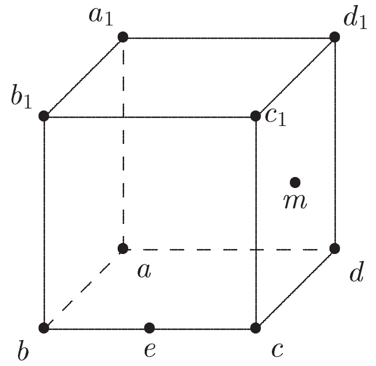
10 The 1998 Final Round

1. Proof that there exist integers a, b and c satisfying $0 < a < b \leq c < 2a$ and $a + b + c = 1998$, and such that $\gcd(a, b, c)$ is maximal. Determine such a triple. Is this triple unique?

2. Given a cube with edges of length 1; given the midpoint e of the edge $[bc]$ and the midpoint m of the face cdd_1c_1 (see figure). Determine the area of the intersection of the cube with the plane determined by a, e and m.



3. Determine all 3×3 magic squares.

   Definition: *A $n \times n$ magic square is a $n \times n$ matrix formed with all the integers 1, 2, ..., n^2 and such that the sums of the elements on each row, on each column and on both diagonals are equal.*

   An example of a 4×4 magic square is

1	15	14	4
12	6	7	9
8	10	11	5
13	3	2	16

4. The figure represents three sides of a billiard table. A white ball is lying at position p_1 and the red ball is lying at position p_2. The dimensions given are expressed in dm. The white ball is played towards the red bal in such a way that it first touches all three sides of the table (see figure). Determine the minimal distance that this white ball covers.