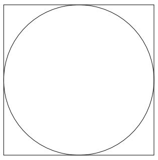
The Niels Henrik Abel Contest 1995–96

Problem 1

If 2 is the average of -3 and x, then x equals

A) -1 B) $-\frac{1}{2}$ C) 1 D) 5 E) 7

Problem 2



If the area of the square 36, then the area of the inscribed curcle is

A) 6π B) 9π C) 12π D) 36π E) 81π

Problem 3

Let $p(x) = x^3 + ax + 1$. If $p(1) = 1$, what is $p(2)$?

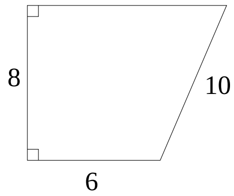
A) 0 B) 1 C) 2 D) 5 E) 7

Problem 4

We have ten boxes. In five of them there are pencils, and in four of them there are pens. Two of the boxes contain both pens and pencils. How many of the boxes contain neither pens nor pencils?

A) None B) 1 C) 2 D) 3 E) 4

Problem 5



What is the area of the quadrilateral?

A) 54 B) 56 C) 60 D) 64 E) 72

Problem 6

If first, a apples are bought at a price of b pounds per apple, and then b apples are bought at a price of a pounds per apple, what then is the average price per apple (in pounds)?

A) $\sqrt{ab}$ B) ab C) $\frac{2ab}{a+b}$ D) $\frac{a+b}{2}$ E) $\frac{a^2+b^2}{a+b}$

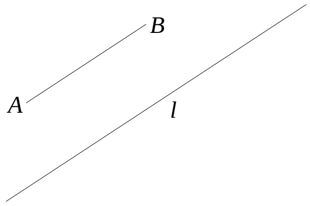
Problem 7

A palindrome is a natural number that remains unchanged when read backwards: eg. 838 and 24842. How many three-digit (0 not allowd as first digit) palindromes exist?

A) 10 B) 81 C) 90 D) 99 E) 100

Problem 8

Let l be a straight line and AB a line segment parallel to l. The line AB has length 10 and the distance from points A and B to l is 5. For how many points P on l does ABP becomes an isosceles triangle (ie. two sides have the same length)?



A) 1 B) 2 C) 3 D) 4 E) 5

Problem 9

A triangle has sides of lengths a, $a+b$, and $a+9b$ where $a, b > 0$. What then is a/b?

A) $\frac{20}{21}$ B) 4 C) 8 D) 20 E) None of these

Problem 10

The commander of the Royal Guard discovers that if he lines up all his soldiers in 2, 3, 4, 5 or 6 rows, there is always one soldier left, but that the soldiers may be lined up in 7 rows without any soldiers left. If n is the least possible number of soldiers so that this is true, what then is the sum of the digits of n?

A) 4 B) 7 C) 8 D) 10 E) None of these

Problem 11

A straight angled triangle has sides of lengths a, b, and c, where c is the longest side. If $c = 4$ and $a + b = \sqrt{19}$, what then is the area of the triangle?

A) $\frac{3}{4}$ B) 1 C) 2 D) $\frac{\sqrt{19}}{2}$ E) None of these

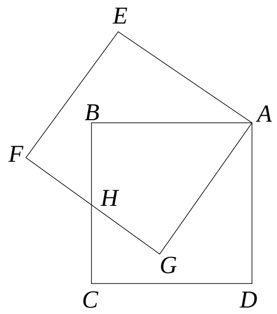
Problem 12

If $x + \frac{1}{x} = 3$, then $x^4 + \frac{1}{x^4}$ equals

A) 7 B) 24 C) 47 D) 72 E) 81

Problem 13

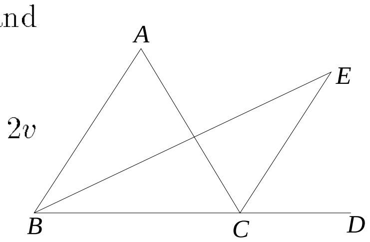
We have two squares $ABCD$ and $AEFG$ with sides of length 1. The point H is the midpoint of both BC and FG. What is the area of $AEFHCD$, ie. the entire figure?



A) $\sqrt{2}$ B) $\dfrac{\sqrt{5}}{2}$ C) $\dfrac{3}{2}$ D) $\dfrac{4}{3}$ E) $\sqrt{3}$

Problem 14

If, on the figure, $\angle ABE = \angle EBC$, $\angle ACE = \angle ECD$ and $v = \angle BEC$, then $\angle BAC$ equals



A) $90° - v$ B) $90° - \dfrac{v}{2}$ C) $2v$ D) $180° - 2v$
E) v

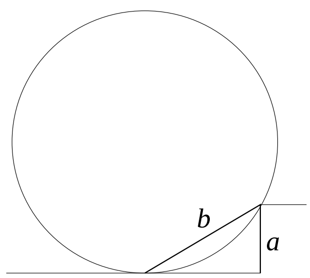
Problem 15

Ole is walking from the parking lot to the cabbin. If he walks 10 kilometres an hour, he will arrive at 18.00. If he walks 15 km/h, he will arrive at 16.00. How fast must he walk in order to arrive at 17.00?

A) 12km/h B) $12\frac{1}{3}$km/h C) $5\sqrt{5}$km/h D) $12\frac{1}{2}$km/h E) 13km/h

Problem 16

A wheel rests againt the edge of a pavement. The height of the pavement is a, and the distance from the point where the wheel rests on the ground to where the wheel meets the pavement is b. What is the radius of the wheel in terms of a and b?



A) $a + b$ B) $\sqrt{a^2 + b^2}$ C) $\sqrt{4b^2 - 7a^2}$ D) $\dfrac{b^2}{2a}$
E) None of these

Problem 17

The number of positive integers dividing 720 (including 1 and 720), is

A) 8 B) 15 C) 21 D) 30 E) 128

Problem 18

How many 7 digit numbers exist, such that all the digits are different and the digits are in an increasing order (zero not allowed as the first digit)? (Eg. 1345689 and 2456789.)

A) 9 B) 18 C) 45 D) 72 E) None of these

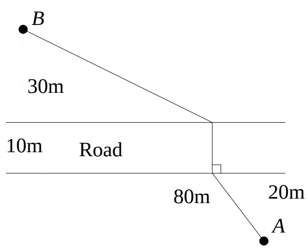
Problem 19

What is $\sqrt{3+2\sqrt{2}}+\sqrt{3-2\sqrt{2}}$?

A) $\sqrt{2}$ B) 2 C) $\sqrt{6}$ D) $2\sqrt{2}$ E) 3

Problem 20

The porcupine Napoleon is walking from his hive at point A to the tray of food at point B. The distance from A to B is 80 metres. In order to reach B, he has to cross a 10 metre wide road. The distance from A to the road is 20 metres, whereas B lies 30 metres from the road. Napoleon is afraid of cars and dares not cross the road straight over (not askew). How far must he walk if he chooses the shortes available path?



A) $10\sqrt{65}$ metres B) $10\sqrt{77}$ metres C) $10+10\sqrt{53}$ metres
D) $10+20\sqrt{2}+20\sqrt{3}$ metres E) None of these