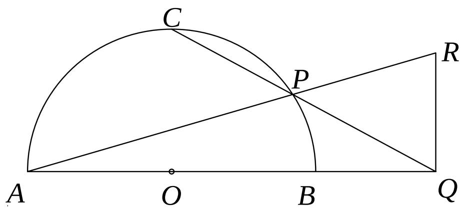
The Nineth Nordic Mathematical Contest

Wednesday 15th March 1994

Time permitted: 4 hours

Oppgave 1

Let AB be the diameter of a circle with centre O. Pick a point C on the circle so that OC is perpendicular to AB. Let P be any point on the circle between C and B, and let the lines CP and AB intersect in Q. Choose R on AP such that RQ and AB are perpendicular to eachother. Prove that $|BQ| = |QR|$.



Oppgave 2

Messages are coded by means of zeros and ones. Only seqences with a maximum of two consecutive zeros and ones are allowed. (Eg. the sequence 011001 is allowed, whereas 011101 is not.) How many sequences are permitted with exactly 12 digits?

Oppgave 3

Let $n \geq 2$ and let $x_1, x_2, \ldots, x_n$ be real numbers such that $x_1 + x_2 + \cdots + x_n \geq 0$ and $x_1^2 + x_2^2 + \cdots + x_n^2 = 1$. Let $M = \max\{x_1, x_2, \ldots, x_n\}$. Prove that

$$M \geq \frac{1}{\sqrt{n(n-1)}}. \tag{1}$$

Decide if equality is possible in (1).

Oppgave 4

Prove that there is an infinite number of non-congurent triangles T such that

1. the lengths of the sides of T are consecutive integers.
2. the area of T is an integer.