VIII Nordic Mathematical Contest

Thursday 17th March 1994

Time permitted: 4 hours

Each problem is worth 5 points.

Problem 1

Let O be a point in the interios of an equilateral triangle ABC with sides of length a. The lines AO, BO, and CO intersect the edges in the points A_1, B_1 og C_1. Prove that

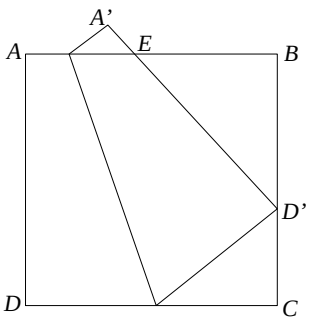
$$|OA_1| + |OB_1| + |OB_2| < a.$$

Problem 2

A finite set S of points in the plane with integer coordinates is a *two-neighbor-set* if for each point (p, q) in S, exactly two of the points $(p+1, q)$, $(p-1, q)$, $(p, q+1)$ and $(p, q-1)$ are in S. For which n does there exist a two-neighbor-set containing exactly n points?

Problem 3

A square sheet $ABCD$ is folded by placing the corner D at a point D' on BC (see figure). Then, AD is moved to $A'D'$ which intersects AB in E. Prove that the circumference of EBD' is half as long as the circumference of the square.



Problem 4

Find all positive integers $n < 200$ such that $n^2 + (n+1)^2$ is a perfect square.