3 Flanders Mathematics Olympiad 1994-1995: Final Round.

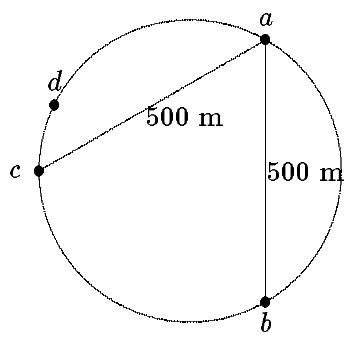
Question 1. Four married couples come together for a game of chess. They draw lots to form four groups of two players. Knowing that:

Beatrice plays against Eddy;
Alice plays against Clara's husband;
Freddy plays against Guy's wife;
Doris plays against Alice's husband;
Guy plays against Eddy's wife,

can one conclude with whom Hubert is married? Give a proof of your answer.

Question 2.
How many real numbers x belonging to $[1, 3]$ have the property that x^2 has the same decimal part as x? Give a proof of your answer.

Question 3.
The points a, b, c and d lie on the edge of a circular lake with radius R, as indicated on the figure. The distances between a and c and between a and b both equal 500 meter. The time a swimmer needs to get from d to c, from d to a and from a to b with the same constant velocity are in proportion to 1, 5 and 7. Determine R.



Question 4.

1. Prove that for all $n \in \mathbb{N}$ and for all $\alpha \in]0, \pi[$:

$$\sum_{k=0}^{n} \sin k\alpha = \frac{\sin \dfrac{(n+1)\alpha}{2} \sin \dfrac{n\alpha}{2}}{\sin \dfrac{\alpha}{2}}.$$

2. Let $G(n)$ be the average length of the diagonals of a regular n-gon ($n \geq 4$), inscribed in a circle of radius 1. Compute:

$$\lim_{n \to +\infty} G(n).$$