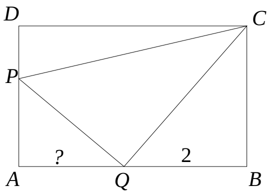
The Niels Henrik Abel Contest 1995–96
Second Round

Problem 1

The numbers from 1 to 1996 are written down — 12345678910111213.... How many zeros are written?

A) 489 B) 699 C) 796 D) 996 E) None of these

Problem 2



On the figure, the quadrilateral $ABCD$ is a rectangle, P lies on AD and Q on AB. The triangles PAQ, QBC and PCD all have the same areas, and $BQ = 2$. How long is AQ?

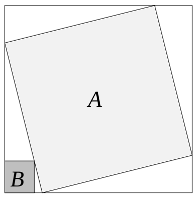
A) $\dfrac{7}{2}$ B) $\sqrt{7}$ C) $2\sqrt{3}$ D) $1+\sqrt{5}$ E) Not uniquely determined

Problem 3

What is the last digit of 17^{1996}?

A) 1 B) 3 C) 5 D) 7 E) 9

Problem 4



Given three squares as in the figure, where the largest square has area 1, and the area A is known. What is the area B of the smallest square?

A) $A/8$ B) $\dfrac{A^2}{2}$ C) $\dfrac{A^2}{4}$ D) $A(1-A)$ E) $\dfrac{(1-A)^2}{4}$

Problem 5

Let $f(x) = \frac{x}{1-x}$ and let a be a real number. If $x_0 = a$, $x_1 = f(x_0)$, $x_2 = f(x_1)$, ..., $x_{1996} = f(x_{1995})$, and $x_{1996} = 1$, what is a?

A) 0 B) $\dfrac{1}{1997}$ C) 1995 D) $\dfrac{1995}{1996}$ E) None of these

Problem 6

How many 11-digit bank-account numbers are there consisting of 1's and 2's only, and such that there are no two consecutive 1's?

A) 64 B) 233 C) 1024 D) 1279 E) 1365

Problem 7

In a class, the teacher discovers that every pupil has exactly three friends in the class, that two friends never have a common friend, and that every pair of two pupils who are not friends they have exactly one common friend. How many pupils are there in the class?

A) 6 B) 9 C) 10 D) 12 E) 17

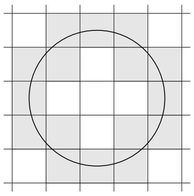
Problem 8

Let $f(x) = x - \frac{1}{x}$. How many different solutions are there to the equation $f(f(f(x))) = 1$?

A) 1 B) 2 C) 3 D) 6 E) 8

Problem 9

We draw a circle with radius 5 on a gridded paper where the grid consist of squares with sides of length 1. The centre of the circle is placed in the middle of one of the squares. All the squares through which the circle passes are coloured. How many squares are coloured? (The figure illustrates this for a smaller circle.)



A) 24 B) 32 C) 40 D) 64 E) None of these

Problem 10

Let $p(x) = x^6 + ax^5 + bx^4 + cx^3 + dx^2 + ex + f$ be a polynomial such that $p(1) = 1$, $p(2) = 2$, $p(3) = 3$, $p(4) = 4$, $p(5) = 5$, and $p(6) = 6$. What is $p(7)$?

A) 0 B) 7 C) 14 D) 49 E) 727