The Niels Henrik Abel Contest 1998–99
Second round
21st January 1999

Problem 1

Let $a \geq b$ be real numbers such that $a^2 + b^2 = 31$ and $ab = 3$. Then, $a - b$ equals

A) 5 B) $\frac{31}{6}$ C) $2\sqrt{6}$ D) $\frac{5}{6}\sqrt{31}$ E) $\frac{5}{6}\sqrt{37}$

Problem 2

Let p be the greatest prime factor of 9991. Then, the sum of the digits of p is

A) 4 B) 10 C) 13 D) 16 E) 28

Problem 3

A bowling contest consists of several series. Mary got 185 points in her previous series and thereby increased her average score per series from 176 to 177 points. How many points would Mary need in her next series to increase her average to 178?

A) 184 B) 185 C) 186 D) 187 E) 188

Problem 4

In a store, there are 7 cases containing 128 apples alltogether. Let N be the greatest number such that one can be certain to find a case with at least N apples. Then, the last digit of N is

A) 0 B) 2 C) 5 D) 7 E) 9

Problem 5

In a perpendicular triangle the perimetre is 60 and the altitude on the hypothenuse is 12. Then, the length of the hypothenuse is

A) 24 B) 25 C) 26 D) 27 E) 28

Problem 6

The number of 3-digit numbers not containing the digit 0 and such that one of the digits is the sum of the two others is

A) 96 B) 100 C) 104 D) 106 E) 108

Problem 7

Let f be a function such that for all integers x and y applies $f(x+y) = f(x) + f(y) + 6xy + 1$ and $f(x) = f(-x)$. Then, $f(3)$ equals

A) 26 B) 27 C) 52 D) 53 E) 54

Problem 8

On a party, there are 6 boys and a number of girls. Two of the girls know exactly four boys each and the remaining girls know exactly two boys each. None of the boys know more than three girls. (We assume that if A knows B, then B will also know A.) Then, the greatest possible number of girls on the party is

A) 6 B) 7 C) 8 D) 9 E) 10 eller mer

Problem 9

In the triangle ABC we have $AB = 5$ and $AC = 6$. The area of the triangle when the angle $\angle ACB$ is as large as possible is

A) 15 B) $5\sqrt{7}$ C) $\frac{7}{2}\sqrt{7}$ D) $3\sqrt{11}$ E) $\frac{5}{2}\sqrt{11}$

Problem 10

The number of pairs of integers (m, n) satisfying the equation

$$m^3 + 6m^2 + 5m = 27n^3 + 9n^2 + 9n + 1$$

is

A) 0 B) 1 C) 2 D) 3 E) Infinitely many