A Half 3-Moufang Quadrangle is Moufang

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Abstract

Recently, we showed in [1] that any 3-Moufang generalized quadrangle is automatically a Moufang quadrangle. In another recent paper, Katrin Tent [2] borrowed an argument of the second author to show that the half Moufang condition implies the Moufang condition for generalized quadrangles. In the present paper we show that this argument can be used to further weaken the hypotheses: we define the half 3-Moufang condition as a kind of greatest common divisor of the 3-Moufang condition and the half Moufang condition and show that it implies the Moufang condition.

1 Introduction, definitions and notation

A generalized quadrangle is a point-line incidence structure $\mathcal{S} = (\mathcal{P}, \mathcal{L}, \mathtt{I})$ containing no ordinary k-gons as subgeometries for $k < 4$, and such that every two members of $\mathcal{P} \cup \mathcal{L}$ are contained in some ordinary quadrangle (called an *apartment*). To avoid trivialities, We will also assume that every point (line) is incident with at least three lines (points). The *automorphism group* $\mathrm{Aut}(\mathcal{S})$ of the generalized quadrangle $\mathcal{S}$ is the group of permutations of $\mathcal{P}$ and of $\mathcal{L}$ that preserve the relation $\mathtt{I}$. Putting $G := \mathrm{Aut}(\mathcal{S})$, we denote the stabilizer of an element $x \in \mathcal{P} \cup \mathcal{L}$ as usual by G_x. For points and lines $x_1, \ldots, x_k$, $k \in \mathbb{N}$, we denote by $G^{[x_1,\ldots,x_k]}$ the stabilizer in G of all elements incident with one of $x_1, \ldots, x_k$.

Let $\mathcal{S} = (\mathcal{P}, \mathcal{L}, \mathtt{I})$ be a generalized quadrangle with automorphism group G. The *incidence graph* is the graph with vertex set $\mathcal{P} \cup \mathcal{L}$ and edges given by $\mathtt{I}$. A *root* is the set of elements of a path of length 4 in the incidence graph. Hence there are two kinds of roots: the ones containing 3 lines, and the ones containing three points. A root $\mathcal{R} = \{y_0, y_1, y_2, y_3, y_4\}$, with $y_0 \mathtt{I} y_1 \mathtt{I} \ldots \mathtt{I} y_4$, is called *Moufang* if the group

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$G^{[y_1,y_2,y_3]}$ acts transitively on the set of apartments containing $\mathcal{R}$. This is equivalent with saying that $G^{[y_1,y_2,y_3]}$ acts transitively on the set of elements incident with y_0 (respectively y_4) different from y_1 (respectively y_3).

A generalized quadrangle is called *Moufang* if all roots are Moufang. A generalized quadrangle is called *half Moufang* if every root of one fixed kind is Moufang. A generalized quadrangle is called 3-*Moufang* if for every path $\{y_0, y_1, y_2, y_3\}$ of length 3, with $y_0 \mathtt{I} y_1 \mathtt{I} \ldots \mathtt{I} y_3$, the group $G^{[y_1,y_2]}$ acts transitively on the set of apartments containing $y_0, \ldots, y_3$.

Generalized quadrangles were introduced by Jacques Tits [4], and he also introduced the Moufang condition in the appendix of [5]. The half Moufang condition was introduced by Thas, Payne and the second author in [3], where the equivalence with the Moufang condition in the finite case was shown. Later on, Richard Weiss and the second author defined the k-Moufang condition for generalized polygons [7] and Thas, Payne and the second author proved in [6] that 3-Moufang is equivalent to Moufang for finite generalized quadrangles.

Recently, Katrin Tent [2] proved in general that the half Moufang condition is equivalent to the Moufang condition, and she used an argument of the second author in order to repair a flaw in an earlier version of her proof. Then, the authors proved in [1] that, again in general, the 3-Moufang condition is equivalent to the Moufang condition (for generalized quadrangles). In the same paper, they showed how the argument of the second author can be adopted to give a very short proof of Tent's result mentioned above. In the present paper, we will apply a variant of that very same argument to further weaken the Moufang condition. We will introduce a condition that is weaker than both the half Moufang condition and the 3-Moufang condition, and therefore we will call it the *half* 3-*Moufang condition*.

First notice that all paths of length 3 in a generalized quadrangles are of the same type. So we cannot restrict on the set of 3-paths in order to weaken the 3-Moufang condition. Instead, we will restrict on the transitivity property of the 3-Moufang condition. More exactly, the *half* 3-*Moufang condition* assures that for one type of root $\mathcal{R} = \{y_0, y_1, y_2, y_3, y_4\}$, with $y_0 \mathtt{I} y_1 \mathtt{I} \ldots \mathtt{I} y_4$, the group $G_{y_0}^{[y_2,y_3]}$ acts transitively on the apartments containing $y_0, \ldots, y_4$.

Our Main result reads now:

Main Result. *Every half* 3-*Moufang generalized quadrangle is a Moufang generalized quadrangle, and vice versa.*

Since the converse is rather trivial to prove, we will only prove that the half 3-Moufang condition implies the Moufang condition.

2 Proof of the Main Result

From now on, we assume that $\mathcal{S} = (\mathcal{P}, \mathcal{L}, \mathtt{I})$ is a generalized quadrangle with automorphism group G, satisfying the half 3-Moufang condition. More exactly, we assume that for all roots $\mathcal{R} = \{y_0, y_1, y_2, y_3, y_4\}$, with $y_0 \mathtt{I} y_1 \mathtt{I} \ldots \mathtt{I} y_4$, and with $y_0, y_2, y_4 \in \mathcal{P}$, the group $G_{y_0}^{[y_2,y_3]}$ acts transitively on the apartments containing $y_0, \ldots, y_4$.

We fix some apartment $\Sigma := \{x_0, x_1, \ldots, x_7\}$, with $x_0 \mathtt{I} x_1 \mathtt{I} \cdots \mathtt{I} x_7 \mathtt{I} x_0$, where $x_0 \in \mathcal{P}$.

2.1 Reduction lemmas

In this subsection, we reduce our main result to proving that certain actions must be independent of certain configurations.

We remark first that all paths $\{x, y, y', z\}$, with $x\mathtt{I}y\mathtt{I}y'\mathtt{I}z$ and $x \in \mathcal{L}$, of length 3 form a single orbit under G, and hence all groups $G_z^{[x,y]}$ are conjugate. The proof is left to the reader, but the arguments follow the lines of the proof of Lemma 2 below.

For two points $x, y \in \mathcal{P}$, the *trace* $\{x, y\}^\perp$ is defined to be the set of all points collinear to both x and y, and the *span* $\{x, y\}^{\perp\perp}$ is the set of all points collinear to all points of $\{x, y\}^\perp$.

Lemma 1 *If in $\mathcal{S}$ the span $\{x, y\}^{\perp\perp}$ of some non-collinear points x, y contains at least 3 elements, then $\mathcal{S}$ is half Moufang.*

Proof We may assume without loss of generality that $\{x, y\} = \{x_2, x_6\}$. Let $x_6' \in \{x_2, x_6\}^{\perp\perp}$, with $x_2 \neq x_6' \neq x_6$. Let x_5' denote the line incident with x_4 and x_6'. As $G_{x_6}^{[x_3,x_4]}$ fixes $\{x_2, x_6\}$, the span $(x_2, x_6)^{\perp\perp}$ has to be stabilized as a set, but as the lines through x_4 are fixed as well, this implies that the span is fixed pointwise, and hence in particular x_6' is fixed. Consider an arbitrary element $g \in G_{x_6,x_6'}^{[x_3,x_4]}$ and choose an element $h \in G^{[x_5',x_6']}$ mapping x_2 to x_6 (h exists by the half 3-Moufang assumption on the root $\{x_0, x_0x_6', x_6', x_5', x_4\}$). The commutator $[g, h]$ clearly belongs to $G_{x_6}^{[x_4,x_5',x_6']}$ and hence is trivial. Consequently $g = g^h \in G^{[x_3,x_4,x_5]}$. ∎

Let Ω denote the set of lines incident with x_0, but distinct from x_1.

Lemma 2 *Let x be any point incident with x_1, $x \neq x_0$, and let y be any point not on x_1 collinear with x. If the action of $G_y^{[x_1,x]}$ on Ω independent of x and y, then $\mathcal{S}$ is half Moufang.*

Proof It suffices to show that there is an element $g \in G^{[x_0,x_1,x_2]}$ mapping x_6 to an arbitrary point z on x_7. Let's start with an arbitrary nontrivial collineation $\alpha \in G_{x_4}^{[x_1,x_2]}$. Then there is a unique point z' on x_5^α collinear with z. Hence, if we denote x_2' the unique point on x_1 collinear with z', then the collineation $\beta \in G_{z'}^{[x_1,x_2']}$ mapping x_7^α to x_7 maps x_6^α to z. The composition $\alpha\beta$ fixes all points on x_1 and — by assumption — it also fixes all lines incident with x_0, since the action of α on Ω must be the inverse of the action of β on Ω. Moreover, $\alpha\beta$ maps x_6 to z. Also, the action of $\alpha\beta$ on the lines through x_2 is the same as the action of β (since α fixes every line through x_2). Interchanging now the roles of x_0 and x_2, we see that the collineation $\gamma \in G_z^{[x_0,x_1]}$ mapping x_3^β back to x_3 has an action on the lines through x_2 inverse to that of $\alpha\beta$, which implies that $\alpha\beta\gamma \in G^{[x_0,x_1,x_2]}$. Since $\alpha\beta\gamma$ maps x_6 to z, the assertion follows. ∎

In order to prove that every half 3-Moufang quadrangle is Moufang, we thus need to show that our choice for x and y does not influence the action of $G_y^{[x_1,x]}$ on Ω. First we will deal with groups of the form $G_y^{[x_1,x_2]}$ where we vary y. For this, we will need Lemma 1. Then we vary x on x_1 and use the argument that repaired Tent's proof alluded to in the introduction.

2.2 The action of $G_y^{[x_1,x_2]}$ on Ω is independent of the choice of y.

Here we prove:

Lemma 3 *Let y be any point not on x_1 collinear with x_2. Then the action of $G_y^{[x_1,x_2]}$ on Ω is independent of y.*

Proof First we note that we may assume y to be incident with x_3. Indeed, this follows immediately from the fact that the group $G_{x_6}^{[x_0,x_1]}$ acts transitively on the lines through x_2 distinct from x_1, and so any group $G_z^{[x_1,x_2]}$ can thus be seen as a conjugate of $G_y^{[x_1,x_2]}$ with $y\mathtt{I}x_3$ under a collineation which does not permute the lines through x_0.

Now, if the action of $G_y^{[x_1,x_2]}$ on Ω were not independent of the choice of y, with y incident with x_3, then we may assume that the action of the group $G_1 := G_{x_4}^{[x_1,x_2]}$ on Ω differs from the action of the group $G_2 := G_{x_0}^{[x_2,x_3]}$ on Ω.

Suppose first that there is an element $\alpha \in G_1 \cup G_2$ such that α commutes with every element of $G_1 \cup G_2$. We claim that G_1 and G_2 must have the same action on Ω. Indeed, if not, then there is a collineation $g_1 \in G_1$ such that its action on Ω is not induced by any element of G_2. Let $g_2 \in G_2$ be such that g_2 maps $x_7^{g_1}$ back to x_7. Then g_1g_2 gives rise to a collineation $g_1g_2 \in G_{x_6,x_6^\alpha}^{[x_2]}$ (because $(x_6^\alpha)^{g_1g_2} = (x_6^{g_1g_2})^\alpha = x_6^\alpha$). If x_6^α were not contained in $\{x_2,x_6\}^{\perp\perp}$, then g_1g_2 would fix at least three points on some line through x_2, implying that g_1g_2 would fix an ideal subquadrangle (i.e., a subquadrangle with the property that every line in $\mathcal{S}$ through a point of the subquadrangle belongs to the subquadrangle). This contradicts the fact that g_1g_2 does not fix all lines through x_0. Hence we have a span of at least three elements, and Lemma 1 concludes the proof in this case (since the current lemma holds for half Moufang quadrangles).

Hence we may assume that the centralizer of $G_1 \cup G_2$ in $G_1 \cup G_2$ is trivial. Note that G_1 and G_2 normalize each other. We claim that G_1 cannot have a commutative action on Ω. Indeed, if G_1 were commutative, then also G_2 would be commutative. If only the identity in G_1 has the same action on Ω as some element of G_2, then G_1 and G_2 centralize each other. But two groups acting regularly on a set Ω and centralizing each other must have the same action on Ω, a contradiction. Hence there is some nonidentity element c_1 in G_1 having the same action on Ω as an element c_2 in G_2. Both c_1, c_2 centralize $G_1 \cup G_2$, again a contradiction with our assumptions. The claim is proved.

Next we claim that only the identity in G_1 has the same action on Ω as some element of G_2. Indeed, suppose by way of contradiction that there is a $\beta_1 \in G_1$ inducing the same action on Ω as some $\beta_2 \in G_2$. Since β_1 cannot lie in the center of $G_1 \cup G_2$, we may suppose there is a $g \in G_1 \cup G_2$ such that the commutator $[\beta_1, g] \neq \mathrm{id}$ (and this is equivalent to the assumption that the action on Ω of that commutator be nontrivial). Suppose $g \in G_2$ — the case $g \in G_1$ is similar, if one interchanges the roles of x_0 and x_4 (noting that the action of G_1 and G_2 on Ω is permutation equivalent with their action on the set of lines though x_4 distinct from x_3). Consider an arbitrary $h \in G_{x_6}^{[x_0,x_1]}$, then g^h induces the same action on Ω as g. It is clear that all the commutators $[\beta_1, g]$, $[\beta_2, g]$ and $[\beta_2, g^h]$ induce the same action on Ω, and each of them fixes all points of x_3. This easily implies $\alpha := [\beta_1, g] = [\beta_2, g] = [\beta_2, g^h]$.

Since the latter fixes the line x_3^h pointwise and since h is arbitrary, we see that $\alpha \neq \mathrm{id}$ fixes all points collinear with x_2. So, the image of x_6 under α must lie in the span of x_2 and x_6 which forces the generalized quadrangle to be half Moufang by Lemma 1. But then the lemma holds, and so the claim is proved.

Hence the regular actions of G_1 and G_2 on Ω normalize each other and share only the identity. This easily implies that they centralize each other, and the actions on Ω are *opposite*, i.e., Ω can be identified with G_2, the group G_1 is anti-isomorphic to G_2 and its action on Ω can be identified with left multiplication in G_2, and the action of G_2 on Ω is right multiplication in G_2.

We conclude that, for arbitrary $y \mathtt{I} x_3$, $y \neq x_2$, the action of $G_y^{[x_0,x_1]}$ on Ω is either the same as the action of G_2 on Ω, or it is opposite.

Suppose both really occur. So for some $y \mathtt{I} x_3$, $y \neq x_2$, the action of $G_1 = G_y^{[x_0,x_1]}$ on Ω is opposite the action of G_2 on Ω, and for some $z \mathtt{I} x_3$, $z \neq x_2$, the action of $G_3 := G_z^{[x_0,x_1]}$ on Ω is the same as the action of G_2 on Ω. Since $G_1 \cap G_2$ is trivial, no nontrivial element of G_2 can fix all points on x_1. This implies that $G_2 \cap G_3$ is trivial. But G_2 and G_3 normalize each other, hence they centralize each other. This means that the action of G_3 — which is the same as the action of G_2 — on Ω centralizes the action of G_2 on Ω, hence this action is commutative! This contradicts a previous claim.

We conclude that all actions of $G_y^{[x_0,x_1]}$ on Ω, $y \mathtt{I} x_3$, $y \neq x_2$, are either the same as the action of G_2 on Ω, or opposite. In particular, the action is independent of y. ∎

2.3 The action of $G_y^{[x_1,x]}$ on Ω is independent of x

Here we prove:

Lemma 4 *If x_2' is an arbitrary point on x_1, $x_2' \neq x_0$, and x_4' is the unique point on x_5 collinear with x_2', then the action of $G_{x_4}^{[x_1,x_2]}$ on Ω coincides with the action of $G_{x_4'}^{[x_1,x_2']}$ on Ω.*

Proof Let U_2 be the permutation group acting on Ω given by the action of $G_{x_4}^{[x_1,x_2]}$. Let x_2' be an arbitrary point on x_1, $x_2' \neq x_0$, and let x_4' be the unique point on x_5 collinear with x_2'. Then we define U_2' as the permutation group on Ω given by the action of $G_{x_4'}^{[x_1,x_2']}$. If we show that $U_2 \equiv U_2'$, then Lemma 2 implies that $\mathcal{S}$ is half Moufang, and hence Moufang by [2]. We assume that $U_2 \neq U_2'$ and seek a contradiction. First we claim that the two different groups U_2 and U_2' cannot have a nontrivial element in common. Indeed, let U_6 be the permutation group acting on Ω the way $G_{x_4}^{[x_6,x_7]}$ does. Then clearly U_6 is conjugate to U_2 since for every $g \in G_{x_4}^{[x_1,x_2]}$ there exists an $h \in G_{x_4}^{[x_6,x_7]}$ (determined by $x_7^g = x_1^{h^{-1}}$) such that $G_{x_4}^{[x_1,x_2]} = G_{x_4}^{[x_6,x_7]gh}$. Similarly U_6 is conjugate to U_2'. Suppose now that there are $g \in G_{x_4}^{[x_1,x_2]}$ and $g' \in G_{x_4'}^{[x_1,x_2']}$ inducing the same action on Ω. For $\alpha \in G_{x_4}^{[x_1,x_2]} \cup G_{x_4'}^{[x_1,x_2']} \cup G_{x_4}^{[x_6,x_7]}$ denote by r_α the corresponding element of $U_2 \cup U_2' \cup U_6$. With this notation $r_g = r_{g'}$. If h is as above, then $U_2 = U_6^{r_g r_h} = U_6^{r_{g'} r_h} = U_2'$, a contradiction. The claim follows.

We now show that the groups U_2 and U_2' also normalize each other. If $u_2 \in U_2$ and $u_2' \in U_2'$, then let $g \in G_{x_4}^{[x_1,x_2]}$ be such that $r_g = u_2$ and similarly let $g' \in G_{x_4'}^{[x_1,x_2']}$

be such that $r_{g'} = u_2'$. Then $g^{g'}$ belongs to $G^{[x_1,x_2]}_{x_4^{g'}}$, which has the same action on Ω as $G^{[x_1,x_2]}_{x_4}$ by Lemma 3. Hence $u_2^{u_2'} \in U_2$ and U_2' normalizes U_2. Similarly, U_2 normalizes U_2'.

Since $U_2 \cap U_2'$ is trivial, it now follows that U_2 and U_2' centralize each other. So, as before, their respective actions on Ω are mutually opposite one another.

We need some more notation now. Note that we may assume that there are at least 4 lines through x_0 otherwise the discussion about U_2 and U_2' having a different action on Ω is absurd. We can thus define two different paths of length 4 both not contained in the apartment Σ by the incidences $x_0 \mathtt{I} \widetilde{x_1} \mathtt{I} \widetilde{x_2} \mathtt{I} \widetilde{x_3} \mathtt{I} x_4$ and $x_4 \mathtt{I} \overline{x_5} \mathtt{I} \overline{x_6} \mathtt{I} \overline{x_7} \mathtt{I} x_0$. Furthermore we denote by $\overline{x_4}'$ the unique point on $\overline{x_5}$ collinear with x_2', and by $\widetilde{x_2}'$ the unique point on $\widetilde{x_1}$ collinear with x_4'. Finally the unique point on x_7 collinear with $\overline{x_4}'$ is denoted p and the the unique point on $\widetilde{x_1}$ collinear with $\overline{x_4}'$ is called q.

Put $\widetilde{\Omega}$ equal to the set of lines through x_0 distinct from $\widetilde{x_1}$. The groups $G^{[\widetilde{x_1},\widetilde{x_2}']}_{x_4'}$ and $G^{[\widetilde{x_1},\widetilde{x_2}]}_{x_4}$ induce opposite actions on $\widetilde{\Omega}$ since there exists a collineation $g \in G^{[x_5,x_6]}_{x_0}$ conjugating $G^{[x_1,x_2']}_{x_4}$ into $G^{[\widetilde{x_1},\widetilde{x_2}']}_{x_4'}$ and $G^{[x_1,x_2]}_{x_4}$ into $G^{[\widetilde{x_1},\widetilde{x_2}]}_{x_4}$.

Also, the group $G^{[\widetilde{x_1},q]}_{\overline{x_4}'}$ induces either the same action on $\widetilde{\Omega}$ as $G^{[\widetilde{x_1},\widetilde{x_2}]}_{x_4}$ or the opposite action, in which case this action coincides with the action of $G^{[\widetilde{x_1},\widetilde{x_2}']}_{x_4'}$ on $\widetilde{\Omega}$. Define $g \in G^{[\overline{x_6},\overline{x_7}]}_{\overline{x_4}'}$ such that $x_1^g = \widetilde{x_1}$, and define $h \in G^{[\overline{x_6},\overline{x_7}]}_{x_4}$ such that $x_1^h = \widetilde{x_1}$. We know from Lemma 3 that g and h have the same action on the set of lines through x_0. But g conjugates $G^{[x_1,x_2']}_{\overline{x_4}'}$ (which induces the same action on Ω as $G^{[x_1,x_2']}_{x_2'}$ by Lemma 3) into $G^{[\widetilde{x_1},q]}_{\overline{x_4}'}$ and h conjugates $G^{[x_1,x_2]}_{x_4}$ into $G^{[\widetilde{x_1},\widetilde{x_2}]}_{x_4}$. Hence the actions of $G^{[\widetilde{x_1},q]}_{\overline{x_4}'}$ and $G^{[\widetilde{x_1},\widetilde{x_2}]}_{x_4}$ on $\widetilde{\Omega}$ are opposite.

We have shown that the actions of $G^{[\widetilde{x_1},q]}_{\overline{x_4}'}$ and $G^{[\widetilde{x_1},\widetilde{x_2}']}_{x_4'}$ on $\widetilde{\Omega}$ coincide. Now let $g' \in G^{[x_1,x_2']}_{x_4'}$ map $\widetilde{x_1}$ to x_7 and let $h' \in G^{[x_1,x_2']}_{\overline{x_4}'}$ map $\widetilde{x_1}$ to x_7. Then, since g' and h' induce the same action on Ω by Lemma 3, and since

$$\left(G^{[\widetilde{x_1},\widetilde{x_2}']}_{x_4'}\right)^{g'} = G^{[x_6,x_7]}_{x_4'} \quad \text{and} \quad \left(G^{[\widetilde{x_1},q]}_{\overline{x_4}'}\right)^{h'} = G^{[p,x_7]}_{\overline{x_4}'},$$

the groups $G^{[x_6,x_7]}_{x_4'}$ and $G^{[p,x_7]}_{\overline{x_4}'}$ induce the same action on the set of lines through x_0. Now let $g'' \in G^{[\overline{x_6},\overline{x_7}]}_{x_4}$ map x_7 to x_1 and let $h'' \in G^{[\overline{x_6},\overline{x_7}]}_{\overline{x_4}'}$ map x_7 to x_1. Again both g'' and h'' induce the same action on the set of lines through x_0. Moreover, we have $(G^{[x_6,x_7]}_{x_4})^{g''} = G^{[x_1,x_2]}_{x_4}$ and $(G^{[p,x_7]}_{\overline{x_4}'})^{h''} = G^{[x_1,x_2']}_{\overline{x_4}'}$. We conclude that the action of U_2 on Ω coincides with that of U_2'.

The lemma is proved. ∎

This now also completes the proof of the Main Result.

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