3 Flanders Mathematics Olympiad 1993-1994: Final Round.

Question 1. Let a be the length of the hypotenuse, and b and c the lengths of the rectangular sides of a rectangular triangle. Determine all real numbers x that satisfy $a^x > b^x + c^x$.

Question 2. Give all solutions (a, b, c) to the equation

$$(a + \sqrt{c})^2 + (b + \sqrt{c})^2 = 60 + 20\sqrt{c}$$

where a, b and c are integers and $c \leq 94$.

Question 3. Two regular tetrahedrons A and B are contained in a cube of volume 1. The vertices of A coincide with the vertices at the ends of a diagonal in the bottom side of the cube and a diagonal of the top side of the cube. The other vertices of the cube are the vertices of B. Compute the volume of $A \cup B$.

Question 4. Consider the real functions f_n defined by $f_1(x) = x$ and $f_n(x) = \sqrt{f_{n-1}(x)} - \frac{1}{4}$, $(n \in I\!N, n \geq 2)$.

(a) Prove that $f_n(x) \leq f_{n-1}(x)$ for all x for which both functions are defined.

(b) For each n, determine the points x belonging to the domain of f_n such that $f_n(x) = x$.