var y	
	6	$\vdash$	$\mathrm{L2} = \mathrm{subst}\,(t, x, \mathrm{L1})$	prem
	7	$\vdash$	$\neg\,(y \in \mathrm{freevar}\,(\mathrm{L1}))$	prem
	8	$\vdash$	$\neg\,(y \in \mathrm{vars}\,(t))$	prem
	9	$\vdash$	$\neg\,(y \in \mathrm{freevar}\,(\mathrm{L2}))$	SubL4 (6, 7, 8)

SubList	1		List A	
	2		List B	
	3		List a	
	4		List b	
	5		Term t	
	6		Var x	
	7	$\vdash$	$A = \mathrm{subst}\,(t, x, a)$	prem
	8	$\vdash$	$B = \mathrm{subst}\,(t, x, b)$	prem
	9	$\vdash$	$(A, B) = \mathrm{subst}\,(t, x, (a, b))$	SubList (7, 8)

SubList2	1		List L	
	2		Term t	
	3		Var x	
	4	$\vdash$	$\neg\,(x \in \mathrm{freevar}\,(L))$	prem
	5	$\vdash$	$L = \mathrm{subst}\,(t, x, L)$	SubList2 (4)

SubEq	1		Term a	
	2		Term b	
	3		Term A	
	4		Term B	
	5		Var x	
	6		Term t	
	7	$\vdash$	$A = \mathrm{subst}\,(t, x, a)$	prem
	8	$\vdash$	$B = \mathrm{subst}\,(t, x, b)$	prem
	9	$\vdash$	$(A = B) = \mathrm{subst}\,(t, x, a = b)$	SubEq (7, 8)

SubPred	1		Pred p	
	2		Term t	
	3		Var x	
	4		List args	
	5		List Args	
	6	$\vdash$	$\mathrm{Args} = \mathrm{subst}\,(t, x, \mathrm{args})$	prem
	7	$\vdash$	$p\,(\mathrm{Args}) = \mathrm{subst}\,(t, x, p\,(\mathrm{args}))$	SubPred (6)

SubTerm	1		Term a	
	2		Var x	
	3		Term t	
	4	$\vdash$	$\neg\,(x \in \mathrm{freevar}\,(a))$	prem
	5	$\vdash$	$a = \mathrm{subst}\,(t, x, a)$	SubTerm (4)

SubVar	1		Term a	
	2		Var x	
	3	$\vdash$	$a = \mathrm{subst}\,(a, x, x)$	SubVar

SubConst	1		Const c	
	2		Term t	
	3		Var x	
	4	$\vdash$	$c = \mathrm{subst}\,(t, x, c)$	SubConst

SubG1	1		Term t	
	2		Var x	
	3		Formula f	
	4	() $\vdash$	$(\forall x : f) = \mathrm{subst}\,(t, x, \forall x : f)$	SubG1

SubG2	1		Term t	
	2		Var x	
	3		Var y	
	4		Formula f	
	5	$\vdash$	$f = \mathrm{subst}\,(t, x, f)$	prem
	6	$\vdash$	$(\forall y : f) = \mathrm{subst}\,(t, x, \forall y : f)$	SubG2 (5)

FreeG	1		Formula f	
	2		Var x	
	3	$\vdash$	$\neg\,(x \in \mathrm{freevar}\,(\forall x : f))$	FreeG

FreeG2	1		Formula f	
	2		Var x	
	3		Var y	
	4	$\vdash$	$\neg\,(x \in \mathrm{freevar}\,(f))$	prem
	5		$\neg\,(x = y)$	prem
	6	$\vdash$	$\neg\,(x \in \mathrm{freevar}\,(\forall y : f))$	FreeG2 (4, 5)

FreeNil	1			Var x	
	2		$\vdash$	$\neg(x \in \text{freevar}\,())$	FreeNil

FreeList	1			List L	
	2			List M	
	3			Var x	
	4		$\vdash$	$\neg(x \in \text{freevar}\,(L))$	prem
	5		$\vdash$	$\neg(x \in \text{freevar}\,(M))$	prem
	6		$\vdash$	$\neg(x \in \text{freevar}\,(L, M))$	FreeList $(4,5)$

VarVar	1			Var x	
	2			Var y	
	3			$\neg(x = y)$	prem
	4		$\vdash$	$\neg(x \in \text{vars}\,(y))$	VarVar

FV1	1			List L1	
	2			List L2	
	3			Term t	
	4			Var x	
	5		$\vdash$	$\text{L2} = \text{subst}\,(t, x, \text{L1})$	prem
	6		$\vdash$	$\neg(x \in \text{vars}\,(t))$	prem
	7		$\vdash$	$\neg(x \in \text{freevar}\,(\text{L2}))$	FV1 $(5,6)$

2 Voorbeeldbewijzen

1			Var x	
2			Term a	
3			List L1	
4			List L2	
5			Pred p	
6		$\vdash$	$\neg(x \in \text{freevar}\,(a))$	prem
7		$\vdash$	$\text{L2} = \text{subst}\,(a, x, \text{L1})$	prem
8	L1	$\vdash$	$(x = a) \Rightarrow p\,(x, a)$	prem
9	$\text{L1}, x = a$	$\vdash$	$p\,(x, a)$	IEl (8)
10		$\vdash$	$a = \text{subst}\,(a, x, x)$	SubVar
11		$\vdash$	$a = \text{subst}\,(a, x, a)$	SubTerm (6)
12		$\vdash$	$(a, a) = \text{subst}\,(a, x, (x, a))$	SubList $(10, 11)$
13		$\vdash$	$p\,(a, a) = \text{subst}\,(a, x, p\,(x, a))$	SubPred (12)
14		$\vdash$	$(a = a) = \text{subst}\,(a, x, x = a)$	SubEq $(10, 11)$
15		$\vdash$	$(\text{L2}, a = a) = \text{subst}\,(a, x, (\text{L1}, x = a))$	SubList $(7, 14)$
16	$\text{L2}, a = a$	$\vdash$	$p\,(a, a)$	Stx a x $(9, 13, 15)$
17		$\vdash$	$a = a$	E
18	L2	$\vdash$	$p\,(a, a)$	SN $(16, 17)$
19	L2	$\vdash$	$\exists x : p\,(x, a)$	ExPxt $(18, 13)$

1			Const a	
2			Var x	
3			List L	
4			List L1	
5			Pred p	
6		$\vdash$	$\text{L1} = \text{subst}\,(a, x, L)$	prem
7	L	$\vdash$	$x = a \Rightarrow p\,(x, a)$	prem
8	$L, x = a$	$\vdash$	$p\,(x, a)$	IEl (7)
9		$\vdash$	$a = \text{subst}\,(a, x, x)$	SubVar
10		$\vdash$	$a = \text{subst}\,(a, x, a)$	SubConst
11	$()$	$\vdash$	$(a, a) = \text{subst}\,(a, x, (x, a))$	SubList $(9, 10)$
12	$()$	$\vdash$	$p\,(a, a) = \text{subst}\,(a, x, p\,(x, a))$	SubPred (11)
13		$\vdash$	$(a = a) = \text{subst}\,(a, x, (x = a))$	SubEq $(9, 10)$
14		$\vdash$	$(\text{L1}, a = a) = \text{subst}\,(a, x, (L, x = a))$	SubList $(6, 13)$
15	$\text{L1}, a = a$	$\vdash$	$p\,(a, a)$	Stx a x $(8, 14, 12)$
16	$()$	$\vdash$	$a = a$	E
17	L1	$\vdash$	$p\,(a, a)$	SN $(15, 16)$
18	L1	$\vdash$	$\exists x : p\,(x, a)$	ExPxt $(17, 12)$