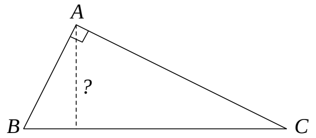
The Niels Henrik Abel Contest 1994–95

Problem 1

What is $\sqrt{1+\left(\frac{3}{4}\right)^2}$?

A) $\dfrac{5}{4}$ B) $\dfrac{11}{8}$ C) $\dfrac{7}{4}$ D) $\dfrac{3}{2}$ E) None of these

Problem 2



In the triangle ABC, the angle $\angle A$ is straight, $AB = 3$ and $AC = 4$. What then is the height from A to BC?

A) 2 B) $\dfrac{12}{5}$ C) $\sqrt{6}$ D) $\dfrac{5}{2}$ E) None of these

Problem 3

The number of digits in $4^8 \cdot 5^{17}$ is

A) 8 B) 10 C) 11 D) 17 E) 23

Problem 4

Which of these five numbers is the smallest?

A) $\dfrac{1994}{1995}$ B) $\dfrac{994}{995}$ C) $\left(\dfrac{1994}{1995}\right)^2$ D) $1-\dfrac{1}{994\cdot 995}$ E) $\sqrt{\dfrac{994}{995}}$

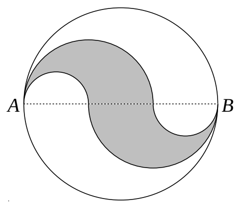
Problem 5

If $\left(1+\frac{1}{n}\right)\left(1-\frac{1}{m}\right)=1$, then m equals

A) $n-1$ B) $n+1$ C) $2n$ D) $\sqrt{n^2+1}$ E) None of these

Problem 6

The line segment AB is divided into three equal parts, and half-circles are drawn as in the fogure. What proportion of the large circle does the coloured region make up?



A) $\dfrac{1}{4}$ B) $\dfrac{4}{13}$ C) $\dfrac{1}{3}$ D) $\dfrac{4}{9}$ E) $\dfrac{1}{2}$

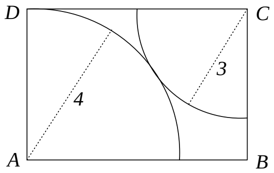
Problem 7

How many of the numbers 1, 2, 3, ..., 1994 cannot be divided by 5 or 7?

A) 1310 B) 1312 C) 1396 D) 1451 E) None of these

Problem 8

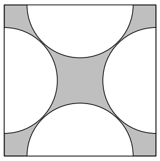
In the rectangle $ABCD$, cricles are drawn around A and C so that the circle aroung A passes through D and the two circles are tangent as on the figure. If the circle around A has radius 4 and the circle around C has radius 3, how long is AB?



A) 4 B) $\sqrt{26}$ C) $3\sqrt{3}$ D) $\sqrt{33}$ E) None of these

Problem 9

Along the edges of a square, four equally large half-circles are drawn so as to touch each other as on the figure. What proportion of the square does the coloured region make up?



A) $1-\dfrac{\pi}{4}$ B) $1-\dfrac{2}{\pi}$ C) $\dfrac{\pi}{8}$ D) $1-\dfrac{\pi}{6}$ E) $\dfrac{\pi}{6}$

Problem 10

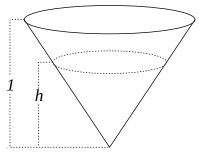
If $x+y=2z$, what is $\frac{x}{x-z}+\frac{y}{y-z}$?

A) 2 B) $\dfrac{(x-y)^2}{(x-z)(y-z)}$ C) $\dfrac{x}{y}+\dfrac{y}{x}$ D) $\dfrac{z}{x}+\dfrac{z}{y}$

E) None of these

Problem 11

We have a circular cone-shaped container of height 1. We fill it half-full of water. What is the height h of the surface?



A) $\dfrac{1}{2}$ B) $\dfrac{2}{3}$ C) $\dfrac{1}{\sqrt[3]{2}}$ D) $\dfrac{\sqrt{3}}{2}$

E) Avhengig av formen, dvs. av radien av toppen.

Problem 12

A man weights sheep, cats, and sacks of potatoes. He finds that himself and one sheep weighs as much as 4 sacks of potatoes. One sheep and two cats weighs as much as 3 sacks of potatoes. One sheep weighs as mush as 4 cats. How many cats does it take for them to weigh as much as the man?

A) 4 cats B) 6 cats C) 7 cats D) 8 cats E) 10 cats

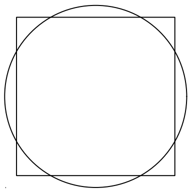
Problem 13

The expression $(a+b+c)^2+(a+b-c)^2+(a-b+c)^2+(b+c-a)^2$ equals

A) $4(ab+ac+bc)$ B) $2(a^2+b^2+c^2+ab+bc+ac)$
C) $4(a^2+b^2+c^2)+8(ab+bc+ac)$
D) $4(a^2+b^2+c^2)$ E) None of these

Problem 14

A square and a circle are placed such that the centre of the circle lies in the centre of the square. The region inside the square but outside the circle has the same area as the region inside the circle but outside the square. What is the quotient between the diameter of the circle and the sides of the square (ie. $\frac{d}{s}$)?



A) $\dfrac{\pi}{3}$ B) $\dfrac{2}{\sqrt{\pi}}$ C) $\dfrac{\pi^2}{8}$ D) $2\sqrt{3}$ E) $\sqrt{\pi}$

Problem 15

Let $f(x)=x(x+1)$. What does $\dfrac{f(x-1)f(x+1)}{f(x)}$ equal?

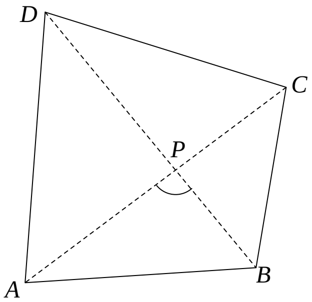
A) $f(x)$ B) $f(x)-1$ C) $f(2x)-3f(x)$ D) $9\cdot f\left(\dfrac{x-1}{3}\right)$
E) $\sqrt{f(x-2)f(x+2)}$

Problem 16

We have ten numbers. The average of these is 20. We remove one of the numbers, changing the average to 19. What number was removed?

A) 20 B) 21 C) 39 D) 40 E) None of these

Problem 17



Ve have a quadrilateral $ABCD$ where the angles $\angle A$, $\angle B$, $\angle C$ and $\angle D$ are known. Let the diagonal AC and BD intersect in the point P. What it the angle $\angle APB$?

A) $90°$ B) $\frac{\angle A + \angle B}{2}$ C) $\angle A + \angle B - \frac{\angle C + \angle D}{2}$
D) $\angle A + \frac{\angle C - \angle D}{2}$ E) Not uniquely determined.

Problem 18

Every day, the professor takes a 2 hour walk to a mountaintop and back down the same route. On his way to and from the mountain he walks 4 km/h, on the way up to the top he walks 3 km/h, whereas on the way down again he walks 6 km/h. How far is the walk?

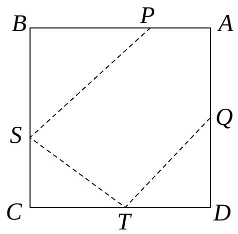
A) 6 B) 8 C) 9 D) 12
E) Not determined, has to know more.

Problem 19

A car salesman sells two types of cars: Alfa and Beta. An Alfa is sold at 40% profit, whereas a Beta is sold at 60% profit. The salesman has calculated that if he sels the same number of each car, his overall profit will be 48%. In reality, he sels 50% more Betas than Alfas. What then is his profit? (If a car is bought for 200,000 and is sold for 300,000, he has 50% profit as the difference is 50% of the price he paid.)

A) 45% B) $46\frac{2}{3}\%$ C) 50% D) 52% E) $53\frac{1}{3}\%$

Problem 20



We have a square $ABCD$ with sides of length 1. We place points P and Q on the edges AB and AD such that $AP = 1/3$ and $AQ = 1/2$. Let S and T be points on BC and CD such that $PS + ST + TQ$ is as short as possible. How long does that make $PS + ST + TQ$?

A) $\sqrt{5}$ B) $\frac{\sqrt{181}}{6}$ C) $3 - \frac{\sqrt{13}}{6}$ D) $\frac{19\sqrt{2}}{12}$ E) $\frac{10 + \sqrt{13}}{6}$