The Niels Henrik Abel Contest 1998–99
First round

Problem 1

The largest integer such that seven times that number is less than 100, is

A) 12 B) 13 C) 14 D) 15 E) 16

Problem 2

The sum of the number of faces, edges and vertices of a cube is

A) 20 B) 22 C) 24 D) 26 E) 28

Problem 3

Which if the following numbers is the greatest?

A) $2^{10} + 2^{-10}$ B) $2^{10} - 2^{-10}$ C) $2^{10} + 10^{-3}$ D) $10^3 + 2^{-10}$
E) $10^3 + 10^{-3}$

Problem 4

If $a + b = 7$ and $2a - b = 17$, then $a - b$ equals

A) 1 B) 3 C) 5 D) 7 E) 9

Problem 5

Per and Kari were walking up the stairs of a tower. Per was constantly 52 steps ahead of Kari. When Per was half-way up the stairs he swaid to Kari: "When I've reached the top, you'll be three times as far up as you are now." The number of stairs in the tower is

A) 104 B) 156 C) 208 D) 256 E) 312

Problem 6

If the points $(2, -3)$, $(4, 3)$ and $(5, \frac{k}{2})$ lie on a line, k equals

A) 6 B) 8 C) 9 D) 12 E) 16

Problem 7

If we add x to the numerator and denominator of both the fractions $\frac{2}{3}$ and $\frac{20}{23}$, the resulting fractions are equal. Then, x is a factor of

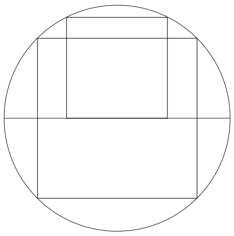
A) 12 B) 15 C) 56 D) 143 E) 170

Problem 8

If $a_0, a_1, a_2, \ldots$ is a sequence of numbers with $a_0 = 1$, $a_1 = 3$ and $a_n^2 - a_{n-1}a_{n+1} = (-1)^n$ for $n \geq 1$, then a_3 equals

A) 10 B) -17 C) 21 D) 33 E) $\dfrac{13}{27}$

Problem 9



Let K_1 be the area of the square inscribed in a semicircle and let K_2 be the are of a square inscribed in the whole circle. Then, the ratio $K_1 : K_2$ equals

A) $\dfrac{1}{2}$ B) $\dfrac{2}{5}$ C) $\dfrac{\sqrt{2}}{3}$ D) $\dfrac{\sqrt{3}}{4}$ E) $\dfrac{1}{\sqrt{5}}$

Problem 10

The smallest prime number dividing $3^{11} + 5^{12}$ is

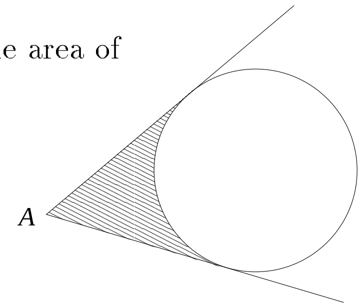
A) 2 B) 3 C) 5 D) $3^{11} + 5^{12}$ E) None of these

Problem 11

If twelve points in the plane are given such that no line passes through three of the points, then the number of lines passing through exactly two of the points is

A) 14 B) 54 C) 66 D) 120 E) 132

Problem 12



The circle in the figure has radius 1 and $\angle A = 60°$. The area of the coloured region is

A) $\sqrt{3} - \dfrac{\pi}{3}$ B) $\dfrac{2}{\sqrt{3}} - \dfrac{\pi}{6}$ C) $\dfrac{\pi}{3} - \dfrac{\sqrt{3}}{6}$

D) $\sqrt{3}\left(1 - \dfrac{\pi}{6}\right)$ E) $\dfrac{\pi}{3}\left(\sqrt{3} - 1\right)$

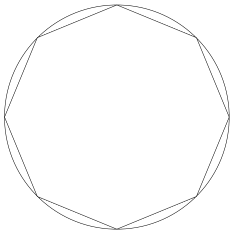
Problem 13

The expression $\frac{\sqrt{3}+1}{\sqrt{2}+1} - \frac{\sqrt{2}(2-\sqrt{2})}{\sqrt{3}-1}$ equals

A) 0 B) -1 C) $\sqrt{3}-\sqrt{2}$ D) $\dfrac{1}{\sqrt{6}+\sqrt{3}-\sqrt{2}-1}$

E) None of these

Problem 14



A regular 8-gon is inscribed in a circle of radius 4. The area of the 8-gon is

A) 48 B) $32\sqrt{2}$ C) $24+16\sqrt{2}$ D) $32+8\sqrt{2}$

E) None of these

Problem 15

Assume that m and n are integer such that $5m+6n=100$. Then, the greatest possible value of mn is

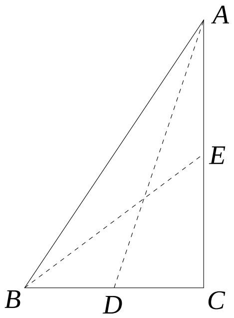
A) 60 B) 70 C) 80 D) 90 E) None of these

Problem 16

If a and b are integers such that $a<b<a^2$ and $ab=89$, then b equals

A) 1 B) -1 C) 89 D) -89 E) Such a b does not exist

Problem 17



In a straight-angled triangle ABC with $\angle C = 90°$, D is the mid-point of BC, and E is the mid-point of AC. If $AD=7$ and $BE=4$, then AB equals

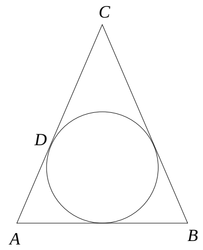
A) $5\sqrt{2}$ B) $5\sqrt{3}$ C) $2\sqrt{13}$ D) $2\sqrt{15}$ E) $2\sqrt{\frac{65}{3}}$

Problem 18

How many of the expressions x^3+y^4, x^4+y^3, x^3+y^3 and x^4-y^4 are positive for all real x and y such that $x>y$?

A) 0 B) 1 C) 2 D) 3 E) 4

Problem 19



In an isosceles triangle ABC, $AC = BC$. The inscribed circle of the triangle has radius 5 and touches AC in the point D with $CD = 12$. Then, AC equals

A) 20 B) $\dfrac{39}{2}$ C) $\dfrac{98}{5}$ D) $\dfrac{200}{11}$ E) $\dfrac{216}{13}$

Problem 20

Let A be a set of N different integers from the set $\{1, 2, 3, \ldots, 100\}$ such that the sum of two different elements of A is never divisible by 10. The greatest possible value of N is

A) 42 B) 45 C) 49 D) 50 E) 55