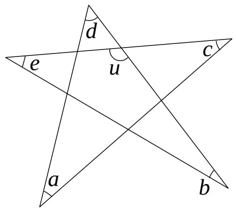
The Niels Henrik Abel Contest 1994–95
Second Round

Problem 1

A walnut-salesman knows that 20% of the nuts are empty. He has found a test for picking these out. This discards 20% of the nuts. However, when cracking the nuts that were discarded, one fourth of them were not empty after all. What proportion of the nuts that passed the test are then empty?

A) 4% B) $6\frac{1}{4}$% C) 8% D) 16% E) None of these

Problem 2



If the angles a, b, c, d, and e are known, what does the angle u equal?

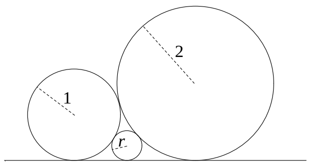
A) $108°$ B) $2a$ C) $3a$ D) $c + d$ E) $a + c + d$

Problem 3

How many natural numbers (ie. 1, 2, 3, ...) equal three times the sum of the digits?

A) None B) 1 C) 2 D) 3 E) 4 or more

Problem 4



To circles with radii 1 and 2 touch each other and a line as in the figure. In the region between the circles and the line, there is a circle with radius r which touches the two circles and the line. What is r?

A) $\frac{1}{3}$ B) $\frac{1}{\sqrt{5}}$ C) $\sqrt{3} - \sqrt{2}$ D) $6 - 4\sqrt{2}$ E) None of these

Problem 5

In how many can you colour the six sides of a cube in black or white? (Do note that the cube is unchanged when rotated?)

A) 7 B) 10 C) 20 D) 30 E) 36

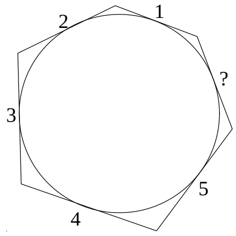
Problem 6

Let the operation $*$ be defined by $a * b = ab + a - b$. Which of the expressions below is wrong, or are they all correct?

A) $a * a = a^2$ B) $a * b = (-b) * (-a)$ C) $(a * b) * a = a^2 * b$
D) $(a * b) * c = (a * c) * b$ E) All four are correct

Problem 7

We have a hexagon such that all its edges touch a circle. If five of the edges have lenghths 1, 2, 3, 4, and 5 as on the figure, how long is the last edge?



A) 1 B) 3 C) $\dfrac{15}{8}$ D) $\sqrt{15}$
E) Not uniquely determined, more than one possibility.

Problem 8

Let f be a function defined on $\mathbf{N}_0 = \{0, 1, 2, 3, \ldots\}$ and with values in $\mathbf{N}_0$, such that for $n, m \in \mathbf{N}_0$ and $m \leq 9$, $f(10n + m) = f(n) + 11m$ and $f(0) = 0$. How many solutions are there to the equation $f(x) = 1995$?

A) Ingen B) 1 C) 2 D) 11 E) Infinitely many

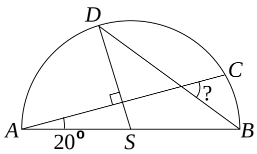
Problem 9

Three married couples arrange a party. They arrive at the party one at a time, the couples not necessarily arriving together. They all, upon arriving, shake the hand of everyone already there, except their own spouse. When everyone has arrived, someone asks all the others how many hands they shook upon arriving, and gets five different answers. How many hands did he himself shake upon arriving?

A) 0 B) 1 C) 2 D) 3 E) 4

Problem 10

We have a half-circle with endpoints A and B and centre S. The points C and D lie on the half-circle such that $\angle BAC = 20°$ and the lines AC and SD are normal to each other. What is the angle between the lines AC and BD?



A) $45°$ B) $55°$ C) $60°$ D) $67\frac{1}{2}°$ E) $72°$