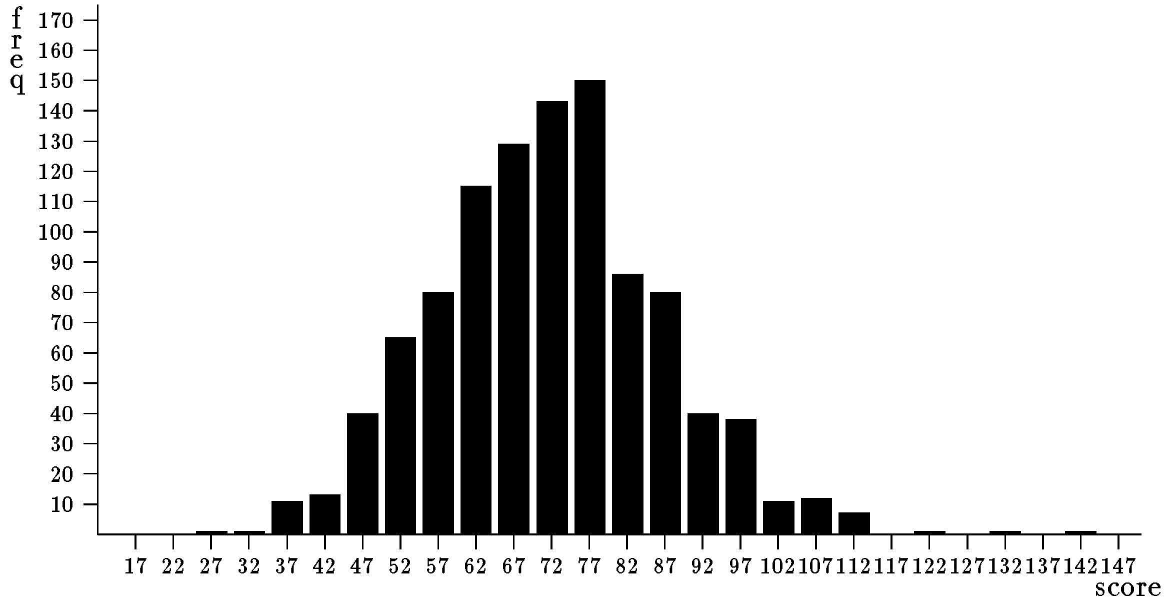
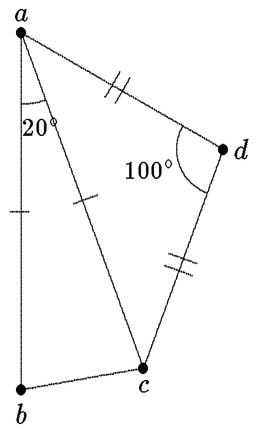
3.2 Histogram of the second round 1996.



Second Round 1995-1996 : all participants

4 Flanders Mathematics Olympiad 1995-1996: Final Round.

1. Let abc and dac be two isosceles triangles. The first has top angle $\hat{a}$ measuring $20°$; the second has top angle $\hat{d}$ measuring $100°$ (see figure).
   Prove that $|ab| = |bc| + |cd|$.



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2. Let P be the set of all primes strictly greater than 5. Determine the greatest common divisor of all integers $p^8 - 1$, where $p \in P$.

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3. Consider the points $1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}, \cdots$ on the real line. Let ℓ be the smallest length such that 5 closed intervals of length ℓ satisfy to cover all these points. Determine ℓ?

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4. The degree of a real polynomial $p(x) = a_n x^n + a_{n-1}x^{n-1} + \cdots + a_1 x + a_0$ is defined as n if $a_n \neq 0$. We write: $\deg(p(x)) = n$.

   (a) Let $\deg(p(x)) \geq 2$. Prove that

   $$\deg(p(x)) = 2 + \deg\left(p(x+1) + p(x-1) - 2p(x)\right).$$

   (b) Let $p(x)$ be a polynomial for which constants r and s ($\in \mathbb{R}$) exist such that

   $$p(x+1) + p(x-1) - rp(x) - s = 0, \quad \text{for all } x \in \mathbb{R}.$$

   Prove that constants a, b and c ($\in \mathbb{R}$) exist such that $p(x) = a + bx + cx^2$.

   (c) Prove that in the preceding case $s = 0$ implies $c = 0$.