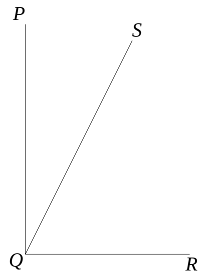
The Niels Henrik Abel Contest 1996–97
First round



On the figure, $\angle PQR$ is 90°, and $\angle RQS$ is 50° greater than $\angle PQS$. Then, $\angle PQS$ equals

A) 20° B) 40° C) 45° D) 50° E) 70°

Problem 1

If a and b are integers such that $b > a$, then the number of integers x such that $a < x < b$ is

A) $b - a - 1$ B) $b - a + 1$ C) $b - a$ D) $b - a - 2$
E) None of these

Problem 2

The integer closest to $\sqrt{\frac{60.1}{0.99} + 3.95}$ is

A) 3 B) 8 C) 9 D) 25 E) 64

Problem 3

The midpoint between $\frac{1}{8}$ and $\frac{7}{12}$ is

A) $\dfrac{1}{2}$ B) $\dfrac{1}{3}$ C) $\dfrac{2}{5}$ D) $\dfrac{11}{48}$ E) $\dfrac{17}{48}$

Problem 4

A car producer makes m cars per week. The production is increased by $n\%$. The number of cars produced per week then by

A) $m + n$ B) $m + \dfrac{n}{100}$ C) $\dfrac{mn}{100}$ D) $m\left(1 + \dfrac{n}{100}\right)$ E) $1 + \dfrac{mn}{100}$

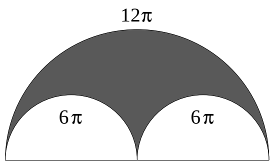
Problem 5

The smallest angle between the hands of a clock at 12:35 is

A) 150° B) 162.5° C) 165° D) 167.5° E) 180°

Problem 6

The lengths of the semicircles are given on the figure. The area of the coloured regin is



A) 18π B) 36π C) 54π D) 72π E) 144π

Problem 7

$1.236 \cdot 10^{15} - 5.23 \cdot 10^{14}$ equals

A) 7.13 B) 71.3 C) -3.994 D) $7.13 \cdot 10^{13}$ E) $7.13 \cdot 10^{14}$

Problem 8

Two lines are placed symmetrically around the line $y = 4$. One line is defined by $y = 3x + 1$. Which equation yields the other line?

A) $y = \frac{1}{3}x + 1$ B) $y = -\frac{1}{3}x + 7$ C) $y = -\frac{1}{3}x + 6$ D) $y = -3x + 6$
E) $y = -3x + 7$

Problem 9

The sum $\frac{2}{1 \cdot 3} + \frac{2}{3 \cdot 5} + \frac{2}{5 \cdot 7} + \cdots + \frac{2}{19 \cdot 21}$ equals

A) $\dfrac{19}{21}$ B) $\dfrac{20}{21}$ C) 1 D) $\dfrac{22}{21}$ E) $\dfrac{8}{7}$

Problem 10

Four persons A, B, C, and D buy a horse for 60 000 kr. A pays half as much as B, C, and D altogether. B pays one third of what A, C, and D pay altogether. C pays one fourth of what A, B, and D pay altogheter. How much does D pay?

A) 12 000 kr B) 13 000 kr C) 15 000 kr D) 17 000 kr E) 20 000 kr

Problem 11

If we add 329 to the three digit number $2x4$, we get $5y3$. If $5y3$ is divisible by three, the greatest possible value of x is

A) 1 B) 4 C) 7 D) 8 E) 9

Problem 12

In a coordinate system, 1 cm is the unit on both axes. The triangle PQR has vertices $P(0,3)$, $Q(4,0)$, and $R(k,5)$, where $0 < k < 4$. The area of the triangle is 8 cm^2. Then, the value of k is

A) 1 B) $\frac{8}{3}$ C) 2 D) $\frac{13}{4}$ E) $\frac{7}{2}$

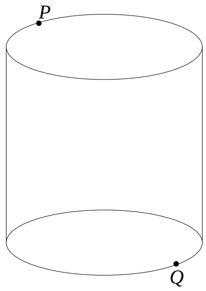
Problem 13

If x and y are positive integers such that $13x + 4y = 100$, then $x + y$ equals

A) 10 B) 12 C) 14 D) 16 E) 18

Problem 14

The figure shows a cylinder with perimeter 6 cm and height 4 cm. A point P on the top edge lies diametrically opposite a point Q on the lower edge. The shortest path (in centimetres) between P and Q along the surface (top, bottom, and side) of the cylinder is



A) 5 B) 7 C) $\sqrt{52}$ D) $4 + \frac{6}{\pi}$ E) $16 + \frac{36}{\pi^2}$

Problem 15

Assume that f is a real function such that $f(x) = f(-x)$ and $f(x+2) = 2f(x)$ for all x. Then, $f(5)$ equals

A) 0 B) 4 C) 32 D) Not uniquely determined
E) No such f exists

Problem 16

The coefficient of x^{99} (ie. the number in front of x^{99}) when expanding the expression

$$(x-1)(x-2)(x-3)\cdots(x-100)$$

is

A) -99 B) -100 C) -4851 D) -4950 E) -5050

Problem 17

In a regular octagon $ABCDEFGH$ (ie. the sides all have the same length, and the angles are all equal) the vertices are being denoted in the clockwise direction. If A has coordinates $(4,0)$, B has the coordinates $(0,4)$, and E has the coordinates (p,q), then $p-q$ equals

A) 4 B) -4 C) -6 D) $4\sqrt{2}$ E) $-4\sqrt{2}$

Problem 18

The number of positive integers n such that n^2+89 is a square is

A) None B) 1 C) 2 D) 3 E) 4 or more

Problem 19

Four balls with radius 10 are placed on a horizontal table such that their centres form a square with sidelengths 20. A fifth ball with radius 10 is placed on top of these so that it touches each of the four balls. How high above the table is the centre of the fifth ball?

A) 24 B) $10\sqrt{6}$ C) $10(1+\sqrt{2})$ D) $10(1+\sqrt{3})$ E) $10(4-\sqrt{2})$