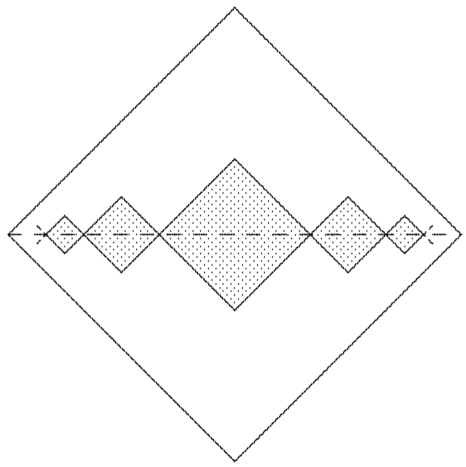
3 The final round.

Vraag 1 *In December, each of the 20 students of the same class sends 10 greeting cards to 10 different fellow class members (different from himself).*

a) Show that there are at least 2 students who send each other a greeting card.

b) Suppose a class consisting of n students, each of them sending m greeting cards to m different fellow class members (different from himself). How do n and m have to be in order to be sure that there will be at least 2 students sending each other a greeting card?

Vraag 2 *A jeweller covers the diagonal of a silver square (of side 1) by putting an "infinite number" of little golden squares on it in the following way:*



- *the sides of the golden squares are always parallel to those of the given (silver) square;*
- *the sides of the golden squares have a length which is exactly half or twice the length of the sides of the neighbouring golden square; moreover, the middle golden square is the largest;*
- *each of the midpoints of the golden squares lies on one diagonal of the silver square and having a distance to one of the vertices (of the silver square) equal to $1/2$, $1/4$, $1/8$, $1/16, \ldots$ of the length of that diagonal;*
- *neighbouring golden squares have one common vertex;*

a) Determine the side-length of the middle golden square.

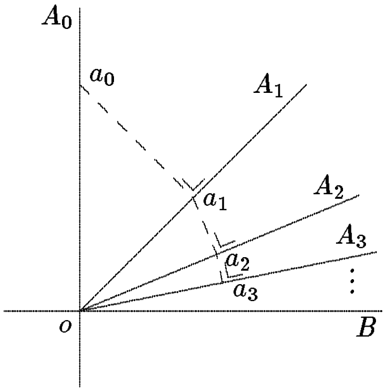
b) Determine the total area covered with gold.

Vraag 3 *Prove the following inequality:*

$$-1 < \left(\frac{a-b}{a+b}\right)^{1993} + \left(\frac{b-c}{b+c}\right)^{1993} + \left(\frac{c-a}{c+a}\right)^{1993} < 1$$

(a, b and c are strictly positive numbers)

Vraag 4 *Consider the following picture:*



A_0 and B are two lines perpendicular to each other and intersecting in o. A_n is the line from o such that the angle between A_{n+1} and B is half of the angle between A_n and B.
a_0 lies on A_0 and satisfies $|oa_0| = 1$ while a_{n+1} is the orthogonal projection of a_n on A_{n+1} ($n \in \mathbb{N}$).

Calculate

$$\lim_{n \to +\infty} |oa_n|.$$