Large maximal partial spreads of the Hermitian variety $\mathrm{H}(5, q^2)$

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We consider the Hermitian variety in 5-dimensions, denoted by $\mathrm{H}(5, q^2)$. This is an example of a finite classical polar space of rank 3. The Hermitian variety $\mathrm{H}(5, q^2)$ contains points, lines and planes of the ambient projective space $\mathrm{PG}(5, q^2)$. The planes contained in $\mathrm{H}(5, q^2)$ are called *generators*.

A *spread* of $\mathrm{H}(5, q^2)$ is a set S of generators such that every point of $\mathrm{H}(5, q^2)$ is contained in exactly one element of S. A spread contains exactly q^5+1 elements. A *partial spread* of $\mathrm{H}(5, q^2)$ is a set S of generators such that every point of $\mathrm{H}(5, q^2)$ is contained in at most one element of S. A partial spread is called *maximal* if no generator of $\mathrm{H}(5, q^2) \setminus S$ can be added to S. Since spreads of $\mathrm{H}(5, q^2)$ does not exist by a result of J. A. Thas ([3]), the natural question is how many elements the largest maximal partial spread contains.

Using counting arguments and the particular geometrical structure, we can improve the known upper bounds ([3] and [2]) and show that a maximal partial spread contains at most q^3+1 elements. Furthermore, from [1], we know that any spread of the symplectic polar space $\mathrm{W}(5, q)$ embedded in $\mathrm{H}(5, q^2)$, constitutes a maximal partial spread of $\mathrm{H}(5, q^2)$, of size q^3+1. Hence, the new upper bound is sharp.

References

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