The Niels Henrik Abel Contest 1997–98
First round

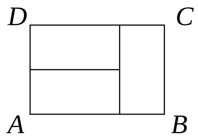
Problem 1

From a divingboard, you jump 1 metre up in the air, fall 5 metres downwards, before swimming 2 metres upwards to the surface. How high above the surface of the water is the divingboard?

A) 1 metre B) 2 metre C) 3 metre D) 4 metre E) 5 metre

Problem 2

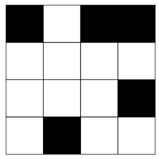
The three smaller rectangles on the figure are congruent (ie. same shape and same size). The length $BC = 1$. What is the length AB?



A) 2 B) $\frac{3}{2}$ C) $\sqrt{3}$ D) $\frac{1+\sqrt{3}}{2}$ E) $\frac{1+\sqrt{5}}{2}$

Problem 3

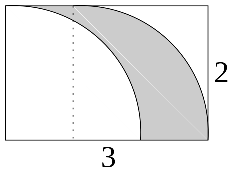
The figure on the right is not symmetric around the mid-point. What is the smallest number of square you have to fill for the figure to be symmetric around the mid-point?



A) 1 B) 2 C) 3 D) 4 E) 5

Problem 4

The figure on the right shows a rectangle with sides of lengths 2 and 3, and with two quarters of circles as shown on the figure. What is the areal of the coloured region?



A) 1 B) 2 C) 3 D) $\pi/2$ E) None of these

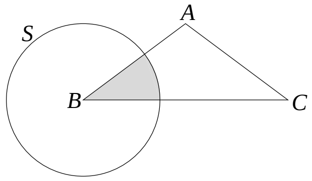
Problem 5

One particular year, the 31 days of January contain exactly four Thursdays and four Sundays. On which day of the week was 1st January that year?

A) Monday B) Tuesday C) Wednesday D) Thursday
E) None of these

Problem 6

Let ABC be an isosceles triangle with $AB = AC = 10$ and $\angle A = 100°$. Let S be a circle with centre B and radius 6. Then, the area of the region which is inside both the circle and the triangle is



A) 10 B) 4π C) $\frac{5\pi}{2}$ D) $\frac{8\pi}{3}$ E) $\frac{\pi^2}{4}$

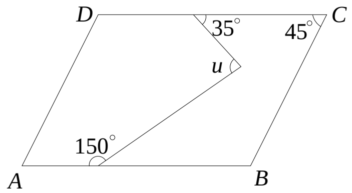
Problem 7

The entrance to a museum is 10 crowns for adults and 5 crowns for children. One Sunday, 50 persons visit the museum, paying a total of 350 crowns in entrance fees. How many of the visitors were adults?

A) 10 B) 20 C) 25 D) 30 E) 40

Problem 8

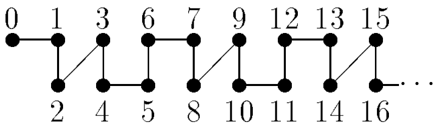
The sides AB and CD are parallel and the sides AD and BC are parallel, making $ABCD$ a parallelogram. If the angles are as given on the figure, what is the angle u?



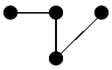
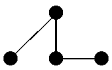
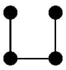
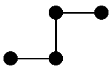
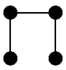
A) 50° B) 60° C) 65° D) 70° E) 75°

Problem 9

We have a repeating pattern as given below.



What does the piece from point 1997 to point 2000 look like?

A)  B)  C)  D)  E) 

Problem 10

The number of integers between 100 and 400 which contain the digit 2 is

A) 130 B) 138 C) 140 D) 154 E) 155

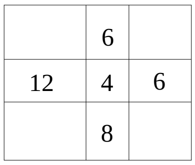
Problem 11

On a dancing course, there are 6 men and 6 women. These are to form 6 pairs, each with one man and one woman. In how many different ways can they make 6 such pairs?

A) 30 B) 120 C) 216 D) 720 E) 46656

Problem 12

A rectangle is divided into mine smaller rectangles as shown on the figure. If the perimeters of the five smaller rectangles are as stated on the figure, what is the perimeter of the entire rectangle?



A) 26 B) 28 C) 36 D) 40 E) 48

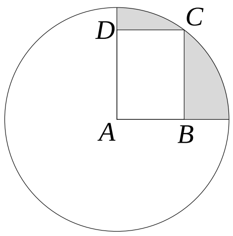
Problem 13

A plane has the velocity 800km/h on the first third of the flying time. The average velocity for the entire flight is 700km/h. What is the average velocity for the last 2/3 of the flying time?

A) 500km/h B) 600km/h C) 625km/h D) 650km/h
E) 750km/h

Problem 14

We have a rectangle $ANCD$ with $AB = 3$ and $AD = 4$. A circle with centre A passes through C. If a denotes the area of the coloured region, then



A) $a < 6$ B) $6 \leq a < 7$ C) $7 \leq a < 8$
D) $8 \leq a < 9$ E) $a \geq 9$

Problem 15

The minister of finance has decided that the country should have coins with values 33 and 60. What the the smallest amount that can be payed using these coins? (Both seller and buyer have sufficient change.)

A) 1 B) 2 C) 3 D) 6 E) 7

Problem 16

Let f be a function on the real number so that $f(2) = 3$ and $f(a+b) = f(a) + f(b) + ab$ for all a and b. Then, $f(11)$ equals

A) 55 B) 66 C) 110 D) 120 E) None of these

Problem 17

In a garden shop, both men and women are working. Exactly one third of the staff each bring one child. One day, each male employee plant 13 trees, each female employee plant 10 trees, and each child plants 6 trees. A total of 159 trees are planted. How many female employees are there?

A) 2 B) 5 C) 7 D) 12 E) 17

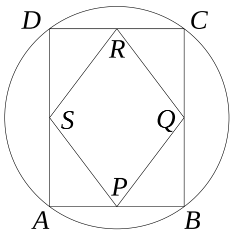
Problem 18

Let $x = \sqrt{1+\sqrt{2}}$. Then, $x + \frac{1}{x}$ equals

A) $1 + \dfrac{1}{x-1}$ B) $\dfrac{x^2}{2}$ C) $\sqrt{2} \cdot x$ D) $\sqrt{5}$ E) None of these

Problem 19

A circle with radius 3 encircles a rectangle $ABCD$. The points P, Q, R, and S are the midpoints of the sides of the rectangle. What is the perimeter of $PQRS$?



A) 5 B) 8 C) 9 D) 12
E) Not uniquely determined

Problem 20

The polynomial $p(x) = x^3 + 2x^2 - 5x + 1$ has three different roots: a, b, c. (Ie., $p(a) = p(b) = p(c) = 0$.) What is $a^3 + b^3 + c^3$?

A) 0 B) 8 C) 27 D) -41 E) $-13\sqrt[3]{2}$