The Niels Henrik Abel Contest 1994

Problem 1

What is the least number of children in a family such that every child has at least one sister and one brother?

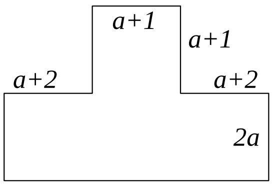
A) 2 B) 3 C) 4 D) 5 E) 6

Problem 2

The expression $\sqrt{8}+\sqrt{18}$ equals

A) 12 B) $\sqrt{54}$ C) $\sqrt{50}$ D) 7 E) $\sqrt{26}$

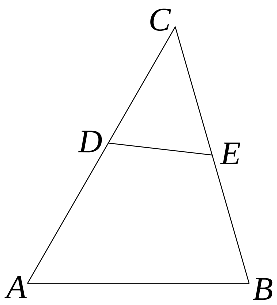
Problem 3



The figure shows a octagon where all angles are straight. With sides the lengths as shown on the figure, what is the area of the octagon?

A) $5a^2+2a+1$ B) $12a+12$ C) $5a^2+10a+1$
D) $7a^2+12a+1$ E) $7a^2+10a+3$

Problem 4



Let ABC be a triangle such that $\angle A = 55°$, $\angle B = 75°$, D lies AC, E lies on BC, and $CD = CE$. What is $\angle CED$?

A) 50° B) 55° C) 60° D) 65° E) 70°

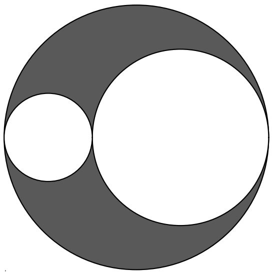
Problem 5

Let $x = -y$, where $y > 0$. Which of the following statements is wrong?

A) $x^2y > 0$ B) $x+y=0$ C) $xy<0$ D) $\frac{1}{x}-\frac{1}{y}=0$ E) $\frac{x}{y}+1=0$

Problem 6

We have three circles with diameters 1, 2, and 3, as shown in the figure. What proportion of the large circle does the collored region make up?



A) $\frac{1}{3}$ B) $\frac{1}{2}$ C) $\frac{2}{3}$ D) $\frac{\sqrt{5}}{3}$
E) None of these

Problem 7

If you cut an A4-sheet on the middle, you get an A5-sheet. The A5-sheet has the same shape as the A4-sheet. What is the quotient between the lengths of the long and the short side of the A4-sheet?

A) $\sqrt{2}$ B) $\frac{3}{2}$ C) $\frac{1+\sqrt{5}}{2}$ D) 2 E) None of these

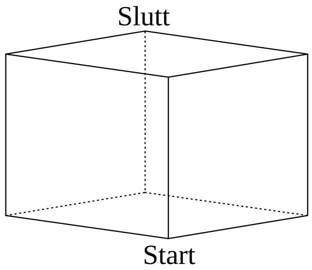
Problem 8

If $x^2 = x + 3$, then x^3 equals

A) $x + 6$ B) $x^2 + 3x + 3$ C) $4x + 3$ D) $4x^2 + 3$ E) $x^2 + 27$

Problem 9

A bug walks inside a cubic room with lengths of sides 1 metre. It starts in one of the lower corners and walks to the opposite corner by the roof. How far must it walk if it chooses the shortest path?



A) 2 B) $\sqrt{5}$ C) 3 D) $1+\sqrt{2}$
E) None of these

Problem 10

If n men can produce n samples of a product by working n hours a day for n days, how many samples will m men produce by working m hours a day form m days?

A) $\frac{n^3}{m^2}$ B) $\frac{m^3}{n^2}$ C) $\frac{n^2}{m^3}$ D) $\frac{m^2}{n^3}$ E) m

Problem 11

Let $y = \frac{1}{1+\frac{y}{x}}$ and $z = \frac{1}{1+\frac{z}{y}}$. If $z = 2$, then x equals

A) -4 B) 2 C) $\dfrac{12}{5}$ D) $\dfrac{16}{5}$ E) 3

Problem 12

Which of the numbers $\sqrt{2}$, $\sqrt[3]{3}$, $\sqrt[4]{4}$, and $\sqrt[5]{5}$ is the least?

A) $\sqrt{2}$ B) $\sqrt[3]{3}$ C) $\sqrt[4]{4}$ D) $\sqrt[5]{5}$ E) Two of them are the least

Problem 13

When multiplying $\left(3x^2 + \frac{2}{x}\right)^3$, we get a term that does not containt x. This term is

A) 6 B) 12 C) 18 D) 36 E) 54

Problem 14

How many different numbers may be written n/m where $n, m \in \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$? (Note that $2/4 = 1/2$.)

A) 34 B) 50 C) 51 D) 63 E) 90

Problem 15

A car drives a certain distance. The first sixth of the distance, the speed is 10 km/h; on the next two thirds, the speed is 20 km/h; and on the last sixth of the distance, the speed is 30 km/h. What is the average speed for the entire distance?

A) 16 km/h B) 18 km/h C) 20 km/h
D) 22 km/h E) 24 km/h

Problem 16

A circular disk is cut into as many pieces as possible using 7 straight lines. How many pieces are attainable?

A) 14 B) 29 C) 35 D) 49 E) 128

Problem 17

If a and b are natural numbers ($a, b \in \{1, 2, 3, \ldots\}$) and $a + b + ab = 54$, then $a + b$ equals

A) 12 B) 14 C) 15 D) 16 E) 17

Problem 18

Let $y = \sqrt{2 + \sqrt{2}}$, $z = \sqrt{2 - \sqrt{2}}$, and $x = y + z$. Then x equals

A) $y\sqrt{2}$ B) $2y$ C) 4 D) $2z$ E) $z\sqrt{2}$

Problem 19

What is the greatest number of lines that can be drawn in the plane and such that each of the lines intersects exactly 4 of the other lines?

A) 5 B) 8 C) 10 D) 16 E) Infinitely many

Problem 20

We define a function f on the integers by $f(x) = x/10$ if x is divisible by 10, and $f(x) = x + 1$ if x is not divisible by 10. Let $a_0 = 1993$ and $a_{n+1} = f(a_n)$. What is the smallest n such that $a_n = 1$?

A) 19 B) 25 C) 52 D) 1992 E) a_n never equals 1