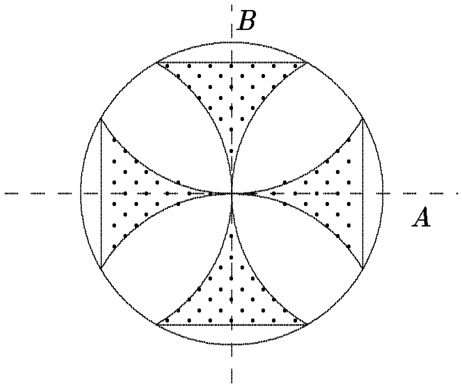
1.3 The final round.

1. Compute the area of the shaded figure, knowing that all arcs come from circles with radius R and knowing that A and B are mutual perpendicular axes of symmetry.



2. If a and b $(a > b)$ are prime numbers, consisting of at least two digits, then $a^4 - b^4$ is divisible by 240. Moreover, 240 is the greatest common divisor of all numbers $a^4 - b^4$ arising in this way.
   Prove this.

3. Determine the probability of the occurrence of the sequence "0123456789" in a code N consisting of 21 digits (N can start with a 0).

4. Assume $f : \mathbb{R}_0^+ \to \mathbb{R}_0^+$ is a strictly decreasing function.

   (a) Assume $a_1, a_2, \ldots, a_k, \ldots$ is a sequence of strictly positive real numbers such that

   $$\forall k \in \mathbb{N}_0 : \quad kf(a_k) \geq (k+1)f(a_{k+1}) \tag{1}$$

   Prove that

   i. $\forall k \in \mathbb{N}_0 : a_{k+1} > a_k$

   ii. $\lim\limits_{k\to+\infty} f(a_k) = 0$

   iii. $\lim\limits_{k\to+\infty} a_k = +\infty$

   (b) Prove: if $\lim\limits_{x\to+\infty} f(x) = 0$, then there exists a sequence $a_1, \ldots, a_k, \ldots$ in $\mathbb{R}_0^+$ satisfying equation (1).